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CORRIGENDUM TO MY PAPER "THE RANKIN-SELBERG METHOD ON CONGRUENCE SUBGROUPS"

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The proof of the theorem in [1] uses the assertion that $\mathcal{D}_\Gamma = \bigcup \alpha_i \mathcal{D}$, $\alpha_i \in SL(2,\mathbb{Q})$. This assertion is incorrect. The correct assertion should be that $\mathcal{D}_\Gamma = K \bigcup \alpha_i \mathcal{D}$, $\alpha_i \in GL(2,\mathbb{Q})$, K a compact set. The proof should be adjusted as follows.

Let

$$\tilde{\mathcal{D}} = \Gamma_\infty \backslash \mathcal{H} - \mathcal{D} = \left(\bigcup_{\gamma \in \Gamma_\infty \backslash \Gamma} \gamma K \right) \bigcup \left(\bigcup_{i=1}^{h} \bigcup_{\substack{\gamma \in \Gamma_\infty \backslash \Gamma \\ \gamma \neq I \text{ or } \alpha_i \neq 1}} \gamma(\alpha_i \mathcal{D}) \right).$$

Equation (6) of [1] should be replaced by

$$\begin{aligned}
(1)\quad R_\infty(F,s) &= \int_0^\infty \int_0^1 [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2} \\
&= \int\int_{\Gamma_\infty \backslash \mathcal{H}} [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2} \\
&= \int\int_{\cup_{\gamma \in \Gamma_\infty \backslash \Gamma} \gamma \mathcal{D}_\Gamma} [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2} \\
&= \int\int_{\cup_{\gamma \in \Gamma_\infty \backslash \Gamma} \gamma (K \cup \cup_{i=1}^{h} \alpha_i \mathcal{D})} [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2} \\
&= \int\int_{\left(\cup_{\gamma \in \Gamma_\infty \backslash \Gamma} \gamma K\right) \cup \left(\cup_{i=1}^{h} \cup_{\substack{\gamma \in \Gamma_\infty \backslash \Gamma \\ \gamma \neq 1 \text{ or } \alpha_i \neq I}} \gamma(\alpha_i \mathcal{D}) \right)} F(z) y^s \frac{dx\,dy}{y^2} \\
&\quad - \int\int_{\tilde{\mathcal{D}}} \psi_\infty(y) y^s \frac{dx\,dy}{y^2} + \int\int_{\mathcal{D}} [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2} \\
&= \int\int_{\cup_{\gamma \in \Gamma_\infty \backslash \Gamma} \gamma K} F(z) y^s \frac{dx\,dy}{y^2} + \int\int_{\cup_{i=1}^{h} \cup_{\substack{\gamma \in \Gamma_\infty \backslash \Gamma \\ \gamma \neq 1 \text{ or } \alpha_i \neq I}} \gamma(\alpha_i \mathcal{D})} F(z) y^s \frac{dx\,dy}{y^2}
\end{aligned}$$

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$$- \int\int_{\tilde{\mathcal{D}}} \psi_\infty(y) y^s \frac{dx\,dy}{y^2} + \int\int_{\mathcal{D}} [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2}$$
$$= I_{\infty,K} + I,$$

where

$$I_{\infty,K}(s) = \int\int_{\cup_{\gamma\in\Gamma_\infty\backslash\Gamma}\gamma K} F(z) y^s \frac{dx\,dy}{y^2}$$
$$= \int\int_{K} F(z) E_\infty(z,s) \frac{dx\,dy}{y^2},$$

and

$$I = \int\int_{\substack{\cup\ \gamma\in\Gamma_\infty\backslash\Gamma\ \gamma(\alpha_i\mathcal{D}) \\ \gamma\neq 1 \text{ or } \alpha_i \neq I}} F(z) y^s \frac{dx\,dy}{y^2} - \int\int_{\tilde{\mathcal{D}}} \psi_\infty(y) y^s \frac{dx\,dy}{y^2}$$
$$+ \int\int_{\mathcal{D}} [F(z) - \psi_\infty(y)]\, y^s \frac{dx\,dy}{y^2}.$$

Now $I = I_{\infty,F}(s) + I_{\infty,F,\psi}(s) + I_{\infty,\psi}(s)$, following the same arguement as in [1]. We have

$$R_\infty(F,s) = I_{\infty,K}(s) + I_{\infty,F}(s) + I_{\infty,F,\psi}(s) + I_{\infty,\psi}(s). \tag{2}$$

To consider $I_{\infty,K} = \int\int_K F(z) E_\infty(z,s) \frac{dx\,dy}{y^2}$ at the general cusp κ, similar to the calculations in equation (13) of [1], we have

$$\begin{aligned} I_{\kappa,K} &= \int\int_{\alpha^{-1}K} f(z) E_\kappa(\alpha z, s) \frac{dx\,dy}{y^2} \\ &= \int\int_{\alpha^{-1}K} F(\alpha z) E_\kappa(\alpha z, s) \frac{dx\,dy}{y^2}. \\ &= \int\int_{K} F(z) E_\kappa(z,s) \frac{dx\,dy}{y^2} \end{aligned} \tag{3}$$

Thus, for any cusp κ, we have

$$R_\kappa(F,s) = I_{\kappa,K}(s) + I_{\kappa,F}(s) + I_{\kappa,F,\psi(s)} + I_{\kappa,\psi}(s), \tag{4}$$

and

$$\vec{R}_\kappa(F,s) = \vec{I}_{\kappa,K}(s) + \vec{I}_{\kappa,F}(s) + \vec{I}_{\kappa,F,\psi(s)} + \vec{I}_{\kappa,\psi}(s), \tag{5}$$

Each term in $\vec{R}_\kappa(F,s)$ has functional equation and analytic continuation.

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References

1. S. Dutta Gupta, *The Rankin-Selberg method on congruence subgroups.*, Illinois J. Math. **44** (2000), 95–103.

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