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On the homology of the Kac-Moody groups and the cohomology of the 3-connective covers of Lie groups

By

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Abstract

Let G be a compact, 1-connected, simple Lie group of exceptional type, g its Lie algebra, and p an odd prime. In this paper, the mod p homology of the Kac-Moody group $K(g^{(1)})$ and the mod p cohomology of the 3-connective cover over G are determined as Hopf algebras over the Steenrod algebra for every case that the integral homology of G has p-torsion.

1. Introduction

In [4], Hamanaka and Hara determined $H_*(\Omega G;\mathbb{F}_3)$ as a Hopf algebra over $\mathcal{A}_3$ for $G = F_4, E_6, E_7$ and E_8 where $\mathcal{A}_p$ is the mod p Steenrod algebra. Moreover, they determined the mod 3 homology map of the adjoint action $\mathrm{Ad}\colon G \times \Omega G \to \Omega G$ for G above except for one equation which is in the case $G = E_6$.

The first purpose of this paper is to determine this remaining equation by computing the mod 3 homology map of the adjoint action $\overline{\mathrm{Ad}}\colon \mathrm{Ad}E_6 \times \Omega E_6 \to \Omega E_6$. Then, by using this result and the result of [4], we determine $H_*(K(g^{(1)});\mathbb{F}_3)$ and $H^*(\tilde{G};\mathbb{F}_3)$ as Hopf algebras over $\mathcal{A}_3$ for G above where g is the Lie algebra of G, $K(g^{(1)})$ is the Kac-Moody group associated with g (see [6], [7] and [8]), and $\tilde{G}$ is the 3-connective cover over G. Also we give a similar result for E_8 at prime 5 by using the result of [5].

This paper is organized as follows. In Section 2, we compute the mod 3 homology map of $\overline{\mathrm{Ad}}$ and complete the computation of the mod 3 homology map of $\mathrm{Ad}\colon E_6 \times \Omega E_6 \to \Omega E_6$. In Sections 3 and 4, we determine the mod p homology of the Kac-Moody group and the mod p cohomology of the 3-connective cover, respectively, as Hopf algebras over $\mathcal{A}_p$ for the cases stated before.

We use the following notation. The subscript of an element of a graded algebra designates the degree. The reduced coproduct of a coalgebra is denoted

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by $\bar{\phi}$. The symbol $*$ is used to indicate the adjoint action as in [4]. (Also see [12].) The mod 3 cohomology and homology are simply denoted by $H^*(\ )$ and $H_*(\ )$.

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2. The adjoint action of $\mathrm{Ad}E_6$ on ΩE_6

Recall from Araki [1], Borel [2] and Petrie [16],

$$H^*(E_6) = \mathbb{F}_3[x_8]/(x_8^3) \otimes \wedge(x_3, x_7, x_9, x_{11}, x_{15}, x_{17}),$$
$$H^*(\mathrm{Ad}E_6) = \mathbb{F}_3[\bar{x}_2, \bar{x}_8]/(\bar{x}_2^9, \bar{x}_8^3) \otimes \wedge(\bar{x}_1, \bar{x}_3, \bar{x}_7, \bar{x}_9, \bar{x}_{11}, \bar{x}_{15}),$$
$$H_*(\Omega E_6) = \mathbb{F}_3[t_2, t_6, t_8, t_{10}, t_{14}, t_{16}, t_{22}]/(t_2^3)$$

as algebras. We choose the same generators as those in Kono [9] and Hamanaka-Hara [4]. For the detail of the coalgebra structures and the $\mathcal{A}_3$-module structures, see [9] and [4].

Let $\pi\colon E_6 \to \mathrm{Ad}E_6$ be the natural projection. Let y_j and $\bar{y}_j$ be the dual elements of the indecomposable classes of x_j and $\bar{x}_j$ respectively. Let $\bar{y}_6$ be the dual element of $\bar{x}_2^3$ with respect to the monomial basis of $H^*(\mathrm{Ad}E_6)$. We can see $\pi_*(y_8) = \bar{y}_8 = \bar{y}_6\bar{y}_2 - \bar{y}_2\bar{y}_6$. (See [14].) Note that $H_*(\mathrm{Ad}E_6)$ is generated by $\bar{y}_1, \bar{y}_2$ and $\bar{y}_6$ as an algebra.

Proposition 1. *The map* $\overline{Ad}_*$ *is given by* $\bar{y}_1 * t_j = 0$ *for any* j *and by the following table*:

	t_2	t_6	t_8	t_{10}	t_{14}	t_{16}	t_{22}
$\bar{y}_2*$	0	t_8	t_{10}	0	t_{16}	κt_6^3	$-\kappa t_8^3$
$\bar{y}_6*$	$-t_8$	$t_8t_2^2$	t_{14}	$-t_{16}$	0	t_{22}	0

where κ *is the same one as that in* [4]. *Moreover,* $\delta = -\kappa \neq 0$ *in Theorem* 2 *of* [4].

Proof. By the dimensional reason and the primitivity, we have $\bar{y}_1 * t_j = 0$ for any j and $\bar{y}_2 * t_2 = \bar{y}_2 * t_{10} = \bar{y}_6 * t_{14} = \bar{y}_6 * t_{22} = 0$. Since $\overline{\mathrm{Ad}}_* \circ (\pi_* \otimes 1) = \mathrm{Ad}_*$, we have $t_{10} = y_8 * t_2 = \bar{y}_8 * t_2 = -\bar{y}_2 * (\bar{y}_6 * t_2)$. Hence we may assume that $t_8 = -\bar{y}_6 * t_2$ and $t_{10} = \bar{y}_2 * t_8$. Then, we can see that $\bar{\phi}(\bar{y}_6 * t_6) = \bar{\phi}(t_8t_2^2)$ and hence $\bar{y}_6 * t_6 = t_8t_2^2$. By applying $\wp^1$ to this, we have $\bar{y}_2 * t_6 = t_8$. Since $(\bar{y}_6 * t_8)\wp^1 = \bar{y}_2 * t_8 = t_{10}$, we can conclude that $\bar{y}_6 * t_8 = t_{14}$. We can see that $t_{16} = \bar{y}_8 * t_8 = \bar{y}_6 * t_{10} - \bar{y}_2 * t_{14}$ while by applying $\wp^1$ to $\bar{y}_6 * t_{14} = 0$, we have $\bar{y}_2 * t_{14} + \bar{y}_6 * t_{10} = 0$. Hence we have $\bar{y}_2 * t_{14} = t_{16}$ and $\bar{y}_6 * t_{10} = -t_{16}$. We have $\bar{y}_8 * t_{10} = \bar{y}_2 * t_{16} = \kappa t_6^3$ and since $(\bar{y}_6 * t_{16})\wp^1 = \bar{y}_2 * t_{16} = \kappa t_6^3$, we have $\bar{y}_6 * t_{16} = t_{22}$. We have $y_8 * t_{16} = \kappa\bar{y}_6 * (t_6^3) - \bar{y}_2 * t_{22}$ while by applying $\wp^1$ to $\bar{y}_6 * t_{22} = 0$, we have $\bar{y}_2 * t_{22} + \kappa\bar{y}_6 * (t_6^3) = 0$. Hence we have $y_8 * t_{16} = \bar{y}_2 * t_{22} = -\kappa\bar{y}_6 * (t_6^3)$. We can see that $\bar{y}_6 * (t_6^3) = t_8^3$ and hence, the proposition is proved. □

3. The homology of the Kac-Moody groups

Let $L(G)$ be the space of free loops on G. Recall that $L(G)$ is the semi-direct product of G and ΩG where the adjoint action $\mathrm{Ad}\colon G\times\Omega G\to\Omega G$ twists the multiplications of G and ΩG. See [4].

Since $K(g^{(1)})$ is the central extension by S^1 of $L(G)$, it is identified as an A_∞-space with the semi-direct product of G and $\Omega\tilde{G}$ where the adjoint action $\widetilde{\mathrm{Ad}}\colon G\times\Omega\tilde{G}\to\Omega\tilde{G}$ twists the multiplications of G and $\Omega\tilde{G}$. See Kac [6] and [7], Kac-Peterson [8]. Accordingly, the Hopf algebra structure over the Steenrod algebra of $H_*(K(g^{(1)});\mathbb{F}_p)$ is determined by that of $H_*(G;\mathbb{F}_p)$, that of $H_*(\Omega\tilde{G};\mathbb{F}_p)$, and the map $\widetilde{\mathrm{Ad}}_*$.

Let $q\colon\tilde{G}\to G$ be the covering projection. Let the generators of $H_*(G)$, $H_*(\Omega G)$, $H_*(E_8;\mathbb{F}_5)$ and $H_*(\Omega E_8;\mathbb{F}_5)$ be as in [4] and Hamanaka-Hara-Kono [5]. Then, we have

$$
\begin{aligned}
H_*(\Omega\tilde{F}_4)&=\mathbb{F}_3[\tilde{t}_{10},\tilde{t}_{14},\tilde{t}_{22},\tilde{u}_{18}]\otimes\wedge(\tilde{u}_{17}),\\
H_*(\Omega\tilde{E}_6)&=\mathbb{F}_3[\tilde{t}_{8},\tilde{t}_{10},\tilde{t}_{14},\tilde{t}_{16},\tilde{t}_{22},\tilde{u}_{18}]\otimes\wedge(\tilde{u}_{17}),\\
H_*(\Omega\tilde{E}_7)&=\mathbb{F}_3[\tilde{t}_{10},\tilde{t}_{14},\tilde{t}_{22},\tilde{t}_{26},\tilde{t}_{34},\tilde{u}_{18},\tilde{u}_{54}]\otimes\wedge(\tilde{u}_{53}),\\
H_*(\Omega\tilde{E}_8)&=\mathbb{F}_3[\tilde{t}_{14},\tilde{t}_{22},\tilde{t}_{26},\tilde{t}_{34},\tilde{t}_{38},\tilde{t}_{46},\tilde{t}_{58},\tilde{u}_{54}]\otimes\wedge(\tilde{u}_{53}),\\
H_*(\Omega\tilde{E}_8;\mathbb{F}_5)&=\mathbb{F}_5[\tilde{t}_{14},\tilde{t}_{22},\tilde{t}_{26},\tilde{t}_{34},\tilde{t}_{38},\tilde{t}_{46},\tilde{t}_{58},\tilde{u}_{50}]\otimes\wedge(\tilde{u}_{49})
\end{aligned}
$$

where $(\Omega q)_*(\tilde{t}_j)=t_j$, $(\Omega q)_*(\tilde{u}_{18})=\kappa t_6^3$, $(\Omega q)_*(\tilde{u}_{54})=t_{18}^3$, $(\Omega q)_*(\tilde{u}_{50})=t_{10}^5$, $(\Omega q)_*(\tilde{u}_{\mathrm{odd}})=0$, $\tilde{u}_{18}\beta=\tilde{u}_{17}$, $\tilde{u}_{54}\beta=\tilde{u}_{53}$, and $\tilde{u}_{50}\beta=\tilde{u}_{49}$. If we note that $(\Omega q)_*$ is injective in even degrees, we can easily determine the $\mathcal{A}_p$-module structures and we can easily see that all generators except for $\tilde{u}_{54}\in H_*(\Omega\tilde{E}_7)$ are primitive and $\bar{\phi}(\tilde{u}_{54})=-\kappa(\tilde{u}_{18}^2\otimes\tilde{u}_{18}+\tilde{u}_{18}\otimes\tilde{u}_{18}^2)$. See Kono [10] and Kono-Kozima [11]. Thus, we are left to determine $\widetilde{\mathrm{Ad}}_*$ for the determination of $H_*(K(g^{(1)});\mathbb{F}_p)$ as a Hopf algebra over $\mathcal{A}_p$ for the cases $(G,p)=(F_4,3)$, $(E_6,3)$, $(E_7,3)$, $(E_8,3)$ and $(E_8,5)$. Let $\widehat{\mathrm{Ad}}\colon \mathrm{Ad}E_6\times\Omega\tilde{E}_6\to\Omega\tilde{E}_6$ be the adjoint action of $\mathrm{Ad}E_6$ on $\Omega\tilde{E}_6$.

Proposition 2.

(i) *The mod 3 homology map $\widehat{\mathrm{Ad}}_*$ is given by $\bar{y}_1*\tilde{t}_{16}=\tilde{u}_{17}$, $\bar{y}_1*\tilde{t}_j=0$ for $j\neq 16$, and $\bar{y}_1*\tilde{u}_j=0$ for $j=17,18$, and by the following table.*

	$\tilde{t}_8$	$\tilde{t}_{10}$	$\tilde{t}_{14}$	$\tilde{t}_{16}$	$\tilde{t}_{22}$	$\tilde{u}_{17}$	$\tilde{u}_{18}$
$\bar{y}_2*$	$\tilde{t}_{10}$	0	$\tilde{t}_{16}$	$\tilde{u}_{18}$	$-\kappa\tilde{t}_8^3$	0	0
$\bar{y}_6*$	$\tilde{t}_{14}$	$-\tilde{t}_{16}$	0	$\tilde{t}_{22}$	0	0	$\kappa\tilde{t}_8^3$

(ii) *For the cases $(G,p)=(F_4,3)$, $(E_6,3)$, $(E_7,3)$ and $(E_8,3)$, $\widetilde{\mathrm{Ad}}_*$ is given by $y_3*\tilde{t}_{14}=-\tilde{u}_{17}$, $y_3*\tilde{t}_j=0$ for $j\neq 14$, $y_7*\tilde{t}_{10}=\tilde{u}_{17}$, $y_7*\tilde{t}_{46}=-\varepsilon\tilde{u}_{53}$, $y_7*\tilde{t}_j=0$ for $j\neq 10,46$, $y_9*\tilde{t}_8=-\tilde{u}_{17}$, $y_9*\tilde{t}_j=0$ for $j\neq 8$, $y_{19}*\tilde{t}_{34}=\varepsilon\tilde{u}_{53}$, $y_{19}*\tilde{t}_j=0$ for $j\neq 34$, and $y_l*\tilde{u}_j=0$ for $l=3,7,8,9,19,20$ and any j, and by the following table where ε is the same one as that in* [4].

	$\tilde{t}_8$	$\tilde{t}_{10}$	$\tilde{t}_{14}$	$\tilde{t}_{16}$	$\tilde{t}_{22}$	$\tilde{t}_{26}$	$\tilde{t}_{34}$	$\tilde{t}_{38}$	$\tilde{t}_{46}$	$\tilde{t}_{58}$
y_8*	$\tilde{t}_{16}$	$\tilde{u}_{18}$	$\tilde{t}_{22}$	$-\kappa\tilde{t}_8^3$	$-\tilde{t}_{10}^3$	$\tilde{t}_{34}$	$-\tilde{t}_{14}^3$	$-\tilde{t}_{46}$	$-\varepsilon\tilde{u}_{54}$	$-\varepsilon\tilde{t}_{22}^3$
$y_{20}*$	–	–	$\tilde{t}_{34}$	–	$-\tilde{t}_{14}^3$	$-\tilde{t}_{46}$	$\varepsilon\tilde{u}_{54}$	$\tilde{t}_{58}$	$\varepsilon\tilde{t}_{22}^3$	$-\tilde{t}_{26}^3$

(iii) *For the case* $(G,p) = (E_8, 5)$, $\widetilde{\mathrm{Ad}}_*$ *is given by* $y_3*\tilde{t}_{46} = -\epsilon\tilde{u}_{49}$, $y_3*\tilde{t}_j = 0$ *for* $j \neq 46$, $y_{11}*\tilde{t}_{38} = \epsilon\tilde{u}_{49}$, $y_{11}*\tilde{t}_j = 0$ *for* $j \neq 38$, *and* $y_l*\tilde{u}_j = 0$ *for* $l = 3, 11, 12$ *and* $j = 49, 50$, *and by the following table where* ϵ *is the same one as that in* [5].

	$\tilde{t}_{14}$	$\tilde{t}_{22}$	$\tilde{t}_{26}$	$\tilde{t}_{34}$	$\tilde{t}_{38}$	$\tilde{t}_{46}$	$\tilde{t}_{58}$
$y_{12}*$	$\tilde{t}_{26}$	$\tilde{t}_{34}$	$\tilde{t}_{38}$	$\tilde{t}_{46}$	$\epsilon\tilde{u}_{50}$	$\epsilon\tilde{t}_{58}$	$-\epsilon^{-1}\tilde{t}_{14}^5$

Proof. By the injectivity of $(\Omega q)_*$ in even degrees, by the results of [4] and [5], and by the result of Section 2, we have the equations of $y_{\mathrm{even}}*$ and $\bar{y}_{\mathrm{even}}*$ on even degree generators. Also we can easily deduce those on odd degree generator. Then, applying suitable cohomology operations, we can easily show the proposition except for the case $(G,p) = (E_6, 3)$ of (ii). For the remaining case, we can similarly deduce the equations of y_3* and y_7*. Then, we can deduce those of y_9* by using $y_9*t = \bar{y}_9*t = (\bar{y}_2\bar{y}_7 - \bar{y}_7\bar{y}_2)*t = \bar{y}_2*(y_7*t) - y_7*(\bar{y}_2*t)$. □

Remark 3. Note that the relation $y_{19} * \tilde{t}_{34} = \varepsilon\tilde{u}_{53}$ in $H_*(\Omega\tilde{E}_7)$ follows from that in $H_*(\Omega\tilde{E}_8)$. Except for this, all relations can be deduced without using inclusions of Lie groups and the computations are completely algebraic.

4. The cohomology of the 3-connective covers

Recall that

$$
\begin{aligned}
H^*(\tilde{F}_4) &= \mathbb{F}_3[\tilde{z}_{18}] \otimes \wedge(\tilde{x}_{11}, \tilde{x}_{15}, \tilde{z}_{19}, \tilde{z}_{23}),\\
H^*(\tilde{E}_6) &= \mathbb{F}_3[\tilde{z}_{18}] \otimes \wedge(\tilde{x}_9, \tilde{x}_{11}, \tilde{x}_{15}, \tilde{x}_{17}, \tilde{z}_{19}, \tilde{z}_{23}),\\
H^*(\tilde{E}_7) &= \mathbb{F}_3[\tilde{z}_{54}] \otimes \wedge(\tilde{x}_{11}, \tilde{x}_{15}, \tilde{x}_{27}, \tilde{x}_{35}, \tilde{z}_{19}, \tilde{z}_{23}, \tilde{z}_{55}),\\
H^*(\tilde{E}_8) &= \mathbb{F}_3[\tilde{z}_{54}] \otimes \wedge(\tilde{x}_{15}, \tilde{x}_{27}, \tilde{x}_{35}, \tilde{x}_{39}, \tilde{x}_{47}, \tilde{z}_{23}, \tilde{z}_{55}, \tilde{z}_{59}),\\
H^*(\tilde{E}_8; \mathbb{F}_5) &= \mathbb{F}_5[\tilde{z}_{50}] \otimes \wedge(\tilde{x}_{15}, \tilde{x}_{23}, \tilde{x}_{27}, \tilde{x}_{35}, \tilde{x}_{39}, \tilde{x}_{47}, \tilde{z}_{51}, \tilde{z}_{59}).
\end{aligned}
$$

Except for the $\mathcal{A}_p$-action on $\tilde{z}_{\mathrm{even}}$, the $\mathcal{A}_p$-module structures of these are easily determined by those of $H_*(\Omega\tilde{G}; \mathbb{F}_p)$. Moreover, we may assume that all generators except for $\tilde{z}_{18} \in H^*(\tilde{E}_6)$, $\tilde{z}_{54} \in H^*(\tilde{E}_7)$, $\tilde{z}_{54} \in H^*(\tilde{E}_8)$, and $\tilde{z}_{50} \in H^*(\tilde{E}_8; \mathbb{F}_5)$ are primitive.

Proposition 4. *We can choose the generators such that*
(i) $\bar{\phi}(\tilde{z}_{18}) = \tilde{x}_9 \otimes \tilde{x}_9$ *where* $\tilde{z}_{18} \in H^*(\tilde{E}_6)$,
(ii) $\bar{\phi}(\tilde{z}_{54}) = \tilde{x}_{27} \otimes \tilde{x}_{27}$ *where* $\tilde{z}_{54} \in H^*(\tilde{E}_7)$,
(iii) $\bar{\phi}(\tilde{z}_{54}) = \tilde{x}_{27} \otimes \tilde{x}_{27} + \tilde{x}_{15} \otimes \tilde{x}_{39} + \tilde{x}_{39} \otimes \tilde{x}_{15}$ *where* $\tilde{z}_{54} \in H^*(\tilde{E}_8)$,

(iv) $\bar{\phi}(\tilde{z}_{50}) = \tilde{x}_{23} \otimes \tilde{x}_{27} + \tilde{x}_{27} \otimes \tilde{x}_{23} - \tilde{x}_{15} \otimes \tilde{x}_{35} - \tilde{x}_{35} \otimes \tilde{x}_{15}$ *where* $\tilde{z}_{50} \in H^*(\tilde{E}_8; \mathbb{F}_5)$.

Proof. We only show (i). The others are similar. By applying the homology suspension to $y_9 * \tilde{t}_8 = -\tilde{u}_{17}$, we have $y_9 * \tilde{y}_9 = -\tilde{b}_{18}$ where $\tilde{y}_9$ and $\tilde{b}_{18}$ are the dual elements of $\tilde{x}_9$ and $\tilde{z}_{18}$ respectively. We can also consider the adjoint action of $\tilde{E}_6$ on itself. Then, we have

$$-\tilde{y}_9^2 = [\tilde{y}_9, \tilde{y}_9] = \tilde{y}_9 * \tilde{y}_9 = y_9 * \tilde{y}_9 = -\tilde{b}_{18}$$

and hence $\tilde{b}_{18} = \tilde{y}_9^2$. We can easily see that this implies (i). □

By this proposition, we can easily determine the $\mathcal{A}_p$-action on $\tilde{z}_{\text{even}}$ and hence, we determine $H^*(\tilde{G}; \mathbb{F}_p)$ as a Hopf algebra over $\mathcal{A}_p$ for every case (G, p) we consider.

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