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IRVING REINER

1924–1986

Biography

Irving Reiner was born February 8, 1924 in Brooklyn, New York. He attended elementary and high school there and began his undergraduate work at Brooklyn College in 1940 with majors in mathematics and physics. His talent for mathematics was demonstrated early when he published his first mathematical paper [1], which appeared in his third year as an undergraduate. In the summer of 1943, he studied at the School of Advanced Instruction and Research in Mechanics at Brown University. It was there that he first met Alex Heller, a fellow student, with whom he later did joint work. He earned his B.A. magna cum laude from Brooklyn College in 1944 and entered the graduate school at Cornell University. His Master of Arts degree was awarded in February of 1945. His master's thesis, written under the direction of Burton W. Jones, was published in 1945 [2]; Jones also directed Reiner's doctoral work; his Ph.D. degree was awarded in June, 1947, and his thesis was published in 1949 [3].

While a graduate student at Cornell, Irving met Irma Moses, also a graduate student of Burton Jones. Irving and Irma were married in August of 1948 and they both came to the University of Illinois that year.

After completing his thesis, Irving Reiner went to the Institute for Advanced Study in Princeton where he met L. K. Hua. They worked together that year and agreed that they would seek employment at the same university in order to continue their collaboration. Both accepted offers from the University of Illinois. As it turned out, this was the only job offer Irv ever accepted. During his thirty-eight years at the University of Illinois, he wrote one hundred seven research papers, survey papers, books, and other scholarly works, many with co-authors. A discussion of his research is given below. He directed the thesis work for seventeen doctoral students. During his career, he received many awards and other recognition for his outstanding work, among these a Guggenheim Fellowship (1962), the Distinguished Alumnus Award from Brooklyn College (1963), and a NATO Senior Fellowship (1977). He received research support every year after 1953 either from the Office of Naval Research or the National Science Foundation. At various times he held visiting appointments at Queen Mary College and King's College of the University of London, the University of Paris, and the University of Warwick, Coventry. He made short term visits to and gave invited addresses at many universities around the world. He was a principal lecturer at mathematical conferences at Carleton College and the University of Sao Paulo, chairman of the organizing committee for the AMS Symposium on Representation Theory of Finite Groups and Related Topics held in 1970 in honor of Professor R. Brauer (resulting in [6E]), co-ordinator of the special year in algebra and algebraic number theory at the University of Illinois in 1981–82, and co-coordinator of the conference on Orders and their Applications held at Oberwolfach in 1984. From 1967 to 1972 he was National Counselor of the honorary mathematics fraternity, Pi Mu Epsilon. He served in various editorial capacities for the Proceedings of the American Mathematical Society, Contemporary Mathematics, and the Illinois Journal of Mathematics.

This rather impersonal recitation of activities and accomplishments indicates the high level of mathematical achievement reached by Irving Reiner. This level was not reached in a vacuum; it was the result of a life dedicated to mathematics. He gave encouragement to everyone in whom he saw some mathematical talent and, in return, he was stimulated by the success of other mathematicians with whom he had contact. Students, who might at first have felt intimidated by his reputation, were always quickly put at ease during their conversations with him about mathematics.

His Ph.D. students comment on his rather firm treatment of them; he was a strong advisor but not one who wrote the student's thesis. He met regularly with his students and required them to write up intermediate results. He would read these write-ups carefully and correct style as well as mathematics. He demanded the clear, precise writing from his students that could be found in all his own work. He treated his students with great respect, as, in fact, he treated everyone. He expected his doctoral students to attend seminars and participate in them on an equal basis with the faculty, to read the literature and to keep up on new developments. Many mathematicians, not only his former students, comment on the encouragement they received from him, especially as young scholars starting out. This was a reflection of his own enthusiasm for mathematical research, and his unselfish interest in having others share his excitement through their own succesess.

An inspection of the list of publications is enough to prove that Irving worked very hard. He would often work in streaks, putting in long hours on many consecutive days and nights. In order to refresh himself after such a period of intensive work, he would like to relax by attending a concert. He and Irma regularly attended the Krannert Center concert hall in Urbana or, for that matter, any concert hall where they happened to be (especially if there was a performance of a Beethoven quartet). During the summers, the vacation of choice would be a couple of weeks in the mountains. Irving loved the long hikes in the Canadian and American Rocky mountains. He always carried his camera on these walks; his wonderful pictures of the mountains and wild flowers are treasured by his family.

Visitors to the Illinois mathematics department were often entertained at the Reiner home. During the 1960s and '70s, when the Reiners' sons David and Peter were still living at home, there was a ping-pong table set up in the sun porch of their home. Irv enjoyed playing a game or two, especially with young visitors to the department who thought themselves good players. Irv was not an aggressive player; he played in a relaxed, gentle way, sometimes carrying on a conversation with his opponent during the game. But somehow he managed to return virtually every ball hit to his side of the table—much to the frustration of the opponent who was concentrating intensely on every shot. Lee Rubel tells of losing such a game to Irv and then attempting to excuse his loss with a remark to the effect that he had not been feeling well earlier that day and was not up to his usual game. Irv was understanding but remarked "I don't recall ever winning a game from a well man."

Mathematical work

It is awesome to consider the record of Irving Reiner's published research. He worked for nearly forty years on some of the most difficult problems in representation theory. I will comment very briefly upon his papers and books, in roughly chronological order.

His first paper [1] appeared while Reiner was still an undergraduate. It is a well-written paper in which he proves certain integer valued functions must take on non-prime values at some positive integer. He obtains a corollary which says that the function $f(x) = 2^{2^x} + k$, k a fixed positive integer, must assume a non-prime value for some positive integer x. His second paper [2] was his master's thesis. In August of 1947 he submitted his doctoral thesis for publication [3]. It was concerned with the values assumed by binary quadratic forms. The proofs made use of Dirichlet characters, L-series, and product representations of zeta-functions. After completing his thesis, he set these analytic methods aside for over thirty years, little knowing that he would return to them in thirteen of his last fourteen papers.

After leaving Cornell with his thesis complete, he went to the Institute for Advanced Study for the year 1947–48. He met L. K. Hua there and began the first of many successful collaborations. Their joint work dealt with classical subgroups of $GL(n, \mathbf{Z})$, the invertible $n \times n$ matrices over the ring of integers. They solved problems dealing with minimal sets of generators by direct matrix computations. Their joint papers [4], [5], [6] were followed by several individual works [7], [8], [9], [10] and the joint paper [11] with J. D. Swift. The results in these papers included a determination of generators for the automorphism groups of $GL(n, \mathbf{Z})$, $PGL(n, \mathbf{Z})$, and $Sp(2n, \mathbf{Z})$. Reiner spent two years 1954–56 at the Institute for Advanced Study in Princeton. During the first year, he participated with Charles W. Curtis, Peter Roquette and others, in a seminar directed toward an understanding of Richard Brauer's still relatively new theory of modular representations of finite groups. He and Curtis produced a set of notes from the seminar and subsequently decided to work together on the book [1B], which would appear some seven years later. In September of 1955, Irving submitted his first paper [12] in representation theory. The main result gave a test to determine if a module is relatively projective or relatively injective.

In late 1955, he wrote a review [1R] of Dieudonne's book on the classical groups. This book probably inspired him to return to some of the problems he had considered earlier. His papers [15], [16] and the joint papers [14], [18], [19] with Joe Landin were concerned with finding generators for $\mathrm{Aut}(GL(n, R))$ where R is a suitably restricted domain. Definitive results, (especially in [16]) were obtained when R is a principal ideal domain, or a skewfield (with the restriction $n \geq 3$), $Z[i]$ and $K[x]$, with K a field and $n = 2$.

In [17], a complete list of invariants of finitely generated ZG lattices was given for the case when G has prime order. Diederichsen (1938) had found the indecomposable ZG lattices but this is not sufficient to determine the decomposable lattices since the Krull-Schmidt theorem does not hold for ZG modules. This paper is of great importance in the development of integral representation theory, as it contains the first complete classification of all finitely generated modules over an integral group ring in a situation where the Krull-Schmidt theorem does not hold.

In [23] and [25] some positive and negative results are given to the following question:

Let R be a Dedekind domain with quotient field K, K' an extension field of K and R' the integral closure of R in K. Suppose that G is a finite group, M and N are RG modules which become isomorphic under the ground ring extension; that is $R'M \cong R'N$ as $R'G$ modules. Does it follow that $M \cong N$?

Questions of this sort reveal some of the subtleties of integral representation theory because in the classical case, $R = K$, $R' = K'$, a well-known theorem of Noether and Deuring says that $K'M \cong K'N$ implies $M \cong N$.

The next four years in the chronology, early 1960 to the middle of 1964, were a period of truly incredible activity. He supervised the thesis work of his first five students, collaborated with Alex Heller on five publications, completed eight individual papers, and produced his part of the classic book [1B]. (Bill Ferguson reminds me that at this time, the teaching load was nine hours per semester.) The papers with Heller made advances on several fronts. In [26], they considered characteristic p representations of abelian groups and showed that the classification problem, in general, was at least as difficult as a well-known unsolved problem in matrix theory. The problem was solvable in small cases and they used it to classify certain representations of the elementary abelian group of type (p, p).

Another problem which was receiving considerable attention at that time was concerned with the classification of those group rings RG which had only a finite number of indecomposable, finitely generated, R-torsion free modules (or RG lattices for short). The number of such indecomposable lattices is denoted by $n(RG)$. In [33] and [35] they proved if G is a p-group and $n(ZG)$ is finite, then G is cyclic of order p or p^2. They did not prove a converse. Alan Troy, in his 1961 thesis written under Reiner's supervision, proved that $n(ZG)$ is finite if G is the cyclic group of order four. This corrected a statement to the contrary in Diederichsen (1938). Then Heller and Reiner improved their result about p-groups to show for any finite group G, if $n(ZG)$ is finite, then for every prime p, the p-Sylow subgroup of G is cyclic of order p or p^2. Finally in his 1962 thesis written under Reiner's direction, Alfredo Jones proved $n(ZG)$ is finite if and only if every Sylow subgroup of G is cyclic of cube free order. Two of Reiner's other students solved related problems: Joseph Oppenheim obtained the classification of indecomposable ZG lattices when G is cyclic of square-free order; Myrna Lee classified the indecomposable ZG lattices for the case G is dihedral of order $2p$, p an odd prime.

Reiner's first student, Lawrence Levy, had already started on a problem when Troy and the others were working on the finiteness of $n(ZG)$. However the strong influence of this period was not entirely lost on Levy. In his 1983 paper (*Journal of Algebra*), Levy classified all finitely generated ZG modules, not just lattices, for the case G is cyclic of square free order.

The book [1B] was completed while Charlie Curtis was at the University of Wisconsin. Irving was fond of telling friends that this was probably the only mathematics book written in a museum and a hotel lobby. Since Chicago was approximately half way between Madison and Urbana, Curtis and Reiner would make day trips there, about once a month, to discuss the progress of the book, meeting first at the Art Institute (which was between their train stations) where the day's work was planned during a stroll through the galleries. When it was time to work on their manuscripts (and later the proofs), they usually found a congenial place in some out-of-the-way corner of the lobby of the Palmer House to spread out their papers. However unconventional that may have been, they succeeded in producing a book which influenced a generation of algebraists. The book contained material on non-semisimple algebras such as quasi-Frobenius algebras, a comprehensive treatment of integral representations, a new account of the theory of induced modules based upon the work of G. Mackey, and an introduction to Brauer's theory of blocks, which had previously not been available in book form.

With the solution of the finite type problem for ZG complete, Reiner turned to the study of Grothendieck groups and K-theory and a study of the papers of Swan on projective modules over group rings. Then he and Heller began an investigation of the representation ring, $a(RG)$, of a group ring RG. As a group, $a(RG)$ is generated by the symbols $[M]$, one for each isomorphism class of RG lattices, subject to the relations induced by direct sums: $[M \oplus N] = [M] + [N]$. The multiplication is induced by the tensor product over R. The coefficient ring R was usually a field of characteristic p or a discrete valuation ring of characteristic zero with finite residue field of characteristic p. This was related to the representation algebra, $A(RG)$, introduced by James Green sometime earlier, by the isomorphism $A(RG) = \mathbf{C} \otimes a(RG)$. Green, Conlon and others had found many cases where $A(RG)$ was a semisimple ring; this property was implied by the condition that $a(RG)$ had no nonzero nilpotent elements. Reiner and his students proved many results concerning existence or nonexistence of nonzero nilpotent elements in $a(RG)$ under various assumptions. The study of $a(RG)$ was viewed as weaker than a classification of the RG modules, but a strong step in that direction. Almost all of [36]–[47] deal with these ideas. In [36] and [38], Heller and Reiner studied Grothendieck groups and Whitehead groups of orders in semisimple algebras. They introduced a "relative G_0" group using torsion modules and established a 4-term exact sequence linking the Whitehead group to the Grothendieck group via a relative G_0-group. This has since been known as the Heller-Reiner exact sequence. These papers were written in the early 1960s when algebraic K-theory was in the formative stages and certainly not widely popular. Their use of this sequence was the among the first uses of long exact sequences, a technique which has since become extensively used.

The visit of T. Y. Lam to the University of Illinois during the summer of 1967 proved to be the beginning of another successful collaboration. In the joint papers [48]–[54], they introduced and studied relative Grothendieck rings, $a(G, H)$. The coefficient ring R is now suppressed from the notation. This relative ring is a homomorphic image of $a(RG)$ with kernel generated by all differences $[X] - [M] - [N]$, where there is an exact sequence

$$0 \to M \to X \to N \to 0$$

of RG modules which is split exact when restricted to RH, H a subgroup of G. They considered the question of when $a(G, H)$ is a free abelian group and obtained relations between $a(G, H)$ and $a(K, H)$ for $H \subset K \subset G$.

In 1968, Reiner gave a survey talk on integral representation theory [3S] at the University of Kentucky; a slightly modified version was the subject of an Invited Hour Address given at the AMS meeting in Chicago that year. The notes from this talk were expanded into a major survey article [5S] which listed results, methods, ideas and problems in integral representation theory. The bibliography of 265 items contained every important paper up to that time. His files contained the reprints of all these papers, and many more, along with his handwritten notes made while reading the papers, including the work of many Russian mathematicians, for whom he had high regard. He was pleased with his short paper [47] which was published in Russian.

During the same year, he wrote a small book [3B] intended as an introduction to matrix theory for students in the calculus sequence. This publication, which is written in the same clear and well-organized style as his other work, stands out from the advanced level texts and many research papers to show his interest in undergraduate education. He took an active interest in curriculum development and was one of the early advocates of the use of computers in undergraduate education. This may come as a surprise to those who know that Irv never used a computer in any of his research. He was fascinated by various results obtained by machine computations. "I must learn more about that someday," he said.

When Stephen Ullom joined the University of Illinois faculty, he brought a general interest in algebraic number theory and class groups in particular. He and Reiner began working together on the class group of an order. The papers [55], [56], [57] and [59] examined ways of computing class groups, $Cl(\Lambda)$, the kernel groups, $D(\Lambda)$, and locally free class groups, $LFP(\Lambda)$, of an order in a semisimple **Q**-algebra. They refined a technique, namely the application of a Mayer-Vietoris sequence to a pull-back diagram, and used it to determine $Cl(ZG)$ for certain metacyclic groups G, and for $G = S_n$ or A_n, $n \leq 5$. Reiner continued the study of this problem in [62] and obtained further results on $D(ZS_n)$ and $D(ZA_n)$ for general n.

About this time, he published *Maximal orders* [4B]. This book came into being as a result of a sequence of graduate courses. It had been his custom for many years to teach a sequence of three one-semester courses: ordinary representation theory, modular representation theory, integral representation theory. The text for these was, of course, Curtis and Reiner [1B]. In the Fall of 1968, he began this sequence, and, because of an unusually large number of attendees, he offered a fourth semester entitled Algebras and Their Arithmetic. The topic was maximal orders. He prepared careful notes for this course, had them typed, and passed them out to the class. The resulting set of notes eventually became the book. After its publication, he said, "This book wrote itself." Here Irv was mistaken—the book did not write itself. It contained a comprehensive account of the application of local methods to noncommutative arithmetic. This theme was first developed in a book by Deuring in 1935, and, with many new methods and results due to Reiner and others, became one of the main topics in the books [1B], [5B] and [6B] as well as his *Maximal orders* [4B]. This book once again demonstrated Irv's instinct to know just what material to include so that the book would be useful to students and researchers alike for many years to come.

In January of 1977, Irving completed work on [65]. This is a remarkable paper in which he uses all the techniques acquired in his years of experience to give a complete classification of ZG lattices for G a cyclic group of order p^2, p a prime. This completed work begun some seventeen years earlier on group rings with only a finite number of indecomposable lattices. But the failure of the Krull-Schmidt theorem in this case means that more than the indecomposable lattices must be classified. To each lattice, he attaches a list of invariants: a genus invariant, a pair of ideal classes from the fields of p and p^2 roots of unity, a unit in a certain finite ring, and a quadratic residue character mod p. He then proves this list completely characterizes the module up to isomorphism. He also gives formulas for the number of indecomposable lattices; the formulas can be evaluated in terms of p only if p is a regular prime. In other cases the orders of certain unit groups appear which are not known explicitly. The methods used in this paper require the explicit computation of certain Ext groups. He applied these results again in [68] to compute certain Ext groups over ZG when G is any cyclic p-group. In a joint paper [66], he and Ed Green applied diagram methods, which had been so successful in the study of finite dimensional algebras of finite type, to give a new proof of a theorem of Jacobinski which gives necessary and sufficient conditions for a commutative order to have finite type.

The advances made in representation theory during the 60's and 70's prompted Curtis and Reiner to begin working on a second book on the subject. Instead of a new edition of [1B], they decided upon a completely new book. As the outline began to take shape, they recognized that the amount of research done since the publication of their first book made this project considerably larger than their first; they planned for two volumes. Eventually they produced two 1500-page manuscripts [5B] and [6B]. The influence of these books has yet to be determined, but it seems clear that these will be the standard reference works in the subject for many years.

After the two short papers [67], [68] were completed, Reiner began the study of a preliminary version of Louis Solomon's paper (Advances in Math., 1977) in which Solomon defines a zeta-function for any **Z**-order in a semisimple **Q**-algebra. For such an order Λ, it is defined as

$$\zeta_\Lambda(s) = \sum_M (\Lambda : M)^{-s}$$

where M runs through all left ideals of Λ having finite index $(\Lambda : M)$. Solomon computed $\zeta_{\mathbf{Z}G}(s)$ for G of prime order p. This was a case Irving knew very well so naturally it interested him. He did the computation a different way and extended it to include the case where G was cyclic of order p^2 [70]. During the time he was working on this paper, Colin Bushnell (Kings's College, London) also was working on the conjectures in Solomon's paper. While Reiner used mainly algebraic methods, Bushnell was using the method of Tate and Weil's reworking of it to prove Solomon's conjecture in a special case. He sent a draft of his work to Reiner. This proved to be a fortunate coincidence; it was just the step Reiner needed since he knew how to reduce the general case to this special case. After an exchange of correspondence, they proved Solomon's conjectures in [69]. This proved to be the beginning of what was to become the most prolific and successful collaboration of Irving's career. They wrote twelve papers and five conference reports/announcements dealing with various partial zeta-functions (the M ranges over a restricted set of left ideals) in both the local and global cases, obtained product formulas, functional equations for the zeta-functions and the closely related L-functions associated with characters of the class group of the order, and analytic continuation of these functions. Much of the earlier work on integral representations, involving methods such as completions and localizations, was qualitative in nature, and the problem of actually computing integral representations remained a difficult one, even for special classes of finite groups. The new joint work with Bushnell added precise quantitative information about ways of counting isomorphism classes in the general case, and has added a new dimension to the theory.

Some of us in the department who, after hearing Irv lecture in seminars for years on modules and class groups, were surprised to hear his equally polished lectures, delivered in the usual organized, thorough manner, on Haar measure, p-adic harmonic analysis, and zeta-functions. These lectures were given as if he had spent his entire career working with these objects. The joint work with Bushnell is regarded by some as his best work. As he approached age sixty, an age when many would be content to take it easy and let the youngsters work on the hard problems, Irv mastered new ideas and methods which led to some of his deepest and most important contributions to the mathematical literature.

Conclusion

Irving Reiner died quietly in his sleep on October 28, 1986, after a long fight against cancer. His strong will to continue working was a source of inspiration to all his friends. After being confined to his home during the last year, he continued to receive visitors, took calls, saw some students, wrote letters, saw to the last details of the publication of [79] and [80], did some reviews for the problem section of the *American Mathematical Monthly*, and worked on [6B]. An indominable courage enabled him to give the impression that his illness was just an annoying problem that was temporarily delaying his return to mathematical research. His determination not to be changed by circumstance enabled him to project such an image of strength that one forgot that he was so terribly handicapped. We miss him very much but we have been enriched by knowing him and are inspired by his example.

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Gerald J. Janusz

Students of Irving Reiner

Lawrence S. Levy, 1961
Alan Troy, 1961
Alfredo Jones, 1962
Joseph H. Oppenheim, 1962
Myrna H.P. Pike, 1962
Lena Pu, 1964
Klaus Roggenkamp
Donald L. Stancl, 1966
Marshall M. Fraser, 1967
Thomas A. Hannula, 1967
Thomas G. Ralley, 1967
Paul R. Wilson, 1967
Janice Rose Zemanek, 1969
William H. Gustafson, 1970
How Ngee Ng, 1974
Gerald H. Cliff, 1975
Alberto G. Raggi-Cardenas, 1984
Hsin-Fong Chen, 1986
Anupam Srivastav

Reiner directed much of the work of K. Roggenkamp, but was not the official thesis advisor since Roggenkamp did not receive his degree from the University of Illinois.

The thesis advisor for M. Fraser was John Eagon who was then an instructor at Illinois; Reiner served as formal advisor and had only a minor role in the direction of the work.

Reiner had agreed to direct the thesis work of A. Srivastav and had introduced Srivastav to the subject matter of his thesis; Stephen Ullom directed Srivastav's thesis work.

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