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Errata

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Due to a printing error, several lowercase psi's (ψ) are missing in the printed versions of two articles in the previous issue (vol. 58, no. 4): "New Degree Spectra of Abelian Groups" by Alexander G. Melnikov (pp. 507–525), and "Grades of Discrimination: Indiscernibility, Symmetry, and Relativity" by Tim Button (pp. 527–553). Corrections for both articles are given below by page number, and the notation appears correctly in the online version of this issue at https://projecteuclid.org/euclid.ndjfl/1506931651. Duke University Press regrets the error.

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The second and third sentences of the second paragraph of Section 2.5 should be read as

> We also fix injective and effective maps $\psi_{\alpha,k}$, $k \in \omega$ (to be used for the operation of substitution), which are different from the $\phi_{\beta,k}$'s and also consistent with Definition 2.5 (i.e., they effectively map the primes used in the corresponding $G(\Sigma)$- or $G(\Pi)$-component H_k to new fresh primes which do not overlap for different k's). In the following, we suppress α in $\psi_{\alpha,k}$.

The statement of Definition 2.11 and the sentence immediately after it should be read as

> Given any finite set S and any finite string $\sigma \in \omega^{<\omega}$, define
>
> $$H_{\sigma,S} = B_{S,\sigma,k}\left(\frac{r_{S,\sigma,k} + r_{S,\sigma,k+1}}{w^{\infty}_{\alpha,k}}\right)_{k\in\omega} B_{S,\sigma,k+1},$$
>
> where $B_{S,\sigma,k} \cong [G(\Sigma^0_\alpha(\sigma(k)))]_{\psi_k}$ if $k \in \mathrm{Age}_S$, and $B_{S,\sigma,k} \cong [G(\Pi^0_\alpha)]_{\psi_k}$ otherwise. Define
>
> $$H_S = \bigoplus_{\sigma\in\omega^{<\omega}} H_{\sigma,S}.$$
>
> Finally, let
>
> $$G_{\mathcal{R}} = \bigoplus_{S\in\mathcal{R}} H_S.$$
>
> We can effectively choose ψ_k to be consistent with Definition 2.5.

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The second sentence of the proof of Lemma 2.12 should be read as

> Consequently, we can apply Lemma 2.6, an effective enumeration of $\omega^{<\omega}$, and the fact that the ψ_k's can be effectively and consistently defined.

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The phrase immediately following the first display in Section 2.6 should be read as

> ...where $B_{S,\sigma,k} \cong [G(\Sigma^0_\alpha(\sigma(k)))]_{\psi_k}$ if $k \in \mathrm{Age}_S$, and $B_{S,\sigma,k} \cong [G(\Pi^0_\alpha)]_{\psi_k}$ otherwise.

The last line of Lemma 2.13 should be read as

> *(Equivalently, $r_{S,\sigma,k}$ is the root of $B_{S,\sigma,k} \cong [G(\Sigma^0_\alpha(\sigma(k)))]_{\psi_k}$.)*

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The phrase immediately following the first display in the proof of Lemma 2.13 should be read as

> ...where $B_{S,\sigma,k} \cong [G(\Sigma^0_\alpha(\sigma(k)))]_{\psi_k}$ if $k \in \mathrm{Age}_S$, and $B_{S,\sigma,k} \cong [G(\Pi^0_\alpha)]_{\psi_k}$ otherwise.

The last sentence of the proof of Claim 2.14 should be read as

> More specifically, the formula says that $x \neq 0$ and $\psi_0(p_\alpha)^\infty | x$, and there exists y such that $\psi_1(p_\alpha)^\infty | y$ and $w^\infty_{\alpha,0} | (x + y)$.

The proof of Claim 2.15 should be read as

> We prove the claim by induction. The case $k = 0$ is Claim 2.14. Suppose that we have produced $\Theta_{k-1}(x, \cdot)$. Consider the pure subgroup generated by the roots of the $B_{S,\sigma,k-1}$-subcomponents and $B_{S,\sigma,k}$-subcomponents. Define $\Theta_k(x, y)$ to be the formula
>
> $$(\exists z)\big(\Theta_{k-1}(x, z) \wedge w^\infty_{\alpha,k-1} \big| (y + z) \wedge \psi_k(p_\alpha)^\infty \big| y\big).$$
>
> By the inductive hypothesis, $z = \sum_{(S,\sigma)\in I} m_{S,\sigma} r_{S,\sigma,k-1}$. Since $\psi_k(p_\alpha)^\infty | y$, we have
>
> $$y = \sum_{(S,\sigma)\in I} t_{S,\sigma} r_{S,\sigma,k},$$
>
> where $t_{S,\sigma}$ are rationals. By the inductive hypothesis, we may assume that $\Theta_{k-1}(x, z)$ contains a conjunct of the form $\psi_{k-1}(p_\alpha)^\infty | z$. Consider the pure closure of the subgroup generated by $r_{S,\sigma,k}$ and $r_{S,\sigma,k-1}$ for various S's and σ's. Note that $w^\infty_{\alpha,k-1} | (y + z)$, and thus, by Lemma 2.8 applied to this pure subgroup we have $t_{S,\sigma} = m_{S,\sigma}$.

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The phrase immediately following the first display on the page should be read as

> ...where $m_{S,\sigma} \in \mathbb{Z} \setminus \{0\}$, and for some S the element $r_{S,\sigma,k}$ is the root of $B_{S,\sigma,k} \cong [G(\Sigma^0_\alpha(\sigma(k)))]_{\psi_k}$.

The statement of Claim 2.16 should be read as

> *For every k we can uniformly produce a Σ^c_α-formula Γ_k such that, for every element of the form $x = \sum_{(S,\sigma)\in I} m_{S,\sigma} r_{S,\sigma,k}$, $G_{\mathcal{R}} \models \Gamma_k$ if and only if for some $m_{S,\sigma} \neq 0$ the corresponding $r_{S,\sigma,k}$ is the root of $B_{S,\sigma,k} \cong [G(\Sigma^0_\alpha(\sigma(k)))]_{\psi_k}$.*

The first display in the proof of Claim 2.16 should be read as

$$A_{S,\sigma,i}\left(\frac{a_{S,\sigma,i}+a_{S,\sigma,i+1}}{\psi_k(v_{\alpha,i})^\infty}\right)_{i\in\omega}A_{S,\sigma,i+1}.$$

The sentence immediately following the second display in the proof of Claim 2.16 should be read as

> Indeed, we may take the formula witnessing Claim 2.14 and replace $w_{\alpha,k-1}$ by $\psi_k(v_{\alpha,i})$ in the formula, and we also replace $\psi_k(p_\alpha)$ by the prime that labels the roots $a_{S,\sigma,i}$ of $A_{S,\sigma,i}$.

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The sentence immediately following the first display on the page should be read as

> Indeed, the first conjunct inside the parentheses guarantees $m_{\sigma,S}=n_{\sigma,S}$, and the second conjunct says that $B_{S,\sigma,k}\cong[G(\Sigma_4^0(\sigma(k)))]_{\psi_k}$ with $\sigma(k)\leq i$.

The beginning of the third sentence in the paragraph immediately preceding the second display on the page should be read as

> Using primes $\psi_k(p_{\alpha-2})$ and $\psi_k(q_{\alpha-2})\ldots$.

The sentence immediately following the second display on the page should be read as

> Furthermore, using a variation of Claim 2.16 with the right choice of primes (e.g., use $\psi_k(v_{\alpha-2,j})$), we can produce a uniform sequence of Σ_3^c-formulae $\{\mathcal{F}_j\}_{j\in\omega}$ such that $\mathcal{F}_j(z,c_j)\wedge\mathcal{Z}(y,z)$ holds if and only if $c_j=\sum_{(S,\sigma,s)}l_{S,\sigma,s}k_{S,\sigma,s,j}$, where $k_{S,\sigma,s,j}$ is the root of $K_{S,\sigma,s,j}$ which is the jth subcomponent of $D_{S,\sigma,s}$ that was used in its definition via the chain operation, counting from its root.

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The phrase immediately preceding the third display on the page should be read as

> However, where $\psi\in\mathscr{L}_1^+$ abbreviates ....

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The proof of Lemma 8.4 should be read as

> (1) Let Γ be the set of all $\mathscr{L}_2^-$-formulas of the form
>
> $$\forall\overline{v}\Big(\bigwedge_{i=1}^n\big[\phi_i(x,x)\wedge\neg\phi_i(x,v_i)\wedge\psi_i(y,y)\wedge\neg\psi_i(y,v_i)\big]\rightarrow\big[\theta(x,y,\overline{v})\leftrightarrow\theta(y,x,\overline{v})\big]\Big)$$
>
> for any $n<\omega$, any $\phi_1,\ldots,\phi_n,\psi_1,\ldots,\psi_n\in\mathscr{L}_2^-$, and any $\theta\in\mathscr{L}_{n+2}^-$. I claim that Γ captures r in any $\mathscr{L}$-structure $\mathcal{M}$.
>
> First, suppose $a\ \mathsf{r}\ b$ in $\mathcal{M}$. Fix some $\gamma\in\Gamma$, and some $\overline{e}\in M^n$. Suppose that
>
> $$\mathcal{M}\models\bigwedge_{i=1}^n\big[\phi_i(a,a)\wedge\neg\phi_i(a,e_i)\wedge\psi_i(b,b)\wedge\neg\psi_i(b,e_i)\big].$$
>
> Then by Lemma 2.2, $e_i\not\approx a$ and $e_i\not\approx b$ for each $1\leq i\leq n$. Since $a\ \mathsf{r}\ b$, Lemma 2.7 tells us that $\mathcal{M}\models\theta(a,b,\overline{e})\leftrightarrow\theta(b,a,\overline{e})$. Hence, $\mathcal{M}\models\gamma(a,b)$ for any $\gamma\in\Gamma$.

Next, suppose $\mathcal{M} \models \gamma(a,b)$ for all $\gamma \in \Gamma$. I claim that the following is a near-correspondence from $\mathcal{M}$ to $\mathcal{M}$:

$$\Pi = \big\{\langle a,b\rangle, \langle b,a\rangle\big\} \cup \big\{\langle x,x\rangle \mid x \not\approx a \text{ and } x \not\approx b\big\}.$$

To show this, fix $n < \omega$, $\theta \in \mathscr{L}^-_{n+2}$ and $\overline{e} \in M^n$ such that $e_i \not\approx a$ and $e_i \not\approx b$ for each $1 \leq i \leq n$. Since each $e_i \not\approx a$ and $e_i \not\approx b$, by Lemma 2.2 there are formulas $\phi_i, \psi_i \in \mathscr{L}^-_2$ for each $1 \leq i \leq n$ such that $M \models \phi_i(a,a) \wedge \neg\phi_i(a,e_i)$ and $\mathcal{M} \models \psi_i(b,b) \wedge \neg\psi_i(b,e_i)$. Conjoining these, we get

$$\mathcal{M} \models \bigwedge_{i=1}^{n} \big[\phi_i(a,a) \wedge \neg\phi_i(a,e_i) \wedge \psi_i(b,b) \wedge \neg\psi_i(b,e_i)\big].$$

Since $\mathcal{M} \models \gamma(a,b)$ for all $\gamma \in \Gamma$, we obtain that, for all $\theta \in \mathscr{L}^-_{n+2}$,

$$\mathcal{M} \models \theta(a,b,\overline{e}) \leftrightarrow \theta(b,a,\overline{e}).$$

By generalizing, Π is a near-correspondence. By the Galois connection of Theorem 4.8, $(\Pi^{\mathbf{i}})^{\mathbf{c}}$ is a relativity on $\mathcal{M}$, and so $a \mathrel{\mathsf{r}} b$.

(2) This follows from Lemmas 7.4 and 8.2.

(3) This is exactly as in Lemma 8.3(3).