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FACTORIZATION OF COMPOSITION OPERATORS THROUGH BLOCH TYPE SPACES

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Introduction

One way to compare function theoretic properties of analytic functions and functional analytic properties of linear operators is through "change of variable" type formulae. The corresponding operators are known as *composition operators*; they have been studied on various classical function spaces, in particular Hilbert function spaces. In this paper we continue investigating composition operators within the framework of general Hardy spaces on the open unit disk.

We are going to identify those composition operators C_φ, say from H^1 to H^1, which allow a canonical factorization $X_1 \to H^\beta$ for some $\beta > 0$ where X_1 is isometrically isomorphic to the classical Bloch space B. More precisely, our result characterizes φ's such that $f \mapsto f' \circ \varphi$ defines a bounded linear map $B \to H^\beta$, and we shall obtain analogous statements for H^p-spaces when $p > 1$ by replacing B with appropriate analytic Lipschitz spaces. As it will turn out, such operators are not only bounded but enjoy nuclearity properties similar to those of diagonal operators $l^\infty \to l^p$.

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Preliminaries

We shall mainly work with classical Hardy spaces

$$H^p \ (0 < p \leq \infty).$$

Recall that H^p consists of all analytic functions f on the open unit disk

$$D := \{z \in \mathbf{C}: |z| < 1\}$$

in the complex plane which satisfy

$$\|f\|_p := \sup_{r<1} \left(\int_{-\pi}^{\pi} |f(re^{it})|^p \frac{dt}{2\pi} \right)^{1/p} < \infty$$

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if $0 < p < \infty$, and which are bounded when $p = \infty$:

$$\|f\|_\infty := \sup_{z \in D} |f(z)| < \infty.$$

With respect to $\|\cdot\|_p$, H^p is a Banach space when $1 \leq p \leq \infty$, and a p-Banach space when $0 < p < 1$.

In what follows, m will always denote normalized Lebesgue measure on the unit circle $\mathbf{T} = \partial D$. We shall frequently identify, through the usual procedure of taking radial limits $t \mapsto \lim_{r \nearrow 1} f(re^{it})$, H^p with a closed subspace of $L^p(m)$; in this way, H^p becomes the closure of all polynomials in $L^p(m)$'s metric topology when $0 < p < \infty$, and in its weak* topology when $p = \infty$. We refer to Duren [4] for this and further results on H^p spaces to be utilized in the sequel without specific reference.

It will be convenient to denote by Φ the set of all analytic functions φ: $D \to \mathbf{C}$ such that $\varphi(D) \subset D$. In other words, Φ is obtained from the unit ball of H^∞ by just deleting the constant functions generated by the elements of $\mathbf{T}$.

Take any $\varphi \in \Phi$. It is a well known consequence of Littlewood's Subordination Principle (Duren [4], p. 10) that, regardless of how we select $0 < p \leq \infty$,

$$f \mapsto f \circ \varphi$$

defines a bounded linear operator

$$C_\varphi \colon H^p \to H^p,$$

the so-called *composition operator* induced by φ. For a discussion of various aspects of this notion, in particular in the Hilbert space setting, we refer to C.C. Cowen's recent survey article [2].

In the sequel, we consider only finite values of p; our interest will mainly be in those composition operators C_φ: $H^p \to H^p$ which have the property that, for some $1 \leq \beta < \infty$, $C_\varphi(H^p)$ is contained in $H^{\beta p}$. It turns out that this is a property depends on φ and β but *not* on p; it is therefore justified to label such composition operators *β-bounded.* There are composition operators with very special properties (e.g., belonging to the Hilbert-Schmidt class when considered as operators $H^2 \to H^2$) which fail to be β-bounded for every $\beta > 1$; see [6] and [7].

Order boundedness

As for Banach spaces, we shall employ standard terminology and notation.

Let X be a Banach space, μ a measure (≥ 0), and p a positive real number. We say that an operator u: $X \to L^p(\mu)$ is *order bounded* if it maps B_X, the unit ball of X, into an order interval of $L^p(\mu)$: we thus require the existence of a non-negative member h of $L^p(\mu)$ such that $|ux| \leq h$ μ-a.e. for each $x \in B_X$.

The question of when a composition operator is order bounded as a map from H^p to $L^q(m)$ was answered by H. Hunziker [6]:

THEOREM 1. *For each $0 < \beta < \infty$ and $\varphi \in \Phi$, the following are equivalent statements.*

(i) *For some $0 < p < \infty$, C_φ exists as an order bounded operator $H^p \to L^{\beta p}(m)$.*
(ii) *For every $0 < p < \infty$, C_φ exists as an order bounded operator $H^p \to L^{\beta p}(m)$.*
(iii) *$(1 - |\varphi|^2)^{-1}$ belongs to $L^\beta(m)$.*
(iv) *For some (all) $0 < r < \infty$, $(\|\varphi^n\|_r)_{n=0}^\infty$ is in the Lorentz sequence space $l^{(r/\beta),r}$.*

Let us say that C_φ is *β-order bounded* when this happens.

For $\beta \geq 1$, examples of β-bounded composition operators which are not β-order bounded can be found in [6]. For $0 < \beta < 1$ the existence of such examples is obvious: just note that every composition operator C_φ is continuous as a map from H^p to $H^{\beta p}$, whereas β-order boundedness of C_φ requires that $m\{|\varphi| = 1\} = 0$.

We also mention that γ-boundedness of C_φ implies β-order boundedness whenever $\gamma > \beta + 1$ [6]. Recently, the second named author [12] proved that this may fail when we only require $\gamma = \beta + 1$.

The case $\beta = 1$ was already investigated by J.H. Shapiro and P.D. Taylor [14]; they proved that condition (iii) of Theorem 1, with $\beta = 1$, characterizes the Hilbert-Schmidt composition operators on H^2. To get this result, only recall that the monomials z^n $(n = 0, 1, 2, \dots)$ form an orthonormal basis in H^2 and that the condition $(\|\varphi^n\|)_n = (\|C_\varphi z^n\|)_n \in l^2$ is just a standard characterization of a Hilbert-Schmidt operator.

This can easily be generalized. Recall that for $\alpha > -1$ the *weighted Dirichlet space* $\mathcal{D}_\alpha$ consists of all analytic functions $f(z) = \sum_{n=0}^\infty a_n z^n$ on D such that

$$\|f\|_{\mathcal{D}_\alpha} := \left(\sum_{n=0}^\infty \frac{|a_n|^2}{(n+1)^{\alpha-1}} \right)^{1/2}$$

is finite. $\mathcal{D}_\alpha$ is a Hilbert space with respect to $\|\cdot\|_{\mathcal{D}_\alpha}$; $\mathcal{D}_1$ is the Hardy space H^2, and $\mathcal{D}_2$ is the classical Bergman space $L_a^2(D, dx\,dy/\pi)$. Since the functions $z \to (n+1)^{(\alpha-1)/2} z^n$ form an orthonormal basis in $\mathcal{D}_\alpha$, we may state:

PROPOSITION 2. *Let $0 < \beta < \infty$ and $\varphi \in \Phi$ be given. C_φ is β-order bounded if and only if $f \to f \circ \varphi$ induces a Hilbert-Schmidt operator $\mathcal{D}_\beta \to H^2$.*

For further characterizations of β-order boundedness via factorization through Hilbert-Schmidt operators see [8].

Remark. It is easy to see that an operator $u\colon l^2 \to L^q(\mu)$ is order bounded if and only if, regardless of how we choose an orthonormal basis (e_n) in l^2, $(\Sigma_n |ue_n|^2)^{1/2}$ exists as an element of $L^q(\mu)$. Using this and arguments to be employed in the proof of Theorem 3 below, one may prove:

Let $\varphi \in \Phi$, $\alpha > 0$, $0 < \beta < \infty$ and $\gamma := \alpha\beta/2$ be given. C_φ is γ-order bounded if and only if it maps $\mathscr{D}_\alpha$ order boundedly to $L^\beta(m)$.

Bloch like spaces

Given $w \in D$, the *point evaluation*

$$\delta_w\colon H^p \to C\colon f \mapsto f(w)$$

is well known to be a bounded linear form. Its norm is known to be

$$\|\delta_w\|_{(H^p)^*} = \left(1 - |w|^2\right)^{-1/p};$$

see e.g. [7]. A slightly less precise result which would still be adequate for the purposes of this paper can be found in [4], p. 36.

Actually, δ_w can be identified with the composition operator whose symbol is the constant function $z \mapsto w$. We shall write

$$\delta_w^{(p)} := \left(1 - |w|^2\right)^{1/p} \cdot \delta_w$$

for the normalized functional in $(H^p)^*$ generated by δ_w.

Fix now $0 < p < \infty$. To conform with established notation (e.g., [15], III.H.27), we denote by $X_{1/p}$ the set of all analytic functions $f\colon D \to \mathbf{C}$ such that

$$\|f\|_{(1/p)} := \sup_{x \in D} (1 - |z|)^{1/p} |f(z)| < \infty.$$

This is a Banach space with norm $\|\cdot\|_{(1/p)}$, and from $\|f\|_{(1/p)} = \sup_{z \in D} |\langle \delta_z^{(p)}, f\rangle|$ we infer that H^p embeds contractively into $X_{1/p}$; by looking at the constant one function we see that the embedding actually has norm one.

Boundedness and compactness of composition operators on spaces $X_{1/p}$ have recently been studied by K. Madigan [9]. We take another direction and relate β-order boundedness of composition operators to a specific mapping property involving these spaces:

THEOREM 3. *Let $0 < \beta < \infty$ be given. The following statements about a function $\varphi \in \Phi$ are equivalent.*

(i) *C_φ is β-order bounded.*

(ii) *For each $1 \leq p < \infty$, $f \mapsto f \circ \varphi$ defines an order bounded operator $X_{1/p} \to L^{\beta p}(m)$.*

(iii) *For some $1 \leq p < \infty$, $f \mapsto f \circ \varphi$ defines an order bounded operator $X_{1/p} \to L^{\beta p}(m)$.*

(iv) *For some, and then all, $1 \leq p < \infty$, $f \mapsto f \circ \varphi$ defines a bounded operator $X_{1/p} \to L^{\beta p}(m)$.*

Naturally, we will denote the operators appearing in (ii), (iii), (iv) by C_φ, too.

Proof. All implications other than (i) $\Rightarrow$ (ii) and (iv) $\Rightarrow$ (i) are trivial, and (i) $\Rightarrow$ (ii) is easy: Suppose that C_φ is β-order bounded, that is, $(1 - |\varphi|^2)^{-1}$ is a member of $L^\beta(m)$. So $(1 - |\varphi|^2)^{-1/p}$ is in $L^{\beta p}(m)$ and it acts as an m-a.e. upper bound for the $|C_\varphi f|$'s when f is in the unit ball of $X_{1/p}$.

The proof of (iv) $\Rightarrow$ (i) is based on a lacunary sequences argument similar to the one used by Arazy-Fisher-Peetre [1, Theorem 16]. Fix $1 \leq p < \infty$ and write $\alpha = p^{-1}$ for notational convenience. We start by showing that the lacunary function

$$f(z) := \sum_{n=0}^{\infty} 2^{\alpha n} z^{2^n} \quad (z \in D)$$

belongs to X_α.

In fact, given $z \in D$, we have

$$\begin{aligned}
\frac{|f(z)|}{1 - |z|} &\leq \left(\sum_{n=0}^{\infty} |z|^n \right) \cdot \left(\sum_{n=0}^{\infty} 2^{n\alpha} |z|^{2^n} \right) \\
&= \sum_{n,k=0}^{\infty} 2^{k\alpha} |z|^{n+2^k} \\
&\leq \sum_{m=1}^{\infty} \sum_{k \leq \log_2 m} 2^{k\alpha} |z|^m \leq C_1 \sum_{m=1}^{\infty} m^\alpha |z|^m \quad (C_1 \leq 2^\alpha/(2^\alpha - 1)) \\
&= C_1 \sum_{m=1}^{\infty} \frac{m^\alpha \cdot m!}{\alpha \cdot (\alpha + 1) \cdot \dots \cdot (\alpha + m)} \\
&\quad \cdot \frac{\alpha \cdot (\alpha + 1) \cdot \ldots \cdot (\alpha + m)}{m!} \cdot |z|^m \\
&= C_1 \sum_{m=1}^{\infty} (\Gamma(\alpha) + \delta_m) \cdot \frac{\alpha \cdot (\alpha + 1) \cdot \dots \cdot (\alpha + m)}{m!} \cdot |z|^m,
\end{aligned}$$

where (δ_m) is a suitable null sequence of scalars. Consequently, there is a constant C_2 such that, for any $z \in D$,

$$\frac{|f(z)|}{1-|z|} \leq C_2 \sum_{m=1}^{\infty} \frac{\alpha \cdot (\alpha+1) \cdot \cdots \cdot (\alpha+m)}{m!} \cdot |z|^m = C_2 \cdot \alpha \cdot \left(\frac{1}{1-|z|}\right)^{\alpha+1}$$

hence

$$\sup_{x \in D} |f(z)| \cdot (1-|z|^2)^{1/p} \leq 2 \cdot \sup_{z \in D} |f(z)| \cdot (1-|z|)^{1/p} < \infty,$$

as asserted.

Set $K := \|f\|_{X_{1/p}}$. Use the Rademacher functions

$$r_n: [0,1] \to \mathbf{R}: t \mapsto \operatorname{sign} \sin(2^n \pi t)$$

to define, for each non-dyadic $t \in [0,1]$,

$$f_t(z) := \sum_{n=0}^{\infty} r_n(t) 2^{n/p} z^{2^n} \quad (z \in D).$$

Clearly, $\|f_t\|_{X_{1/p}} = K$ for each t. Khinchin's Inequality tells us that for each $0 < q < \infty$ there are constants $A_q, B_q > 0$ such that, however we choose finitely many scalars $a_1, \ldots, a_l$,

$$A_q \cdot \left(\sum_{k=1}^{l} |a_k|^2\right)^{1/2} \leq \left(\int_0^1 \left|\sum_{k=1}^{l} r_k(t) a_k\right|^q dt\right)^{1/q} \leq B_q \cdot \left(\sum_{k=1}^{l} |a_k|^2\right)^{1/2}.$$

Using this with $C := A_{\beta p}^{\beta p}$ we get

$$\begin{aligned} K^{\beta p} &\geq \int_0^1 \|C_\varphi f_t\|_{\beta p}^{\beta p}\, dt \\ &= \int_{-\pi}^{\pi} \int_0^1 \left|\sum_{n=0}^{\infty} r_n(t) 2^{n/p} \varphi(e^{i\theta})^{2^n}\right|^{\beta p} dt \frac{d\theta}{2\pi} \\ &\geq C \cdot \int_{-\pi}^{\pi} \left(\sum_{n=0}^{\infty} \left|2^{n/p} \varphi(e^{i\theta})^{2^n}\right|^2\right)^{\beta p/2} \frac{d\theta}{2\pi} \\ &\geq C \cdot \int_{-\pi}^{\pi} \left(\sum_{n=0}^{\infty} 2^{2n} |\varphi(e^{i\theta})|^{p \cdot 2^{n+1}}\right)^{\beta/2} \frac{d\theta}{2\pi} \quad (\text{since } p \geq 1). \end{aligned}$$

Write $I_n := \{k \in \mathbf{N}_0 \colon 2^n - 1 \le k < 2^{n+1} - 1\}$ $(n \in \mathbf{N}_0)$. For $0 < r < 1$, we find that

$$\begin{aligned} \sum_{n=0}^{\infty} 2^{2n} r^{2^{n+1}} &= \frac{1}{2} \cdot \sum_{n=0}^{\infty} \sum_{k \in I_n} w^{n+1} \cdot (r^2)^{2^n} \\ &\ge \frac{1}{2} \cdot \sum_{n=0}^{\infty} \sum_{k \in I_n} (k+1) \cdot r^{2k} \\ &= \frac{1}{2} \cdot \frac{1}{(1-r^2)^2}. \end{aligned}$$

Thus, with $\sigma := C \cdot 2^{-\beta/2}$,

$$K^{\beta p} \ge \sigma \cdot \int_{-\pi}^{\pi} \left(\frac{1}{1 - |\varphi(e^{i\theta})|^{2p}} \right)^{\beta} \frac{d\theta}{2\pi}.$$

It follows that $(1 - |\varphi|^2)^{-\beta}$ is m-integrable. QED

For $\alpha > 0$, the *Bloch type space* $\mathscr{B}_\alpha$ consists of all analytic functions $f\colon D \to C$ whose derivative belongs to X_α. By $f \mapsto \|f'\|_{(\alpha)}$ a seminorm with one dimensional kernel is defined on $\mathscr{B}_\alpha$; the corresponding normed quotient $\mathscr{B}_\alpha/\mathbf{C}$ is a Banach space, and $f \mapsto f'$ induces an isometric isomorphism of $\mathscr{B}_\alpha/\mathbf{C}$ onto X_α. The *classical Bloch space* is the space $\mathscr{B}_1$, and what corresponds to X_1^0 inside of $\mathscr{B}_1$ is known as the "*little Bloch space*". It is known (Duren [4], p. 74) that in case $0 < \alpha < 1$ membership in $\mathscr{B}_\alpha$ can be characterized in terms of a Lipschitz condition of order $1 - \alpha$; more precisely, an equivalent seminorm on $\mathscr{B}_\alpha$ is given by

$$f \mapsto \sup \left\{ \frac{|f(z) - f(w)|}{|z - w|^{1-\alpha}} : z, w \in D,\ z \ne w \right\};$$

see also K. Zhu [16].

COROLLARY 4. *Given $\varphi \in \Phi$ and $0 < \beta < \infty$, the following are equivalent.*

(i) *C_φ is β-order bounded.*

(ii) *For some, and then all, $1 \le p < \infty$, $f \mapsto f' \circ \varphi$ defines a bounded operator $\mathscr{B}_{1/p} \to H^p$.*

(iii) *For some, and then all, $1 \le p < \infty$, $f \mapsto f' \circ \varphi$ defines an order bounded operator $\mathscr{B}_{1/p} \to L^p(m)$.*

Recall that a Banach space operator $u\colon X \to Y$ is (*absolutely*) *p-summing* (for $1 \le p < \infty$) if it takes weak l^p sequences $(x_n)_n$ in X into strong l^p

sequences $(ux_n)_n$ in Y. In other words, we require $\sum_{n=1}^{\infty} \|ux_n\|^p$ to be finite whenever $\sum_{n=1}^{\infty} |\langle x^*, x_n\rangle|^p$ converges for each x^* from X^*, the dual of X. The operator $u\colon X \to Y$ is p-summing if and only if there is a probability measure μ such that u admits a factorization

$$u\colon X \xrightarrow{w} Z_\infty \xrightarrow{j} Z_p \xrightarrow{v} Y,$$

where Z_∞ is a subspace of $L^\infty(\mu)$, Z_p is Z_∞'s closure in $L^p(\mu)$, j is induced by the formal identity $j_p\colon L^\infty(\mu) \to L^p(\mu)$, and v and w are suitably chosen operators. The operator u is called *p-integral* if a factorization

$$k_Y u\colon X \xrightarrow{w} L^\infty(\mu) \xrightarrow{j_p} L^p(\mu) \xrightarrow{v} Y^{**}$$

is available, k_Y being the canonical embedding $Y \hookrightarrow Y^{**}$. A still smaller class consists of all *p-nuclear* operators: u is p-nuclear if it factors

$$u\colon X \xrightarrow{w} l^\infty \xrightarrow{\Delta} l^p \xrightarrow{v} Y$$

where Δ is a diagonal operator induced by a scalar l^p-sequence.

Thus formal inclusions $L^\infty(\mu) \to L^p(\mu)$ and diagonal operators $l^\infty \to l^p$ appear as the prototypes of all p-integral and p-nuclear operators, respectively.

A classical theorem due to A. Grothendieck [5] informs us that $u\colon X \to L^1(\mu)$ is order bounded if and only if it is 1-integral. The following extension to the case $1 < p < \infty$ goes back to L. Schwartz and S. Kwapień: if $u\colon X \to L^p(\mu)$ is order bounded, then it is p-integral, and if u's adjoint, u^*, is p-summing, then u is order bounded. The converse is known to fail in both cases. We refer to A. Pietsch's monograph [11] for details on summing, integral and nuclear operators.

It is well known that given $1 \le p < \infty$, every operator from an L^∞-space into an L^p-space is r-integral, where $r = 2$ when $1 \le p \le 2$ and $r > p$ when $2 < p < \infty$, and that this is best possible for general operators.

The situation changes if we consider composition operators. It was already shown in [8] that, for a certain range of p's and β's, βp-summability of the adjoint of a composition operator $C_\varphi\colon H^p \to H^p$ is equivalent to β-order boundedness. The next result shows that, in the present context, β-order bounded composition operators display a property known from diagonal operators: once they are defined, the appropriate nuclearity is automatic.

COROLLARY 5. *Suppose that $p \ge 1$ and $\beta p \ge 1$. If $\varphi \in \Phi$ is such that C_φ exists as a bounded operator $X_{1/p} \to H^{\beta p}$, then this operator is βp-nuclear.*

Proof. It is a consequence of Theorem 3 that C_φ is certainly βp-summing as an operator $X_{1/p} \to H^{\beta p}$. But since $X_{1/p}$ is isomorphic to l^∞ (cf. [15],

p. 90), this operator is actually βp-integral. If $\beta p = 1$, then we can settle the case by using the fact that H^1 is a separable dual space and so has the Radon-Nikodym property; cf. Diestel-Uhl [3], p. 79. In the general case, we may proceed as follows. Consider

$$X_{1/p}^0 := \left\{ f \in X_{1/p} \colon \lim_{|z| \to 1-} \left(1 - |z|^2\right)^{1/p} \cdot |f(z)| = 0 \right\}.$$

This is a closed subspace of $X^{1/p}$, more precisely, it is the closure of H^p in $X_{1/p}$. It was proved by L.A. Rubel and A.L. Shield [13] that $(X_{1/p}^0)^{**} = X_{1/p}$ and that $X_{1/p}^0$ has a separable dual; see also K. Zhu [16]. By a result of Persson ([10], Theorem 5), C_φ is βp-nuclear as an operator $X_{1/p}^0 \to H^{\beta p}$. But any βp-nuclear representation of this operator is also a βp-nuclear representation of $C_\varphi \colon X_{1/p} \to H^{\beta p}$. QED

Let us conclude by presenting the following problem. If we write an analytic function $f \colon D \to \mathbf{C}$ as a power series $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$, then we see that in general a composition operator C_φ must have the form

$$(*) \qquad C_\varphi = \sum_{n=0}^{\infty} \mu_n \otimes \varphi^n,$$

where $\mu_n \colon f \mapsto \hat{f}(n)$ is understood as a functional on the underlying domain space; to investigate the mode of convergence is part of discussing properties of the operator. It is plausible to expect that, in the situation of Corollary 4, $(*)$ should be a βp-nuclear representation. But, as can easily be verified, each μ_n is a bounded linear form on $X_{1/p}$ with norm of order $O(n^{1/p})$, so that we see from (iv) of Theorem 1 that $(*)$ doesn't supply us automatically with a βp-nuclear representation whenever $\varphi \in \Phi$ is such that C_φ maps $X_{1/p}$ to $H^{\beta p}$. So we may ask what a "natural" βp-nuclear representation of an operator as "natural" as a composition operator $C_\varphi \colon X_{1/p} \to H^{\beta p}$ looks like.

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