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On the Hopf algebra structure of the mod 3 cohomology of the exceptional Lie group of type E_6

By

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1. Introduction

Kono-Mimura [9] and Toda [13] determine $H^*(E_6; \mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$ the mod 3 Steenrod algebra. Kono [7] determines $H^*(AdE_6; \mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$ and simultaneously gives a new method to determine $H^*(E_6; \mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$. It is, however, very difficult to determine $\bar{\mu}^*(x_{17})$ where x_{17} is the generator of degree 17 in $H^*(E_6; \mathbf{Z}_3)$. (For a Hopf algebra, the reduced coproduct map is denoted by $\bar{\mu}^*$ in this paper.) In fact, a direct method is not found until now.

The main purpose of this paper is to give a direct method of the determination of $\bar{\mu}^*(x_{17})$. At the same time, $H^*(\tilde{E}_6; \mathbf{Z}_3)$ is determined as a Hopf algebra over $\mathscr{A}_3$ where $\tilde{E}_6$ is the 3-connective cover over E_6. For our purpose, in §2, we shall define five maps (which we call in this paper the adjoint maps) and state their properties. It should be emphasized that among these maps, what bring us improvement essentially are

$$\hat{\mathrm{ad}} : AdE_6 \wedge E_6 \to E_6$$

and

$$\check{\mathrm{ad}} : AdE_6 \wedge \tilde{E}_6 \to \tilde{E}_6.$$

In §3, we shall determine $H^*(AdE_6; \mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$ by a slightly different way from that of Kono [7]. In §4, we shall determine $H^*(E_6; \mathbf{Z}_3)$ as a Hopf algebra over $\mathscr{A}_3$. In §5, the last section, we shall prove the following.

Theorem 1.1. *As a Hopf algebra over $\mathscr{A}_3$, $H^*(\tilde{E}_6; \mathbf{Z}_3)$ is given as follows. As an algebra,*

$$H^*(\tilde{E}_6; \mathbf{Z}_3) = \mathbf{Z}_3[\tilde{y}_{18}] \otimes \Lambda(\tilde{x}_9, \tilde{x}_{11}, \tilde{x}_{15}, \tilde{x}_{17}, \tilde{y}_{19}, \tilde{y}_{23})$$

where $\deg \tilde{x}_k = \deg \tilde{y}_k = k$. The coproducts are given by $\bar{\mu}^*(\tilde{y}_{18}) = \tilde{x}_9 \otimes \tilde{x}_9$ and $\bar{\mu}^*(z) = 0$ $(z = \tilde{x}_k, \tilde{y}_{19}, \tilde{y}_{23})$. The cohomology operations are given by

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z	$\tilde{x}_9$	$\tilde{x}_{11}$	$\tilde{x}_{15}$	$\tilde{x}_{17}$	$\tilde{y}_{18}$	$\tilde{y}_{19}$	$\tilde{y}_{23}$
βz	0	0	0	0	$\tilde{y}_{19}$	0	0
$\wp^1 z$	0	$\tilde{x}_{15}$	0	0	0	$\tilde{y}_{23}$	0

and by $\wp^9 \tilde{y}_{18} = \tilde{y}_{18}^3$, $\wp^{3^j} \tilde{y}_{18} = 0$ $(j \neq 2)$ *and* $\wp^{3^j} z = 0$ $(z = \tilde{x}_k, \tilde{y}_{19}, \tilde{y}_{23}; j \geq 1)$.

Throughout this paper, we use $\mathbf{Z}_3$ as the coefficient ring of homology and cohomology unless mentioned.

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2. Adjoint maps

Let $p : \tilde{E}_6 \to E_6$ be the covering projection and $w : E_6 \to AdE_6$ the natural projection. We define five maps

$$\bar{\mathrm{ad}} : AdE_6 \rtimes AdE_6 \to AdE_6,$$

$$\mathrm{ad} : E_6 \rtimes E_6 \to E_6,$$

$$\hat{\mathrm{ad}} : AdE_6 \rtimes E_6 \to E_6,$$

$$\tilde{\mathrm{ad}} : \tilde{E}_6 \rtimes \tilde{E}_6 \to \tilde{E}_6,$$

$$\check{\mathrm{ad}} : AdE_6 \rtimes \tilde{E}_6 \to \tilde{E}_6$$

as follows where $A \rtimes B$ denotes the half smash product $(A \times B)/(A \times \{e\})$ for based spaces A and B.

We define $\bar{\mathrm{ad}}$ by $\bar{\mathrm{ad}}(x, y) = xyx^{-1}$. Similarly we define ad by $\mathrm{ad}(x, y) = xyx^{-1}$. For the definition of ad, we can define $\hat{\mathrm{ad}}$ as the map which makes the following diagram commute:

$$\begin{tikzcd} E_6 \rtimes E_6 \arrow[r, "\mathrm{ad}"] \arrow[d, "w \rtimes 1"'] & E_6 \\ AdE_6 \rtimes E_6. \arrow[ur, "\hat{\mathrm{ad}}"'] & \end{tikzcd}$$

We define $\tilde{\mathrm{ad}}$ as the map which makes the following diagram commute up to homotopy:

$$\begin{tikzcd} \tilde{E}_6 \rtimes \tilde{E}_6 \arrow[r, "\tilde{\mathrm{ad}}"] \arrow[d, "p \rtimes p"'] & \tilde{E}_6 \arrow[d, "p"] \\ E_6 \rtimes E_6 \arrow[r, "\mathrm{ad}"'] & E_6. \end{tikzcd}$$

Note that $\tilde{\mathrm{ad}}$ certainly exists and is unique up to homotopy since $\tilde{E}_6 \wedge \tilde{E}_6$ is 3-connected. Similarly, we define $\check{\mathrm{ad}}$ as the map which makes the following diagram commute up to homotopy:

$$\begin{tikzcd}
AdE_6 \wedge \tilde{E}_6 \arrow[r, "\check{\mathrm{ad}}"] \arrow[d, "1 \wedge p"'] & \tilde{E}_6 \arrow[d, "p"] \\
AdE_6 \wedge E_6 \arrow[r, "\hat{\mathrm{ad}}"'] & E_6 .
\end{tikzcd}$$

Also $\check{\mathrm{ad}}$ exists and is unique up to homotopy since $AdE_6 \wedge \tilde{E}_6$ is 3-connected.

In connection with these maps, note that there are the following two homotopy-commutative diagrams which we need later:

$$(2.1)\qquad \begin{tikzcd}
\tilde{E}_6 \wedge \tilde{E}_6 \arrow[r, "\tilde{\mathrm{ad}}"] \arrow[d, "(w \circ p) \wedge 1"'] & \tilde{E}_6 \\
AdE_6 \wedge \tilde{E}_6, \arrow[ur, "\check{\mathrm{ad}}"'] &
\end{tikzcd}
\qquad
\begin{tikzcd}
AdE_6 \wedge \tilde{E}_6 \arrow[r, "\check{\mathrm{ad}}"] \arrow[d, "1 \wedge (w \circ p)"'] & \tilde{E}_6 \arrow[d, "w \circ p"] \\
AdE_6 \wedge AdE_6 \arrow[r, "\bar{\mathrm{ad}}"'] & AdE_6 .
\end{tikzcd}$$

The homotopy-commutativity of the first one is showed by

$$\begin{aligned}
p \circ \check{\mathrm{ad}} \circ \{(w \circ p) \wedge 1\} &\simeq \hat{\mathrm{ad}} \circ (1 \wedge p) \circ \{(w \circ p) \wedge 1\} \\
&= \hat{\mathrm{ad}} \circ (w \wedge 1) \circ (p \wedge p) \\
&= \mathrm{ad} \circ (p \wedge p)
\end{aligned}$$

and the uniqueness of the homotopy class of $\tilde{\mathrm{ad}}$. The other is similar.

The following proposition is partly due to Kono-Kozima [8] and Hamanaka [5] and is proved in the same manner. Let β_* and $\wp^1_*$ be the dual maps of β and $\wp^1$ respectively. Denote $a * a' = f_*(a \otimes a')$ for $f = \bar{\mathrm{ad}}, \mathrm{ad}, \hat{\mathrm{ad}}, \tilde{\mathrm{ad}}$ or $\check{\mathrm{ad}}$.

Proposition 2.1. (i) *$a * a'$ is primitive if a' is primitive.*
(ii) $\beta_*(a * a') = (\beta_* a) * a' + (-1)^{|a|} a * (\beta_* a')$.
(iii) $\wp^1_*(a * a') = (\wp^1_* a) * a' + a * (\wp^1_* a')$.
(iv) $(aa') * a'' = a * (a' * a'')$.
(v) *Let $*$ mean $\bar{\mathrm{ad}}_*$, ad_* or $\tilde{\mathrm{ad}}_*$. If a is primitive, then* $a * a' = aa' + (-1)^{|a|\,|a'|+1} a'a$.

3. AdE_6

According to Araki [2], as an algebra

$$H^*(AdE_6) = \mathbf{Z}_3[\bar{x}_2, \bar{x}_8]/(\bar{x}_2^9, \bar{x}_8^3) \otimes \Lambda(\bar{x}_1, \bar{x}_3, \bar{x}_7, \bar{x}_9, \bar{x}_{11}, \bar{x}_{15})$$

where $\deg \bar{x}_k = k$, $\beta \bar{x}_1 = \bar{x}_2$, $\wp^1 \bar{x}_3 = \bar{x}_7$ and $\beta \bar{x}_7 = \bar{x}_8$. We shall determine the Hopf algebra structure of $H^*(AdE_6)$.

It is clear that $\bar{\mu}^*(\bar{x}_k) = 0$ $(k = 1, 2)$.

Kono [7] shows that we may put

$$\bar{\mu}^*(\bar{x}_3) = \bar{x}_2 \otimes \bar{x}_1$$

and hence

$$\bar{\mu}^*(\bar{x}_k) = \bar{x}_2^3 \otimes \bar{x}_{k-6} \quad (k = 7, 8)$$

using a inclusion $i : SU(6) \hookrightarrow E_6$ such that $i_* : \pi_3(SU(6)) \cong \pi_3(E_6)$. Note that i induces $\bar{i} : SU(6)/\mathbf{Z}_3 \to AdE_6$. Moreover according to Baum-Browder [3],

$$H^*(SU(6)/\mathbf{Z}_3) = \mathbf{Z}_3[\bar{\xi}_2]/(\bar{\xi}_2^3) \otimes \Lambda(\bar{\xi}_1, \bar{\xi}_3, \bar{\xi}_7, \bar{\xi}_9, \bar{\xi}_{11})$$

as an algebra where $\deg \bar{\xi}_k = k$ and $\bar{\mu}^*(\bar{\xi}_k) = \bar{\xi}_2 \otimes \bar{\xi}_{k-2}$ $(k = 3, 9)$.

Next, we shall determine $\bar{\mu}^*(\bar{x}_9)$. According to Kono [7], we can choose $\bar{x}_9$ such that

$$\begin{aligned}(3.1) \qquad \bar{\mu}^*(\bar{x}_9) = {} & \alpha_1 \bar{x}_2 \otimes \bar{x}_7 + \alpha_2 \bar{x}_2^3 \otimes \bar{x}_3 \\ & + \alpha_1 \bar{x}_2^4 \otimes \bar{x}_1 + (\alpha_2 - \alpha_1)\bar{x}_8 \otimes \bar{x}_1 \qquad (\alpha_k \in \mathbf{Z}_3).\end{aligned}$$

In the following, we shall show that we may put $\alpha_1 = 1$. According to Borel [4], as an algebra

$$H^*(E_6) = \mathbf{Z}_3[x_8]/(x_8^3) \otimes \Lambda(x_3, x_7, x_9, x_{11}, x_{15}, x_{17})$$

where $\deg x_k = k$, $\wp^1 x_3 = x_7$, $\beta x_7 = x_8$ and $w^*(\bar{x}_k) = x_k$ $(k = 3, 7, 8, 9, 11, 15)$. By Kudo's transgression theorem [11], we have as an algebra

$$H^*(\tilde{E}_6) = \mathbf{Z}_3[\tilde{y}_{18}] \otimes \Lambda(\tilde{x}_9, \tilde{x}_{11}, \tilde{x}_{15}, \tilde{x}_{17}, \tilde{y}_{19}, \tilde{y}_{23})$$

where $\deg \tilde{x}_k = \deg \tilde{y}_k = k$, $\beta \tilde{y}_{18} = \tilde{y}_{19}$, $\wp^1 \tilde{y}_{19} = \tilde{y}_{23}$ and $p^*(x_k) = \tilde{x}_k$ $(k = 9, 11, 15, 17)$. Let $\tilde{SU}(6)$ be the 3-connective cover over $SU(6)$. Then, we have as an algebra

$$H^*(\tilde{SU}(6)) = \mathbf{Z}_3[\tilde{\zeta}_{18}] \otimes \Lambda(\tilde{\xi}_5, \tilde{\xi}_9, \tilde{\xi}_{11}, \tilde{\zeta}_7, \tilde{\zeta}_{19})$$

where $\deg \tilde{\xi}_k = \deg \tilde{\zeta}_k = k$. Furthermore, according to Nishimura [12], we can choose $\tilde{\zeta}_{18}$ such that $\bar{\mu}^*(\tilde{\zeta}_{18}) = \tilde{\xi}_9 \otimes \tilde{\xi}_9$.

Note that i induces $\tilde{i} : \tilde{SU}(6) \to \tilde{E}_6$. We can easily check that we can choose $\tilde{y}_{18}$ such that $\tilde{i}^*(\tilde{y}_{18}) = \tilde{\zeta}_{18}$. Hence we have

$$(3.2) \qquad \bar{\mu}^*(\tilde{y}_{18}) = \tilde{x}_9 \otimes \tilde{x}_9$$

and $\tilde{i}^*(\tilde{x}_9) = \tilde{i}^* \circ p^* \circ w^*(\bar{x}_9) = \pm \tilde{\xi}_9 \neq 0$. Consider the following homotopy-commutative diagram:

$$\begin{array}{ccccc} \tilde{SU}(6) & \longrightarrow & SU(6) & \longrightarrow & SU(6)/\mathbf{Z}_3 \\ \downarrow{\scriptstyle \tilde{i}} & & \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle \bar{i}} \\ \tilde{E}_6 & \xrightarrow[p]{} & E_6 & \xrightarrow[w]{} & AdE_6. \end{array}$$

Since $\bar{\mu}^*(\bar{\xi}_9) = \bar{\xi}_2 \otimes \bar{\xi}_7$, we may put $\alpha_1 = 1$.

To determine α_2, we need to compute $\bar{\mathrm{ad}}^*(\bar{x}_9)$ and dualize it to homology. By (3.1), we have

$$\bar{\mathrm{ad}}^*(\bar{x}_9) = 1 \otimes \bar{x}_9 + \bar{x}_2 \otimes \bar{x}_7 + \alpha_2 \bar{x}_2^3 \otimes \bar{x}_3 + (\text{OTHER TERMS}).$$

Let $\bar{a}_k$ be the dual element of $\bar{x}_k$ and $\bar{a}_6$ be that of $\bar{x}_2^3$ with respect to the monomial basis. Then we have

$$\begin{aligned} \bar{a}_9 &= \bar{a}_2 * \bar{a}_7 \\ &= \alpha_2 \bar{a}_6 * \bar{a}_3. \end{aligned}$$

For a Hopf space G, put $P_*(G) = \{a \in H_*(G), a \text{ is primitive}\}$. Applying $\wp_*^1$ for $\bar{a}_6 * \bar{a}_7 \in P_{13}(AdE_6) = 0$, we have $\bar{a}_2 * \bar{a}_7 + \bar{a}_6 * \bar{a}_3 = 0$. Hence we have $\bar{a}_9 = \bar{a}_2 * \bar{a}_7 = -\bar{a}_6 * \bar{a}_3$ and thus $\alpha_2 = -1$. Accordingly we have

$$\begin{aligned} \bar{\mu}^*(\bar{x}_9) = \bar{x}_2 \otimes \bar{x}_7 &- \bar{x}_2^3 \otimes \bar{x}_3 \\ &+ \bar{x}_2^4 \otimes \bar{x}_1 + \bar{x}_8 \otimes \bar{x}_1. \end{aligned}$$

Next, we shall determine $\bar{\mu}^*(\bar{x}_{11})$. According to Kono [7], we can choose $\bar{x}_{11}$ such that

$$\begin{aligned} \bar{\mu}^*(\bar{x}_{11}) = \alpha'(&\bar{x}_2 \otimes \bar{x}_9 - \bar{x}_2^2 \otimes \bar{x}_7 - \bar{x}_2^4 \otimes \bar{x}_3 \\ &+ \bar{x}_8 \otimes \bar{x}_3 - \bar{x}_2^5 \otimes \bar{x}_1 + \bar{x}_2\bar{x}_8 \otimes \bar{x}_1) \qquad (\alpha' \in \mathbf{Z}_3). \end{aligned}$$

Hence we have

$$\bar{\mathrm{ad}}^*(\bar{x}_{11}) = 1 \otimes \bar{x}_{11} + \alpha' \bar{x}_2 \otimes \bar{x}_9 + (\text{OTHER TERMS})$$

and

$$\bar{a}_{11} = \alpha' \bar{a}_2 * \bar{a}_9.$$

To determine α', it suffices to show the following lemma. Let $\tilde{a}_k$, $\tilde{b}_k$ be the dual elements of $\tilde{x}_k$, $\tilde{y}_k$ respectively with respect to the monomial basis.

Lemma 3.1. *We can choose $\tilde{a}_{11}$ such that $\tilde{a}_{11} = \tilde{a}_2 * \tilde{a}_9$ and $\tilde{b}_{18} = \tilde{a}_7 * \tilde{a}_{11}$.*

It follows that $\bar{a}_{11} = \bar{a}_2 * \bar{a}_9$ by (2.1) and the above lemma. Thus $\alpha' = 1$ and hence we obtain

$$\begin{aligned} \bar{\mu}^*(\bar{x}_{11}) = \bar{x}_2 \otimes \bar{x}_9 &- \bar{x}_2^2 \otimes \bar{x}_7 - \bar{x}_2^4 \otimes \bar{x}_3 \\ &+ \bar{x}_8 \otimes \bar{x}_3 - \bar{x}_2^5 \otimes \bar{x}_1 + \bar{x}_2\bar{x}_8 \otimes \bar{x}_1. \end{aligned}$$

Proof of lemma 3.1. By (3.2) we have $\tilde{\mathrm{ad}}^*(\tilde{y}_{18}) = 1 \otimes \tilde{y}_{18} - \tilde{x}_9 \otimes \tilde{x}_9$ and hence we get $\tilde{b}_{18} = -\tilde{a}_9 * \tilde{a}_9$. Using (2.1), we obtain $\tilde{b}_{18} = -\bar{a}_9 * \tilde{a}_9$. Substituting $\bar{a}_9 = \bar{a}_2 * \bar{a}_7 = \bar{a}_2\bar{a}_7 - \bar{a}_7\bar{a}_2$, we have

$$\begin{aligned}\tilde{b}_{18} &= -(\bar{a}_2\bar{a}_7) * \tilde{a}_9 + (\bar{a}_7\bar{a}_2) * \tilde{a}_9 \\ &= \bar{a}_7 * (\bar{a}_2 * \tilde{a}_9)\end{aligned}$$

since $\bar{a}_7 * \tilde{a}_9 \in P_{16}(\tilde{E}_6) = 0$. Accordingly we may put as desired.

Finally, we shall determine $\bar{\mu}^*(\bar{x}_{15})$. We can choose $\bar{a}_{15}$ such that $\wp_*^1\bar{a}_{15} = \bar{a}_{11}$. In fact, we may put $\bar{a}_{15} = \bar{a}_6 * \bar{a}_9$. Hence we have $\wp^1\bar{x}_{11} = \bar{x}_{15}$. We can determine $\bar{\mu}^*(\bar{x}_{15})$ by applying $\wp^1$ for $\bar{\mu}^*(\bar{x}_{11})$.

Remark 3.2. Kono [7] determines $\bar{\mu}^*(\bar{x}_k)$ $(k = 9, 11, 15)$ using β-operation.

Thus we have determined the Hopf algebra structure of $H^*(AdE_6)$. Simultaneously, we can easily determine the cohomology operations in it.

4. E_6

We shall determine the Hopf algebra structure of $H^*(E_6)$. It is obvious that $\bar{\mu}^*(x_k) = 0$ $(k = 3, 7, 8, 9)$. According to §3, we have $\bar{\mu}^*(x_k) = x_8 \otimes x_{k-8}$ $(k = 11, 15)$. Hence we are left to determine $\bar{\mu}^*(x_{17})$.

According to Ishitoya-Kono-Toda [6], we can choose x_{17} such that

$$\bar{\mu}^*(x_{17}) = \delta x_8 \otimes x_9 \qquad (\delta \in \mathbf{Z}_3).$$

Hence we have

$$\mathrm{ad}^*(x_{17}) = 1 \otimes x_{17} + \delta(x_8 \otimes x_9 - x_9 \otimes x_8)$$

and $a_{17} = \delta a_8 * a_9$ where a_k is the dual element of x_k with respect to the monomial basis.

To determine δ, we need the following.

Lemma 4.1. *We can put* $a_{17} = -\bar{a}_6 * a_{11}$.

Proof. We may put $\tilde{b}_{19} = \bar{a}_8 * \tilde{a}_{11}$ since

$$\begin{aligned}\beta_*(\bar{a}_8 * \tilde{a}_{11}) &= (\beta_*\bar{a}_8) * \tilde{a}_{11} + \bar{a}_8 * (\beta_*\tilde{a}_{11}) \\ &= \bar{a}_7 * \tilde{a}_{11} \\ &= \tilde{b}_{18}.\end{aligned}$$

We can easily compute that

$$\bar{\mathrm{ad}}^*(\bar{x}_8) = 1 \otimes \bar{x}_8 + \bar{x}_2^3 \otimes \bar{x}_2 - \bar{x}_2 \otimes \bar{x}_2^3$$

and hence

$$\bar{a}_8 = -\bar{a}_2 * \bar{a}_6 \tag{4.1}$$
$$= \bar{a}_6\bar{a}_2 - \bar{a}_2\bar{a}_6.$$

Accordingly we have

$$\tilde{b}_{19} = (\bar{a}_6\bar{a}_2 - \bar{a}_2\bar{a}_6) * \tilde{a}_{11}$$
$$= \bar{a}_6 * (\bar{a}_2 * \tilde{a}_{11}) - \bar{a}_2 * (\bar{a}_6 * \tilde{a}_{11})$$
$$= -\bar{a}_2 * (\bar{a}_6 * \tilde{a}_{11}).$$

Hence we may put $\tilde{a}_{17} = -\bar{a}_6 * \tilde{a}_{11}$. By the definition of $\check{\mathrm{ad}}$, we have $a_{17} = -\bar{a}_6 * a_{11}$.

Applying $\wp^1_*$ for $\bar{a}_6 * a_{15} \in P_{21}(E_6) = 0$, we have $\bar{a}_2 * a_{15} + \bar{a}_6 * a_{11} = 0$ and hence we get $a_{17} = \bar{a}_2 * a_{15}$. Substituting (4.1) for $\bar{a}_8 * a_9$, we have

$$\bar{a}_8 * a_9 = \bar{a}_6 * (\bar{a}_2 * a_9) - \bar{a}_2 * (\bar{a}_6 * a_9)$$
$$= \bar{a}_6 * a_{11} - \bar{a}_2 * a_{15}$$
$$= -a_{17} - a_{17}$$
$$= a_{17}.$$

By the definition of $\hat{\mathrm{ad}}$, we have $a_8 * a_9 = a_{17}$. Thus, we may put $\delta = 1$ and hence we obtain

$$\bar{\mu}^*(x_{17}) = x_8 \otimes x_9.$$

Besides we can easily determine the cohomology operations in $H^*(E_6)$.

5. $\tilde{E}_6$

In §3, we have $\bar{\mu}^*(\tilde{y}_{18}) = \tilde{x}_9 \otimes \tilde{x}_9$. It is clear that $\bar{\mu}^*(\tilde{x}_k) = 0$ $(k = 9, 11, 15, 17)$ and $\bar{\mu}^*(\tilde{y}_k) = 0$ $(k = 19, 23)$. Checking the cohomology operations is easy.

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