Ann. Funct. Anal. 7 (2016), no. 2, 254–260
http://dx.doi.org/10.1215/20088752-3475508
ISSN: 2008-8752 (electronic)
http://projecteuclid.org/afa

ADDITIVE MAPS PRESERVING THE SEMI-FREDHOLM DOMAIN IN SPECTRUM

WEIJUAN SHI[1] and GUOXING JI[2*]

Communicated by M. Ptak

Abstract. Let $\mathcal{X}$ be an infinite-dimensional complex Banach space and let $\mathcal{B}(\mathcal{X})$ be the algebra of all bounded linear operators. In this article we show that an additive surjective map φ on $\mathcal{B}(\mathcal{X})$ preserving the semi-Fredholm domain in spectrum is an automorphism or an antiautomorphism on $\mathcal{B}(\mathcal{X})$.

1. Introduction

Preserver problems aim to characterize those linear or nonlinear maps on operator algebras preserving certain properties, subsets, or relations. One of the most famous problems in this direction is Kaplansky's problem (see [6]) asking whether every surjective unital invertibility-preserving linear map between two semisimple Banach algebras is a Jordan homomorphism. This problem was first solved in the finite-dimensional case. J. Dieudonné in [4] and Marcus and Purves in [7] proved that every unital invertibility-preserving linear map on a complex matrix algebra is either an inner automorphism or an inner antiautomorphism. This result was later extended to the algebra of all bounded linear operators on a Banach space by A. R. Sourour in [10] and to von Neumann algebras by B. Aupetit in [1]. As we know, spectrum is a very fundamental and key concept in operator theory. Hence many authors (see [1], [3], [5]) are interested in preserver problems related to the spectrum as well as to certain parts of the spectrum. For example, in [3], Cui and Hou showed that additive maps on standard operator algebras preserving parts of the spectrum are either isomorphisms or anti-isomorphisms. It is known

Received Mar. 12, 2015; Accepted Jun. 29, 2015.
*Corresponding author.
2010 *Mathematics Subject Classification.* Primary 47B48; Secondary 47A10.
Keywords. semi-Fredholm domain, additive map, additive preserver.

that the notion of semi-Fredholm operators have close relationship with compact operators. Several authors have studied the linear maps which preserve Fredholm operators, semi-Fredholm operators, or upper semi-Weyl operators in both directions (see [2], [8]). They showed that such maps preserve the ideal of compact operators in both directions, and their induced maps on the Calkin algebra are Jordan automorphisms. For example, in [2], the authors discuss the linear surjective maps preserving upper semi-Weyl operators, and show that their induced maps on the Calkin algebra are Jordan automorphisms. However, the problem of determining the structure of the maps itself hasn't been solved. In this paper, we combine the spectrum with semi-Fredholm domain, and consider an additive map which preserves the intersection of semi-Fredholm domain and spectrum. How does the semi-Fredholm domain in spectrum influence the structure of automorphisms on the algebra of all bounded linear operators on a Banach (or Hilbert) space? Let $\mathcal{X}$ be a complex infinite-dimensional Banach space, and let $\mathcal{B}(\mathcal{X})$ be the algebra of all bounded linear operators on $\mathcal{X}$. We will show that the semi-Fredholm domain in spectrum is an invariant of an automorphism or an anti-automorphism on $\mathcal{B}(\mathcal{X})$.

Let $T \in \mathcal{B}(\mathcal{X})$. We denote by $\mathcal{X}^*$, T^*, $N(T)$, and $R(T)$ the dual space of $\mathcal{X}$, the conjugate operator, null space, and range of T, respectively. For a closed subspace M of $\mathcal{X}$, $\dim M$ is the dimension of M and $\operatorname{codim} M$ is the dimension of the complemented subspace of M. Recall that an operator T is called *upper semi-Fredholm* if $\dim N(T) < \infty$ and if $R(T)$ is closed, while T is called a *lower semi-Fredholm operator* if $\operatorname{codim} R(T) < \infty$. The semi-Fredholm operator is upper semi-Fredholm or a lower semi-Fredholm operator. If both $\dim N(T) < \infty$ and $\operatorname{codim} R(T) < \infty$ are finite, then T is said to be a Fredholm operator. If T is upper (lower) semi-Fredholm, then we define $\operatorname{ind} T$ the index of T denote by $\operatorname{ind}(T) = \dim N(T) - \operatorname{codim} R(T)$. The operator T is Weyl if it is Fredholm of index zero. Denote the spectrum and the point spectrum of T by

$$\sigma(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is not invertible}\},$$
$$\sigma_p(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is not injective}\};$$

the semi-Fredholm domain, the Fredholm domain, and the Weyl domain of T are defined by

$$\rho_{SF}(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is semi-Fredholm operator}\},$$
$$\rho_e(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is Fredholm operator}\},$$
$$\rho_w(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is Weyl operator}\}.$$

The semi-Fredholm domain in the spectrum of T will be denoted by $\sigma_1(T)$.

That is,

$$\sigma_1(T) = \big\{\lambda \in \sigma(T) : T - \lambda I \text{ is semi-Fredholm operator}\big\}.$$

Then we obtain that $\sigma_1(T) = \sigma(T) \cap \rho_{SF}(T)$. In the present article, we characterize an additive surjective map φ on $\mathcal{B}(\mathcal{X})$ which preserves the semi-Fredholm domain in spectrum, that is $\sigma_1(\varphi(T)) = \sigma_1(T)$ for all $T \in \mathcal{B}(\mathcal{X})$. We obtain that such a surjective map is an automorphism or an antiautomorphism on $\mathcal{B}(\mathcal{X})$.

2. Main results

First, we fix some notation. Let h be a ring automorphism of $\mathbb{C}$. A mapping $A : \mathcal{X} \to \mathcal{X}$ will be called *h-quasilinear* if it is additive and if the relation $A(\lambda x) = h(\lambda)Ax$ holds for all complex numbers λ and all $x \in \mathcal{X}$. For $x \in \mathcal{X}$, $f \in \mathcal{X}^*$, we denote by $x \otimes f$ the bounded linear rank 1 operator defined by $(x \otimes f)y = f(y)x$, $\forall y \in \mathcal{X}$. For a subset M of $\mathcal{X}$, $\bigvee\{M\}$ denotes the closed subspace spanned by M. Now, we give our main result.

Theorem 2.1. *Let $\varphi : \mathcal{B}(\mathcal{X}) \to \mathcal{B}(\mathcal{X})$ be a surjective additive map. If $\sigma_1(\varphi(T)) = \sigma_1(T)$ for all $T \in \mathcal{B}(\mathcal{X})$, then either*

1. *there is an invertible operator $A \in \mathcal{B}(\mathcal{X})$ such that $\varphi(T) = ATA^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$, or*
2. *there is a bounded invertible linear operator $C : \mathcal{X}^* \to \mathcal{X}$ such that $\varphi(T) = CT^*C^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$. In this case, $\mathcal{X}$ must be a reflexive space.*

For the Fredholm domain (Weyl domain) in spectrum, we can get the following result. Denote

$$\sigma_2(T) = \big\{\lambda \in \sigma(T) : T - \lambda I \text{ is Weyl operator}\big\},$$
$$\sigma_3(T) = \big\{\lambda \in \sigma(T) : T - \lambda I \text{ is Fredholm operator}\big\}.$$

Corollary 2.2. *Let $\varphi : \mathcal{B}(\mathcal{X}) \to \mathcal{B}(\mathcal{X})$ be a surjective additive map; then the following statements are equivalent:*

1. *$\sigma_1(\varphi(T)) = \sigma_1(T)$ for all $T \in \mathcal{B}(\mathcal{X})$,*
2. *$\sigma_2(\varphi(T)) = \sigma_2(T)$ for all $T \in \mathcal{B}(\mathcal{X})$,*
3. *$\sigma_3(\varphi(T)) = \sigma_3(T)$ for all $T \in \mathcal{B}(\mathcal{X})$,*
4. *there is an invertible operator $A \in \mathcal{B}(\mathcal{X})$ such that $\varphi(T) = ATA^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$ or there is a bounded invertible linear operator $C : \mathcal{X}^* \to \mathcal{X}$ such that $\varphi(T) = CT^*C^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$. The last case occurs only if $\mathcal{X}$ is a reflexive space.*

It is known that the Fredholm index is a very important concept for the Fredholm operator, and thus we also discuss the idea that φ preserves the part of the semi-Fredholm domain with nonpositive (nonnegative) index in spectrum. In this case, it implies that $\operatorname{ind}(\varphi(T)) = \operatorname{ind}(T)$ if T is a semi-Fredholm operator; it follows that the second case in Theorem 2.1 cannot occur if $\mathcal{X}$ is not reflexive, or if $B(\mathcal{X})$ contains a semi-Fredholm operator with nonzero index.

Let

$$\sigma_4(T) = \big\{\lambda \in \sigma(T) : T - \lambda I \text{ is semi-Fredholm operator and } \operatorname{ind}(T - \lambda I) \leq 0\big\},$$
$$\sigma_5(T) = \big\{\lambda \in \sigma(T) : T - \lambda I \text{ is semi-Fredholm operator and } \operatorname{ind}(T - \lambda I) \geq 0\big\}.$$

Then we have the following result.

Corollary 2.3. *Let $\varphi : \mathcal{B}(\mathcal{X}) \to \mathcal{B}(\mathcal{X})$ be a surjective additive map, then the following statements are equivalent:*

1. *$\sigma_4(\varphi(T)) = \sigma_4(T)$ for all $T \in \mathcal{B}(\mathcal{X})$,*
2. *$\sigma_5(\varphi(T)) = \sigma_5(T)$ for all $T \in \mathcal{B}(\mathcal{X})$,*

(3) *there is an invertible operator* $A \in \mathcal{B}(\mathcal{X})$ *such that* $\varphi(T) = ATA^{-1}$ *for all* $T \in \mathcal{B}(\mathcal{X})$ *or there is a bounded invertible linear operator* $C : \mathcal{X}^* \to \mathcal{X}$ *such that* $\varphi(T) = CT^*C^{-1}$ *for all* $T \in \mathcal{B}(\mathcal{X})$*. The last case cannot occur if* $\mathcal{X}$ *is not reflexive, or if* $B(\mathcal{X})$ *contains a semi-Fredholm operator with nonzero index.*

The proof will be given only for Theorem 2.1, but also works for Corollary 2.2 and 2.3. First, recall that invertible operators are Fredholm of index zero, and so are their rank 1 perturbations. In order to give the proof, we begin with a lemma.

Lemma 2.4. *Let* $A, B \in \mathcal{B}(\mathcal{X})$*. If* $\sigma_1(A+F) = \sigma_1(B+F)$ *for all rank* 1 *operator* $F \in \mathcal{B}(\mathcal{X})$*, then* $A = B$*.*

Proof. For any nonzero vector $x \in \mathcal{X}$, $N = \{f \in \mathcal{X}^* \mid f(x) = 1\}$, fix a scalar $\alpha \in \mathbb{C}$ such that $\alpha > \|A\| + \|B\|$. For any $f \in N$, we define an operator

$$F_f = (A - \alpha I)x \otimes f.$$

Then $F_f x = Ax - \alpha x$, and hence $\alpha \in \sigma(A - F_f)$. So we have $\alpha \in \sigma_1(A - F_f)$ from the fact that $A - F_f - \alpha I$ is a semi-Fredholm operator; it follows that $\alpha \in \sigma_1(B - F_f)$. It is known that $B - F_f - \alpha I$ is a Fredholm operator of index zero, and thus $\alpha \in \sigma_p(B - F_f)$. We obtain that there exists a nonzero vector y_f such that $(B - F_f)y_f = \alpha y_f$. Then

$$y_f = f(y_f)(B - \alpha I)^{-1}(A - \alpha I)x.$$

Let $y = (B-\alpha I)^{-1}(A-\alpha I)x$; then we know that $(B-F_f)y = \alpha y$ for any $f \in N$. We assert that y and x is linearly dependent, otherwise if y and x is linearly independent, then there exists some $f_0 \in N$ such that $f_0(y) = 0$. It implies that $By = \alpha y$, which is in contradiction to the fact that $\alpha > \|A\| + \|B\|$. It follows that $(B - F_f)x = \alpha x$. Consequently, $Ax = Bx$. From the arbitrariness of x, we obtain that $A = B$. □

Proof of Theorem 2.1. We will prove the theorem in four steps.

Step 1: φ *is injective.*

Let $\varphi(T) = 0$ for some $T \neq 0$. Then we can find a vector $x_0 \in \mathcal{X}$ such that $Tx_0 \neq 0$. So there exists $f \in \mathcal{X}^*$ such that $f(x_0) = 1$ and $f(Tx_0) \neq 0$. Fix a scalar λ_0 such that $|\lambda_0| > \|T\|$. We define an operator $S \in B(\mathcal{X})$:

$$S = (Tx_0 + \lambda_0 x_0) \otimes f.$$

Then $(S - T)x_0 = \lambda_0 x_0$. It follows that $\lambda_0 \in \sigma_1(S - T)$. Thus

$$\lambda_0 \in \sigma_1\big(\varphi(S - T)\big) = \sigma_1\big(\varphi(S)\big) = \sigma_1(S).$$

But $\sigma_1(S) = \{\lambda_0 + f(Tx_0)\}$, a contradiction.

Step 2: φ *preserves rank* 1 *operators in both directions.*

Let $P \in B(\mathcal{X})$ be a rank 1 operator and let $\varphi(P) = Q$. We will prove that Q is rank 1. Assume that rank $Q > 1$; then there exist two linearly independent vectors $y_1, y_2 \in R(Q)$, and hence we can choose two linearly independent vectors $x_1, x_2 \in \mathcal{X}$ such that $Qx_1 = y_1$, $Qx_2 = y_2$. Let $M = \bigvee\{x_1, x_2, y_1, y_2\}$; then

$\mathcal{X} = M \oplus K$, where K is a closed subspace. Fix a scalar λ_0 such that $|\lambda_0| > 2\|Q\|$. There are three cases for $\dim M$.

(1) If $\dim M = 2$, then $M = \bigvee\{x_1, x_2\}$. We define an operator $R \in B(H)$ by

$$\begin{cases} Rx_1 = \lambda_0 x_1 - y_1, \\ Rx_2 = \lambda_0 x_2 - 2y_2, \\ Rz = 0 \quad \forall z \in K. \end{cases}$$

Then we have $\lambda_0 \in \sigma(R+Q) \cap \sigma(R+2Q)$ and $\lambda_0 \notin \sigma(R)$.

(2) If $\dim M = 3$, then without loss of generality, let $M = \bigvee\{x_1, x_2, y_1\}$, and suppose that $y_2 = \xi_1 x_1 + \xi_2 x_2 + \xi_3 y_1$, where at least one of $\xi_1, \xi_2, \xi_3 \in \mathbb{C}$ is nonzero. If $\xi_1 \neq 0$, we define an operator $R \in B(\mathcal{X})$ by

$$\begin{cases} Rx_1 = \lambda_0 x_1 - y_1, \\ Rx_2 = \lambda_0 x_2 - 2y_2, \\ Ry_1 = x_2, \\ Rz = 0 \quad \forall z \in K. \end{cases}$$

Then $\lambda_0 \in \sigma(R+Q) \cap \sigma(R+2Q)$. We claim that $R - \lambda_0 I$ is injective. If $x \in N(R - \lambda_0 I)$, then $Rx = \lambda_0 x$. Let $x = x_m + x_k$, where $x_m \in M$, $x_k \in K$. Then $Rx_m = Rx = \lambda_0 x_m + \lambda_0 x_k$; this implies that $x_k = 0$ and that $x = x_m \in M$. So let $x = ax_1 + bx_2 + cy_1$, where $a, b, c \in \mathbb{C}$. We get

$$0 = (R - \lambda_0 I)x = -ay_1 - 2by_2 + cx_2 - c\lambda_0 y_1.$$

Then

$$-2b\xi_1 x_1 + (c - 2b\xi_2)x_2 - (a + 2b\xi_3 + c\lambda_0)y_1 = 0.$$

It follows that $a = b = c = 0$ since the vectors x_1, x_2, y_1 are linearly independent and $\xi_1 \neq 0$. This shows that $R - \lambda_0 I$ is invertible, which means that $\lambda_0 \notin \sigma(R)$.

If $\xi_1 = 0$, then $\xi_2 \neq 0$ since the vectors y_1, y_2 are linearly independent. We define an operator $R \in B(\mathcal{X})$ by

$$\begin{cases} Rx_1 = \lambda_0 x_1 - y_1, \\ Rx_2 = \lambda_0 x_2 - 2y_2, \\ Ry_1 = x_1, \\ Rz = 0 \quad \forall z \in K. \end{cases}$$

Then we also obtain $\lambda_0 \in \sigma(R+Q) \cap \sigma(R+2Q)$ and $\lambda_0 \notin \sigma(R)$.

(3) If $\dim M = 4$, then $M = \bigvee\{x_1, x_2, y_1, y_2\}$. We define an operator $R \in B(\mathcal{X})$ by

$$\begin{cases} Rx_1 = \lambda_0 x_1 - y_1, \\ Rx_2 = \lambda_0 x_2 - 2y_2, \\ Ry_1 = x_1, \\ Ry_2 = x_2, \\ Rz = 0 \quad \forall z \in K. \end{cases}$$

Then we have $\lambda_0 \in \sigma(R+Q) \cap \sigma(R+2Q)$. Using a similar proof of the above, we can get $\lambda_0 \notin \sigma(R)$.

Therefore, there exists a finite-rank operator $R \in B(\mathcal{X})$ and λ_0 satisfying $|\lambda_0| > 2\|Q\|$, $\lambda_0 \in \sigma(R+Q) \cap \sigma(R+2Q)$ and $\lambda_0 \notin \sigma(R)$. We know that $R+Q-\lambda_0 I$ and $R+2Q-\lambda_0 I$ are both semi-Fredholm operators, and thus

$$\begin{aligned}\lambda_0 &\in \sigma_1(R+Q) \cap \sigma_1(R+2Q)\\ &= \sigma_1(R_0+P) \cap \sigma_1(R_0+2P)\\ &\subseteq \sigma(R_0+P) \cap \sigma(R_0+2P).\end{aligned}$$

Here $R_0 \in B(\mathcal{X})$ satisfying $R = \varphi(R_0)$ and it does exist since φ is surjective. By Theorem 1 in [5], we must have $\sigma(R_0+P) \cap \sigma(R_0+2P) \subseteq \sigma(R_0)$, and then $\lambda_0 \in \sigma(R_0)$. This implies that $\lambda_0 \in \sigma_1(R_0)$ since $\lambda_0 \in \sigma_1(R_0+P)$ and P is finite-rank, and thus $\lambda_0 \in \sigma_1(R)$. This contradicts the fact that $\lambda_0 \notin \sigma(R)$. Therefore, we have rank $Q = 1$. Since φ is bijective and φ^{-1} has the same property as φ, it follows that φ preserves the set of operators of rank 1 in both directions. By using Theorem 3.3 in [9], we can see that there is a ring automorphism $h : \mathbb{C} \to \mathbb{C}$, and there are either h-quasilinear bijective mappings $A : \mathcal{X} \to \mathcal{X}$ and $C : \mathcal{X}^* \to \mathcal{X}^*$ such that

$$\varphi(x \otimes f) = Ax \otimes Cf \quad \text{for all } x \in \mathcal{X} \text{ and } f \in \mathcal{X}^*,$$

or there are h-quasilinear bijective mappings $A : \mathcal{X} \to \mathcal{X}^*$ and $C : \mathcal{X}^* \to \mathcal{X}$ such that

$$\varphi(x \otimes f) = Cf \otimes Ax \quad \text{for all } x \in \mathcal{X} \text{ and } f \in \mathcal{X}^*.$$

Step 3: φ preserves idempotents of rank 1 and their linear spans in both directions.

Let F is an idempotent of rank 1. Then $\sigma_1(F) = \{1\}$, and so $\sigma_1(\varphi(F)) = \{1\}$. This implies that $\varphi(F)$ is an idempotent of rank 1. Moreover, we have that φ is h-quasilinear on rank 1 operators by step 2. So we get that φ preserves idempotents of rank 1 and their linear spans in both directions.

Therefore, it follows from the Main Theorem in [9] that: (1) there is an invertible operator $A \in \mathcal{B}(\mathcal{X})$ such that $\varphi(T) = AFA^{-1}$ for all finite rank operators $F \in \mathcal{B}(\mathcal{X})$, or (2) there is a bounded invertible linear operator $C : X^* \to X$ such that $\varphi(T) = CT^*C^{-1}$ for all finite rank operators $F \in \mathcal{B}(\mathcal{X})$. In this case $\mathcal{X}$ must be a reflexive space.

Step 4: we extend the result of Step 3 to $\mathcal{B}(\mathcal{X})$. There is an invertible operator $A \in \mathcal{B}(\mathcal{X})$ such that $\varphi(T) = ATA^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$ or there is a bounded invertible linear operator $C : X^* \to X$ such that $\varphi(T) = CT^*C^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$.

Assume that (1) holds. Let $T \in \mathcal{B}(\mathcal{X})$ and for any rank 1 operator F, we have

$$\begin{aligned}\sigma_1(T+F) &= \sigma_1\big(\varphi(T)+\varphi(F)\big)\\ &= \sigma_1\big(\varphi(T)+AFA^{-1}\big)\\ &= \sigma_1\big(A\big(A^{-1}\varphi(T)A+F\big)A^{-1}\big)\\ &= \sigma_1\big(A^{-1}\varphi(T)A+F\big).\end{aligned}$$

Then we obtain that $T = A^{-1}\varphi(T)A$ by the Lemma 2.4. Consequently, $\varphi(T) = ATA^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$.

If (2) holds, then we similarly have that $\varphi(T) = CT^*C^{-1}$ for all $T \in \mathcal{B}(\mathcal{X})$. $\square$

Acknowledgments. The authors would like to thank the referees for their valuable comments and suggestions. The research of both authors was supported by National Natural Science Foundation of China grants No. 11371233 and No. 114711200 and by the Fundamental Research Funds for the Central Universities grant No. GK201301007.

References

1. B. Aupetit, *Spectrum-preserving linear mapping between Banach algebra or Jordan-Banach algebras*, J. Lond. Math. Soc. (2) **62**, no. 3, (2000), 917–924. MR1794294. DOI 10.1112/S0024610700001514. 254
2. X. Cao and S. Chen, *Linear maps between operator algebras preserving certain spectral functions*, Banach J. Math. Anal. **8** (2014), no. 1, 39–46. Zbl 1290.47038. MR3161680. 255
3. J. Cui and J. Hou, *Additive maps on standard operator algebras preserving parts of the spectrum*, J. Math. Anal. Appl. **282** (2003), no. 1, 266–278. Zbl 1042.47027. MR2000343. DOI 10.1016/S0022-247X(03)00146-X. 254
4. J. Dieudonné, *Sur une généralisation du groupe orthogonal à quatre variables*, Arch. Math. (Basel) **1** (1949), 282–287. MR0029360. 254
5. A. A. Jafarian and A. R. Sourour, *Spectrum-preserving linear maps*, J. Funct. Anal. **66** (1986), no. 2, 255–261. Zbl 0589.47003. MR0832991. DOI 10.1016/0022-1236(86)90073-X. 254, 259
6. I. Kaplansky, "Algebraic and analytic aspects of opertors algebras" in *Conference Board of the Mathematical Sciences Regional Conference Series in Mathematics*, Amer. Math. Soc., Providence, 1970. MR0312283. 254
7. M. Marcus and R. Purves, *Linear transformations on algebras of matrices: The invariance of the elementary symmetric functions*, Canad. J. Math. **11** (1959), 383–396. Zbl 0086.01704. MR0105425. 254
8. M. Mbekhta and P. Šemrl, *Linear maps preserving semi-Fredholm operators and generalized invertibility*, Linear Multilinear Alg. **57** (2009), no. 1, 55–64. Zbl 1201.47015. MR2536067. DOI 10.1080/03081080701559500. 255
9. M. Omladič and P. Šemrl, *Additive mappings preserving operators of rank one*, Linear Algebra Appl. **182** (1993), 239–256. Zbl 0803.47026. MR1207085. DOI 10.1016/0024-3795(93)90502-F. 259
10. A. R. Sourour, *Invertibility preserving linear maps on $\mathcal{L}(X)$*, Trans. Amer. Math. Soc. **348** (1996), no. 1, 13–30. Zbl 0843.47023. MR1311919. DOI 10.1090/S0002-9947-96-01428-6. 254

[1]School of Mathematics and Information Science, Shaanxi Normal University, Xi'an, 710062, China.

E-mail address: shiweijuan1016@163.com

[2]School of Mathematics and Information Science, Shaanxi Normal University, Xi'an, 710062, China.

E-mail address: gxji@snnu.edu.cn