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ABOUT LOCALLY m-CONVEX ALGEBRAS WITH DENSE FINITELY GENERATED IDEALS

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Abstract. It is well known, as a consequence of a theorem of Richard Arens, that a commutative Fréchet locally m-convex algebra E with unit does not have dense finitely generated ideals. We shall see that this result can no longer be true if E is not complete and metrizable. We observe that the same is true for the theorem of Arens; that is, this theorem can no longer be true if E is not complete and metrizable. Moreover, several conditions for a unital commutative (not necessarily complete) locally m-convex algebra are given, for which all maximal ideals have codimension one.

1. Preliminaries

Let E be a unital topological algebra over $\mathbb{C}$ (the field of complex numbers) with separately continuous multiplication (in short, a topological algebra).

Let B be a complex commutative unital algebra. We denote by $\mathfrak{M}^{\#}(B)$ the set of all nonzero multiplicative linear functionals on B provided with the weak-star topology w^*. When B is a topological algebra, $\mathfrak{M}(B)$ denotes the topological subspace of $\mathfrak{M}^{\#}(B)$ consisting of all nonzero multiplicative continuous linear functionals on B provided with the weak-star topology w^*.

A locally pseudoconvex algebra is a topological algebra with a base of neighborhoods of zero consisting of balanced and pseudoconvex sets, that is, of sets U for which $\mu U \subset U$, whenever $|\mu| \leq 1$ and $U + U \subset \lambda U$ for some $\lambda \geq 2$. The topology of a locally pseudoconvex algebra can be defined by a family

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$\{p_\lambda : \lambda \in \Lambda\}$ of k_λ-homogeneous seminorms with $k_\lambda \in (0,1]$ (a seminorm p is called *k-homogeneous* ($k \in (0,1]$) if $p(\mu a) = |\mu|^k p(a)$ for any $a \in E$ and any scalar μ). A locally pseudoconvex algebra E is called *locally m-pseudoconvex* if every seminorm in the family $\{p_\lambda : \lambda \in \Lambda\}$ is submultiplicative, and it is called *locally convex* if $k_\lambda = 1$ for each $\lambda \in \Lambda$. A locally convex algebra is called *m-convex* if every seminorm in the family $\{p_\lambda : \lambda \in \Lambda\}$ is submultiplicative; that is, $p_\lambda(xy) \leq p_\lambda(x)p_\lambda(y)$ for each $\lambda \in \Lambda$. A metrizable and complete algebra is called a *Fréchet algebra*.

Let I be a (nonempty) upward-directed set with the partial order "$\prec$." So, for any $\alpha, \beta \in I$ there is a $\gamma \in I$ such that $\alpha \prec \gamma$ and $\beta \prec \gamma$. Let $(E_\alpha)_{\alpha\in I}$ be a family of algebras, and for every $\alpha, \beta \in I$ with $\alpha \prec \beta$ let

$$f_{\alpha\beta} : E_\beta \to E_\alpha$$

be a homomorphism such that

(1) $f_{\alpha\alpha} = \mathrm{id}_{E_\alpha}$ for every $\alpha \in I$

and

(2) $f_{\alpha\gamma} = f_{\alpha\beta} \circ f_{\beta\gamma}$ for any $\alpha, \beta, \gamma \in I$ such that $\alpha \prec \beta \prec \gamma$.

The family of algebras $(E_\alpha)_{\alpha\in I}$ with the maps $f_{\alpha\beta}$ defined above is called a *projective system of algebras*, and it is denoted by $(E_\alpha, f_{\alpha\beta})$.

Now, consider the Cartesian product algebra $F = \prod_{\alpha\in I} E_\alpha$ and the subset of F:

$$E = \big\{x = (x_\alpha) \in F : x_\alpha = f_{\alpha\beta}(x_\beta) \text{ if } \alpha \prec \beta \text{ in } I\big\}.$$

By definition of the algebra operations in F and the hypothesis for the maps $f_{\alpha\beta}$, E is a subalgebra of F. The algebra E is called the *projective limit algebra* of the given projective system, and it is denoted by

$$E = \varprojlim E_\alpha.$$

Given a projective system of algebras, we have a family of homomorphisms $\{\pi_\alpha : \alpha \in \Lambda\}$, such that

$$\pi_\alpha : E \to E_\alpha$$

for every $\alpha \in I$, where π_α is the restriction to E of the canonical projection map of F to E_α.

A projective system of topological algebras is a projective system of algebras $(E_\alpha, f_{\alpha\beta})$, where the E_α are topological algebras and the maps $f_{\alpha\beta}$ are continuous homomorphisms.

We endow $E = \varprojlim E_\alpha$ with the initial topology τ, defined by the maps π_α, and we call (E, τ) the *projective limit topological algebra* (see [3, p. 84]).

The following very important theorem is known as the *Arens–Michael theorem*, (see [3, pp. 88–90]).

Theorem 1.1. *Every Fréchet locally m-convex algebra E is (within a topological algebraic isomorphism) the projective limit of a sequence of Banach algebras $\widehat{E_n}$, where $\widehat{E_n}$ denotes the completion of $E/\ker(\|\cdot\|_n)$.*

Now, if we consider a Fréchet locally m-convex algebra E and for each $n \in \mathbb{N}$ the completion $\widehat{E_n}$ of $E/\ker(\|\cdot\|_\alpha)$, then

$$E = \varprojlim \widehat{E_n}.$$

This expression is called the *Arens–Michael decomposition of E.*

2. About a theorem of Arens

Let E be a commutative topological algebra with unit e. In what follows, we will give some definitions which are necessary to understand the well-known theorem of Arens (see [5]):

(i) $(x_1, x_2, \ldots, x_n) \in E^n$ is *regular* if the ideal

$$x_1E + x_2E + \cdots + x_nE$$

is equal to E.

(ii) $(x_1, x_2, \ldots, x_n) \in E^n$ is *topologically regular* if the closure of the ideal

$$x_1E + x_2E + \cdots + x_nE$$

is equal to E.

It is easy to see that if $(x_1, x_2, \ldots, x_n) \in E^n$ is regular, then $(x_1, x_2, \ldots, x_n)$ is topologically regular. The opposite implication is not true, in general.

In this section, E_α denotes the completion of $E/\ker(\|\cdot\|_\alpha)$, and e_α denotes the unital element in E_α.

Theorem 2.1 (Arens)**.** *Let E be a commutative, Fréchet m-convex algebra with unit e, and let $x_1, x_2, \ldots, x_N \in E$ such that $(\pi_n(x_1), \ldots, \pi_n(x_N))$ is regular in E_n for each $n \in \mathbb{N}$. Then, $(x_1, x_2, \ldots, x_N)$ is regular in E.*

Corollary 2.2. *Let E be a commutative, Fréchet m-convex algebra with unit e. Then, there are no dense finitely generated ideals in E.*

Proof. We suppose that there exists a dense finitely generated ideal I; then $\overline{I} = E$, and hence there exists $(y_i) \subset I$ such that $y_i \to e$. If $u_1, u_2, \ldots, u_n$ are the generators of I, then $y_i = x_1^i u_1 + \cdots + x_n^i u_n$ for each i, so $x_1^i u_1 + \cdots + x_n^i u_n \to e$ and by continuity of π_k we have that $\pi_k(x_1^i)\pi_k(u_1) + \cdots + \pi_k(x_n^i)\pi_k(u_n) \to e_k$. Now, as E_k is a Banach algebra, the set of invertible elements $\mathrm{Inv}(E_k)$ of E_k is open. Therefore, there is a neighborhood $O(e_k)$ of e_k such that $O(e_k) \subset \mathrm{Inv}(E_k)$ and by continuity of π_k there exists $m \in E$ such that $\pi_k(y_i) \in O(e_k) \subset \mathrm{Inv}(E_k)$ for every $i > m$. Hence, there exists $v \in E_k$ such that

$$v\big(\pi_k(x_1^i)\pi_k(u_1) + \cdots + \pi_k(x_n^i)\pi_k(u_n)\big) = e_k.$$

Hence, we conclude that $(\pi_k(u_1), \ldots, \pi_k(u_n))$ is regular for each k, and then by Theorem 2.1 $(u_1, \ldots, u_n)$ is regular in E. Therefore, $e \in I$ and $I = E$, which is a contradiction. □

For an algebra E, let $\operatorname{rad} E$ denote the topological radical, and let $\operatorname{comm} E$ denote the commutator ideal of E, that is, the closure of the two-sided ideal of E generated by the set $\{ab - ba : a, b \in E\}$. When $E/\operatorname{rad} E$ is commutative, E is called *topologically almost commutative*. In [1, Theorem 2], M. Abel and K. Jarosz generalized the previous result. They showed that if E is a complex unital locally m-pseudoconvex Fréchet algebra such that $\operatorname{comm} E \neq E$ or E is almost topologically commutative, then every proper finitely generated two-sided ideal in E is contained in a closed maximal two-sided ideal of E and hence there are no dense finitely generated two-sided ideals in E. We shall see that we cannot remove completeness or metrizability from the hypothesis of the theorem of Arens (or from [1, Theorem 2]).

Example 2.3. Let $\mathcal{E}$ be the entire function algebra on the complex plane endowed with the topology induced by the norm $\|f\|_N = \max_{|z|\leq N} |f(z)|$, where $f \in \mathcal{E}$ and N is a positive integer.

Let $f \in \mathcal{E}$ be nonconstant; then there exists $z_1 \in \mathbb{C}, |z_1| > N$ such that $|f(z_1)| > \|f\|_N$. We consider $c \in \mathbb{C}$ such that $\|f\|_N < c < |f(z_1)|$ and $F : \mathcal{E} \to \mathbb{C}$ defined by $F(f) = f(z_1)$; F is a linear multiplicative functional on $\mathcal{E}$; and

$$Z(F) = \big\{f \in \mathcal{E} : f(z_1) = 0\big\}$$

is a maximal ideal of $\mathcal{E}$. We have that F is not continuous since if we define $f_n = (\frac{f}{c})^n, n = 1, 2, \ldots,$ we obtain that $\|f_n\|_N \to 0$ and $F(f_n) \to \infty$. Hence, $Z(F)$ is not closed, and it is a dense maximal ideal. Moreover,

$$Z(f) = \big\{f \in \mathcal{E}\colon f(z) = (z - z_1)h(z) \text{ with } h \in \mathcal{E}\big\};$$

hence, we have that $Z(f) = (z - z_1)\mathcal{E}$ and $\overline{(z - z_1)\mathcal{E}} = \mathcal{E}$ and so $\mathcal{E}$ with this topology has dense finitely generated ideals. From Corollary 2.2 it follows that $(\mathcal{E}, \|\cdot\|_N)$ is not complete; therefore, we observe that we cannot remove the condition of completeness in the theorem of Arens.

Proposition 2.4. *Let E be a commutative m-convex algebra with unit e and $\mathfrak{M}(E)$ compact. Then, the following sentences are equivalent:*

1. *E does not have proper dense finitely generated ideals.*
2. *If $x_1, \ldots, x_n \in E$ and $\sum_{i=1}^n |f(x_i)| > 0$ for each $f \in \mathfrak{M}(E)$, then there exist elements $y_1, \ldots, y_n \in E$ such that $\sum_{i=1}^n x_i y_i = e$.*
3. *Every maximal ideal of E is closed.*

W. Żelazko [4, p. 297, Proposition 2] has the same result assuming completeness of E; we shall give the proof that (1) implies (2). The proofs of the other implications are the same as in the paper mentioned.

(1) implies (2). If we consider $x_1, \ldots, x_n$ such that $\sum_{i=1}^n |f(x_i)| > 0$ for each $f \in \mathfrak{M}(E)$, then $x_1, \ldots, x_n$ are not contained in any closed maximal ideal. We affirm that the ideal, I, generated by $x_1, \ldots, x_n$ is dense since if we assume that I is not dense, then $\bar{I} \neq E$ and then $E/\bar{I}$ is a commutative m-convex algebra such that $\mathfrak{M}(E/\bar{I}) \neq \emptyset$ (see [5, p. 95]). We can consider the quotient map $\pi : E \to E/\bar{I}$ which is continuous and $F \in \mathfrak{M}(E/\bar{I})$. We define $f = F \circ \pi$; f is by definition an element of $\mathfrak{M}(E)$ such that $|f(x_i)| = 0$ for every $i \in \{1, \ldots, n\}$, which is a

contradiction. So, since I is a dense finitely generated ideal, we have that $I = E$, and we conclude that there exist elements $y_1, \ldots, y_n \in E$ such that $\sum_{i=1}^{n} x_i y_i = e$.

Corollary 2.5. *Let E be a commutative, locally m-convex algebra with unit e; $\mathfrak{M}(E)$ is compact and without proper dense finitely generated ideals. If $x_1, \ldots, x_N \in E$ is such that $(\pi_\alpha(x_1), \ldots, \pi_\alpha(x_N))$ is regular in E_α for each $\alpha \in \Lambda$, then $(x_1, \ldots, x_N)$ is regular in E.*

Proof. Let $y_1, \ldots, y_N \in E_\alpha$ be such that $\pi_\alpha(x_1)y_1 + \cdots + \pi_\alpha(x_N)y_N = e_\alpha$. Then by Proposition 2.4, it is enough to prove that $f(x_i) \neq 0$ for some $i \in \{1, \ldots, N\}$, for each $f \in \mathfrak{M}(E)$. If we assume that there exists $F \in \mathfrak{M}(E)$ such that $F(x_i) = 0$ for each $i \in \{1, \ldots, N\}$, then we define $F_1 : E/\ker(\| \cdot \|_\alpha) \to \mathbb{C}$, $F_1(\pi_\alpha(x)) = F(x)$. Clearly, F_1 is a continuous, linear multiplicative function, and hence it can be extended continuously to E_α; that is, there exists a function $\widehat{F_1} \in \mathfrak{M}(E_\alpha)$ such that $\widehat{F_1}(\pi_\alpha(x_1)y_1 + \cdots + \pi_\alpha(x_N)y_N) = 0$ (by definition of $\widehat{F_1}$), but since $\pi_\alpha(x_1)y_1 + \cdots + \pi_\alpha(x_N)y_N = e_\alpha$, we have that

$$\widehat{F_1}\big(\pi_\alpha(x_1)y_1 + \cdots + \pi_\alpha(x_N)y_N\big) = 1,$$

which is a contradiction. So we conclude that $f(x_i) \neq 0$ for some $i \in \{1, \ldots, N\}$ and for every $f \in \mathfrak{M}(E)$. □

Corollary 2.6. *Let E be a commutative m-convex algebra with unit and $\mathfrak{M}(E)$ compact. Suppose that one of the following holds:*

(1) *E has maximal ideals which are not closed.*
(2) *$\mathfrak{M}(E) \neq \mathfrak{M}^{\#}(E)$.*

Then, the theorem of Arens for E does not follow.

Proof. We suppose that E satisfies (1). Then, by Proposition 2.4, we have that E has proper dense finitely generated ideals. Now, if (2) is true, then there are in E maximal ideals which are not closed. From there, the assumption of Corollary 2.2 does not follow, and hence the theorem of Arens does not follow. □

Corollary 2.7. *Let E be a commutative, Fréchet, m-convex algebra with unit e. If $\mathfrak{M}(E)$ is compact, then $\mathfrak{M}(E) = \mathfrak{M}^{\#}(E)$.*

Proof. Since the theorem of Arens is satisfied, by Corollary 2.6 we have that every maximal ideal of E is closed and that $\mathfrak{M}(E) = \mathfrak{M}^{\#}(E)$. □

A unital topological algebra (E, τ) is called a *Q-algebra* if the set of invertible elements is open. If (E, τ) is a Q-algebra, then E does not have proper dense finitely generated ideals.

We remember that if E is a unital topological algebra, then the spectrum of $x \in E$ is defined by

$$\sigma(x) = \{\lambda \in \mathbb{C} : x - \lambda e \text{ is not invertible}\}.$$

Proposition 2.8. *Let (E, τ) be a commutative, m-convex algebra with unit e and $\mathfrak{M}^{\#}(E)$ compact. Then the following sentences are equivalent:*

(1) *There exists any topology $\tau^\star$ stronger than τ such that $(E, \tau^\star)$ is a Q-algebra.*

(2) *If* $x_1, \ldots, x_n \in E$ *and* $\sum_{i=1}^n |f(x_i)| > 0$ *for each* $f \in \mathfrak{M}^\#(E)$*, then there exist elements* $y_1, \ldots, y_n \in E$ *such that* $\sum_{i=1}^n x_i y_i = e$*.*

(3) *Every maximal ideal of* E *is of codimension one.*

Proof. (1) implies (2). Let $x_1, \ldots, x_n \in E$ be such that $\sum_{i=1}^n |f(x_i)| > 0$ for every $f \in \mathfrak{M}^\#(E)$. Then, $x_1, \ldots, x_n$ is not contained in a maximal ideal. Thus, the ideal I generated by $x_1, \ldots, x_n$ is not proper; that is, $I = E$.

(2) implies (3). Since we have that if $x \in E$ such that $|f(x)| > 0$ for every $f \in \mathfrak{M}^\#(E)$, then there exists $y \in E$ such that $xy = e$. We consider $\lambda \in \sigma(x)$. Then $x - \lambda e$ is not invertible, and there exists $f \in \mathfrak{M}^\#(E)$ such that $f(x - \lambda e) = 0$; it is $f(x) = \lambda$. So, $\sigma(x) = \widehat{x}(\mathfrak{M}^\#(E))$, where $\widehat{x}(\mathfrak{M}^\#(E) = \{f(x) : f \in \mathfrak{M}^\#(E)\}$, and, as $\mathfrak{M}^\#(E)$ is compact, we have by [2, p. 54, Theorem 2] that every maximal ideal is of codimension one.

(3) implies (1). If $\lambda \in \sigma(x)$, then $x - \lambda e$ is not invertible and $x - \lambda e \in M$, where M is a maximal ideal of codimension one.

Then there exists $f \in \mathfrak{M}^\#(E)$ such that $f(x) = \lambda$. So, $\sigma(x) = \widehat{x}(\mathfrak{M}^\#(E))$ for each element $x \in E$, and hence by [2, p. 54, Theorem 2], there exists τ^* stronger than τ under which E is a Q-algebra. □

We note by [2, p. 54, Theorem 2] that the condition $\mathfrak{M}^\#(E)$ compact in the hypothesis in the previous proposition can be changed by the condition $\widehat{x}(\mathfrak{M}^\#(E))$ bounded for every $x \in E$ or by $\widehat{x}(\mathfrak{M}^\#(E))$ compact for every $x \in E$.

Proposition 2.8 allows us to give the following reformulation of Theorem 2 of [2].

Corollary 2.9. *The properties (i)–(vi) are equivalent for any commutative unital* m*-convex algebra* (E, τ)*.*

(i) E *is an* m*-convex* Q*-algebra under some topology* τ^**.*

(ii) E *is an advertibly complete* m*-convex algebra under some topology* τ^**, and the space* $\mathfrak{M}^\#(E)$ *is compact in the weak-star topology.*

(iii) *The space* $\mathfrak{M}^\#(E)$ *is compact in the weak-star topology, and for each element* $x \in E$ *the spectrum* $\sigma(x) = \widehat{x}(\mathfrak{M}^\#(E))$*.*

(iii)′ $\mathfrak{M}^\#(E)$ *is compact in the weak-star topology and for each* $(x_1, \ldots, x_n) \equiv \bar{x}$*,* $\sigma(\bar{x}) = \widehat{\bar{x}}(\mathfrak{M}^\#(E))$*, where*

$$\sigma(\bar{x}) = \big\{(\lambda_1, \ldots, \lambda_n : (x_1 - \lambda_1 e, \ldots, x_n - \lambda_n e) \textit{ is not regular}\big\}$$

and

$$\widehat{\bar{x}}\big(\mathfrak{M}^\#(E)\big) = \big\{\big(f(x_1), \ldots, f(x_n)\big) : f \in \mathfrak{M}^\#(E)\big\}.$$

(iv) E *is spectrally bounded and* $\sigma(x) = \widehat{x}(\mathfrak{M}^\#(E))$*.*

(v) *For each* $x \in E$ *the spectrum* $\sigma(x)$ *is compact and* $\sigma(x) = \widehat{x}(\mathfrak{M}^\#(E))$*.*

(vi) E *is an* m*-convex* Q*-algebra under some topology* $\tau^\star$ *stronger than* τ*.*

Furthermore, each one of these properties implies the following:

(vii) *Every maximal ideal of* E *is of codimension one.*

Proof. It is enough to prove that (iii) and (iii)′ are equivalent. Clearly, (iii)′ implies (iii). We consider $(\lambda_1, \ldots, \lambda_n) \in \sigma(\bar{x})$; then the ideal generated by $(x_1 -$

$\lambda_1 e, \dots, x_n - \lambda_n e)$ is contained in a maximal ideal M which is closed since, by (i), E is a Q-algebra and it is of codimension one by (vii). Then the projection

$$F : E \to E/M \cong \mathbb{C}$$

is a linear multiplicative functional of E. Therefore, $F(x_i - \lambda_i) = 0$ for each $i \in \{1, \dots, n\}$ and $F(x_i) = \lambda_i$ for each $i \in \{1, \dots, n\}$.

Now, we consider $(\lambda_1, \dots, \lambda_n) \in \widehat{\bar{x}}(\mathfrak{M}^{\#}(E))$; then $\lambda_i = F(x_i)$ for some $F \in \mathfrak{M}^{\#}(E)$ and $i \in \{1, \dots, n\}$. We affirm that $(x_1 - F(x_1)e, \dots, x_n - F(x_n)e)$ is not regular, since if $(x_1 - F(x_1)e, \dots, x_n - F(x_n)e)$ is regular, then there exist $y_1, \dots, y_n \in E$ such that

$$\sum_{i=1}^{n} \big(x_i - F(x_i)e\big) y_i = e.$$

Hence, $0 = F(\sum_{i=1}^{n} (x_i - F(x_i)e) y_i) = 1$, which is a contradiction. So,

$$(\lambda_1, \dots, \lambda_n) \in \sigma(\bar{x}). \qquad \square$$

Next, we show an example of a commutative, complete m-convex algebra with dense finitely generated ideals, which was studied by W. Żelazko [4]. This is an example that shows that metrizability is needed in the hypothesis of the theorem of Arens.

Example 2.10. We consider

$$D = \Big\{(z_1, z_2) \in \mathbb{C}^2 : \frac{1}{2} \le |z_1|^2 + |z_2|^2 \le 1\Big\}$$

and

$$D_0 = \Big\{(z_1, z_2) \in \mathbb{C}^2 : \frac{1}{2} \le |z_1|^2 + |z_2|^2 < 1\Big\}.$$

Let E be the algebra of continuous functions on D and holomorphic on D_0. We define a seminorm $\|\cdot\|_\alpha$ for each convergent sequence

$$\alpha = \big\{(z_1^{(n)}, z_2^{(n)})\big\} \subset D \quad \text{such that} \quad \lim_n (z_1^{(n)}, z_2^{(n)}) = (z_1, z_2), \quad z_1 \ne z_i, i = 1, 2$$

and

$$\|f\|_\alpha = \begin{cases} \sup_n |f(z_1^{(n)}, z_2^{(n)})|, & (z_1, z_2) \in D \setminus D_0, \\ \max(\sup_n |f(z_1^{(n)}, z_2^{(n)})|, \sup_n |f \frac{f(z_1, z_2^n) - f(z_1, z_2)}{z_2^n - z_2}|, \sup_n |\frac{f(z_1^n, z_2) - f(z_1, z_2)}{z_1^n - z_1}|) \\ \quad \text{in another case.} \end{cases}$$

Then E is a complete m-convex algebra with the topology endowed by the seminorms $\{\|f\|_\alpha\}_\alpha$. Since every holomorphic function in D_0 can be extended only to one holomorphic function on

$$D_1 = \big\{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 < 1\big\},$$

we have that $\mathfrak{M}^{\#}(E) = \overline{D_1}$ and $\mathfrak{M}(E) = D$. Moreover, Corollary 2.6 follows since $\mathfrak{M}(E)$ is compact. Thus, E has dense finitely generated ideals.

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