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Blurring: An Approach to Conflation

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Abstract I consider the phenomenon of *conflation*—treating distinct things as one—and develop logical tools for modeling it. These tools involve a purely consequence-theoretic treatment, independent of any proof or model theory, as well as a four-valued valuational treatment.

1 Conflation

So-called *Frege puzzles*—cases in which one thing (in some very generous sense of "thing") is treated, at least in some way, as multiple—have proved a remarkably fruitful topic of philosophical discussion. These cases (Hesperus/Phosphorus cases, woodchuck/whistle-pig cases, Paderewski cases, etc.) seem to tie together a number of important issues in epistemology, metaphysics, semantics, pragmatics, rationality, probability, and so forth.

Oddly, the reverse phenomenon—when multiple things (in some very generous sense of "thing") are treated, at least in some way, as one—is not so thoroughly explored, despite raising issues precisely parallel to those raised by Frege puzzles. I will call this phenomenon *conflation*; I suspect it is low-hanging fruit, in terms of philosophical payoff.[1]

In the remainder of this section, I give some examples of what I have in mind, argue that the phenomenon of *propositional conflation*—treating multiple propositions as one—gives a common currency for all these examples, and defend three desiderata for a logical treatment of conflation. In the rest of the article, I develop and deploy logical tools for understanding conflation that meet these desiderata. Section 2 develops these tools in a purely consequence-theoretic setting, without appeal to any particular proof theory or model theory; Section 3 goes on to develop a particularly flexible valuational model theory (inspired by Humberstone [16]) that is of use here, but should also be of wider use.

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1.1 Some examples The main running example in this article is the case of Fred and the ant farm, taken from Camp [4], [5]:

> Fred has an ant colony in his kitchen in which there are two big ants, which we will call "Ant A" and "Ant B". Big ants make every effort to avoid conflict and so our two ants arrange to split their time between running around on the surface performing feats of strength, and napping down in the bowels of the ant colony. Fred never catches on to the fact that there are in fact two big ants and decides to name "the" big ant "Charley" [5, p. 692].

This is a clear case of conflation: Fred treats the two ants as one. He assigns the name "Charley" as an ordinary name, with (what he takes to be) a single referent; he buys just enough big-ant food for one big ant; he infers from perceptions of Ant A to conclusions he draws on in interacting with Ant B; and so on.

The case, though, has some features that are not necessary for conflation in my sense: first, Fred has conflated *individuals*, rather than, say, properties or propositions; and second, he has conflated them because he is *mistaken* about how many big ants there are, rather than, say, because he cannot be bothered to keep track of the number, or in order to throw an investigator off the scent. Conflation does not require either of these.

For example, consider our usual practice around the properties of *weight* and *mass*. Although these are distinct properties, we often, at least in nontheoretical contexts, treat them as one. For example, consider my baking scale. Like any scale, it responds to the weight of what is on it. But it reports its results in grams: a unit of mass. One way to understand this, I suppose, is as *assuming* a certain gravitation, and calculating from the force applied to its pan the mass that must be on top of it to apply that amount of force via gravity. But this is certainly not how I am thinking of the scale when I use it; I am simply ignoring the difference between weight and mass, treating them as a single feature of the pile of sugar I am measuring.

Most of the time, I reckon, many of us are like this: we treat weight and mass as one property, although they are distinct. This too is a case of conflation, of treating what is multiple as one. But it is unlike Fred's case in a few respects. First, we are not conflating *individuals* but *properties*. Just as Frege puzzles arise for properties no less than for individuals, so too conflation is a possibility for properties no less than for individuals. Second, we (many of us, anyway) are not *mistaken* the way Fred is: we know perfectly well that mass and weight are distinct, and we can distinguish them when we think it worthwhile to do so. It is just that, most of the time, it is not worth the trouble, and so we justifiably do not bother.

These are not the only reasons to conflate. Consider too weight-loss ads' frequent conflation of *health* with *slenderness*. This is also a conflation of properties rather than individuals, but it is engaged in for commercial gain, rather than for efficiency's sake (like our conflation of weight with mass) or from a mistaken belief (like Fred's conflation of Ant A with Ant B).

1.2 Propositional conflation Conflation of individuals and of properties results in conflation of *propositions*.[2] As Fred conflates Ant A with Ant B, calling them both "Charley," so too he conflates the proposition *Ant A is healthy* with the proposition *Ant B is healthy*. If he did not conflate these propositions, if he did not treat them as one, that *in itself* would be a way of failing to conflate Ant A and Ant B, of failing to treat them as one. Conflation of individuals always brings with it conflation of propositions in just this way.

Similarly, as weight-loss ads conflate health with slenderness, so too they conflate the proposition *Ant A is healthy* with the proposition *Ant A is slender*. If they did not conflate these propositions, if they did not treat them as one, that *in itself* would be a way of failing to conflate health with slenderness, of failing to treat them as one. Conflation of properties always brings with it conflation of propositions in just this way.

Conflation of individuals or of properties, then, brings with it conflation of propositions. But not all conflation of propositions comes this way. Propositions can be conflated *directly*, without any conflation of individuals or properties. For example, consider *scope conflations*, of the sort we help some of our logic students overcome. It is a cognitive achievement to see the distinction between $\forall\exists$ scope and $\exists\forall$ scope, and a further achievement to keep the distinction present to mind. Letting the distinction slip (or not seeing it in the first place) leads to treating distinct propositions as one: conflation. Similarly, the medieval distinction between *necessity of the consequent* and *necessity of the consequence*, what we would today commonly notate as the difference between $A \to \Box B$ and $\Box(A \to B)$, can be lost track of, resulting in conflation.[3] But neither of these scope conflations can be understood as conflation of anything smaller than the full propositions involved.

Conflation of propositions thus provides a common currency for exploring many kinds of conflation: conflation of individuals, of properties, and of propositions directly. In the logical approach to follow, I will focus exclusively on propositional conflation, intending to treat conflation of individuals or properties by first implicitly reading such conflation up to the propositional level.[4]

1.3 Desiderata In this section, I will develop and defend three desiderata for a logical treatment of conflation. A logical treatment, as I will pursue it here, is a construction that begins with three ingredients: first, a language (or a space of propositions) $\mathcal{L}$; second, a consequence relation $\vdash$ on $\mathcal{L}$; and third, an equivalence relation $\approx$ on $\mathcal{L}$. From these, the treatment should generate a new consequence relation $\vdash^{\approx}$ on $\mathcal{L}$.

Here is the idea: $\mathcal{L}$ is the background language we are working within. This should be an *unconflated* language; a logical treatment should show how to generate conflation from it. For example, in the case of Fred, $\mathcal{L}$ should contain separate propositions about Ant A and Ant B. It should not contain any propositions about Charley; if such propositions are wanted, they should be generated from the treatment itself. (One option for this will appear in Section 2.4.) Similarly, $\vdash$ gives the *unconflated* truths and validities in $\mathcal{L}$. It is not intended to be a *logical* consequence relation in any sense; it can contain a rich theory of the case being explored, extending to material validities of all sorts (see, e.g., Brandom [3], Sellars [28]). Unconflated truths can be taken up into $\vdash$ as theorems (consequences of no premises), but $\vdash$ can contain plenty more besides.[5]

I will call any pair of subsets of $\mathcal{L}$ a *sequent*, and write the sequent $\langle\Gamma, \Delta\rangle$ as $[\Gamma \rhd \Delta]$, dropping the outer brackets where they do not aid clarity. I will call the first member of a sequent the sequent's *antecedent* and the second its *succedent*. Sequents represent arguments in $\mathcal{L}$; the antecedent of a sequent is the set of the argument's premises, and the succedent the set of its conclusions. A *consequence* *relation* $\vdash$ is any set of sequents; these are to be understood as the arguments that are valid according to $\vdash$. I will write "$\Gamma \vdash \Delta$" to mean $[\Gamma \rhd \Delta] \in \vdash$, and use other standard abbreviations (e.g., writing $\Gamma, A \vdash \Delta, \Delta'$ for $\Gamma \cup \{A\} \vdash \Delta \cup \Delta'$).[6]

Finally, $\approx$ is the relation that tells us which members of $\mathscr{L}$ should be treated as one; it is where the conflation itself enters the approach. Note that this must be an equivalence relation. Treating things *as one* is a very strong connection between them indeed, and anything weaker than an equivalence relation will not capture this. (The most likely worry is about transitivity, but conflation must be transitive: if $A \approx B$ and $A \not\approx C$, then this in itself is a way in which B and C are treated differently, so it must be that $B \not\approx C$.) In the case of Fred, for example, we should have $C(\text{Ant A}) \approx C(\text{Ant B})$, for all propositional contexts $C(\,)$, since Fred conflates each such pair.

An approach should take these three ingredients and yield a consequence relation $\vdash^{\approx}$ on $\mathscr{L}$, to be understood as the *conflated* consequence relation. This is what $\vdash$ looks like through the lens provided by $\approx$.

Given this setup, the best approaches to conflation will exhibit three features. I will call these *intersubstitutivity*, *validity preservation*, and *conservativity*.

1.3.1 Intersubstitutivity Conflation is treating things as *one*, not drawing any distinctions between them. So our conflated consequence relation $\vdash^{\approx}$ should draw no distinctions between any propositions related by $\approx$; they should be *intersubstitutable* for each other. That is, we want that for any $\Gamma, \Delta \subseteq \mathscr{L}$ and $A, B \in \mathscr{L}$, if $A \approx B$, then:

- $\Gamma, A \vdash^{\approx} \Delta$ iff $\Gamma, B \vdash^{\approx} \Delta$, and
- $\Gamma \vdash^{\approx} A, \Delta$ iff $\Gamma \vdash^{\approx} B, \Delta$.

If this intersubstitutivity property is not realized, then we have not fully captured what it is to conflate A with B.

1.3.2 Validity preservation Drawing new distinctions *invalidates* arguments; it does not make formerly invalid arguments become valid. Conversely, treating multiple things as one only *closes off* possibilities for how the world can be; it does not open up new possibilities. (Dually, it only adds new possibilities for proof; it does not break any existing proofs.)

We should expect, then, that conflation will not invalidate any antecedently valid arguments. An approach to conflation should only *add* validities, not remove any. As such, the second desideratum is *validity preservation*: that $\vdash \subseteq \vdash^{\approx}$.

A similar desideratum is given in [4], there called "inferential charity." Camp offers a "[b]umper sticker: [conflation] is a defect in one's 'powers of discrimination' which entails no defect of pure reason" (p. 42). Because of this, he argues, a treatment of conflation that declares certain arguments invalid simply because they involve a conflation would lead us to overcriticize people like Fred. When Fred infers "Charley is warm and happy" from "Charley is warm" and "Charley is happy," he is inferring *validly*, despite his conflation. The problem, if there is one, is not with Fred's *reasoning* at all. A treatment of conflation that finds fault with Fred's reasoning here is missing the point. While I focus here on validity rather than reasoning, I find Camp's arguments on this score compelling. A treatment of conflation should preserve unconflated validities.[7]

1.3.3 Conservativity Say that $A \in \mathscr{L}$ is *properly conflated* if and only if there is some $B \neq A$ such that $A \approx B$. It is the properly conflated propositions that represent conflations; the rest of the propositions, each member in an equivalence cell by itself, should be left alone.

Conservativity gives us a sense in which these other propositions are to be left alone. (It is related to, but quite distinct from, the familiar notion of a *conservative extension*.) An approach is conservative if and only if: whenever $\Gamma \vdash^{\approx} \Delta$ but $\Gamma \nvdash \Delta$, then there is some $A \in \Gamma \cup \Delta$ that is properly conflated. If no proposition in a sequent is properly conflated, and the sequent is not $\vdash$-valid, then a conservative approach cannot have the sequent come out $\vdash^{\approx}$-valid.

The reason for demanding conservativity is simple: it gives us some reassurance that the approach in question is *only* dealing with the conflation at hand, not adding validities willy-nilly. Note, for example, that an approach that always yields the *universal* consequence relation for $\vdash^{\approx}$ (the relation according to which every argument is valid) will meet the first two desiderata. But this is clearly a bad approach; it tells us nothing at all about particular conflations, instead collapsing *all* distinctions without sensitivity. It is conservativity that allows us to rule such an approach out.

2 Blurring

I call the treatment I will recommend *blurring*. For the purposes of blurring, $\mathcal{L}$ can be arbitrary: any set.[8] Similarly, $\vdash$ can be any consequence relation on $\mathcal{L}$, and $\approx$ can be any equivalence relation on $\mathcal{L}$. So blurring is really quite general: it can apply to any language, subject to any consequence relation, conflated in any way.

The first step in blurring is to lift $\approx$ from a relation on $\mathcal{L}$ to a relation on subsets of $\mathcal{L}$, and then to a relation on sequents. This lifted relation is then used to move from $\vdash$ to $\vdash^{\approx}$. I consider each step in turn. For clarity, I will sometimes write $\approx_{\mathcal{L}}$ for the relation on $\mathcal{L}$, $\approx_{\{\mathcal{L}\}}$ for the relation on $\wp\mathcal{L}$, and $\approx_{[\triangleright]}$ for the relation on sequents, but I will often omit the subscripts and simply write $\approx$, allowing context to disambiguate.

2.1 Lifting $\approx$ First, to lift $\approx$ from $\mathcal{L}$ to $\wp\mathcal{L}$. The most intuitive way to do this works with a notion of "blurred subset."

Definition 1 (Blurred subsets) For $\Gamma, \Gamma' \subseteq \mathcal{L}$, Γ is a *blurred subset* of Γ' (written $\Gamma \precsim \Gamma'$) if and only if for all $\gamma \in \Gamma$, there is some $\gamma' \in \Gamma'$ such that $\gamma \approx_{\mathcal{L}} \gamma'$.

Note that replacing $\approx_{\mathcal{L}}$ with $=$ in the definiens of the above definition gives the usual understanding of subset. That is, $\Gamma \precsim \Gamma'$ if and only if: Γ would have been a subset of Γ' if the things actually related to each other by $\approx_{\mathcal{L}}$ had been identical; or, if and only if someone engaging in the conflation on $\mathcal{L}$ described by $\approx_{\mathcal{L}}$ would call Γ a subset of Γ'. From here, it is straightforward to get blurring on subsets of $\mathcal{L}$. Just as equality of sets can be understood as two-way subsethood, blurring is understood as two-way blurred subsethood.

Definition 2 ($\approx$ on sets) For $\Gamma, \Gamma' \subseteq \mathcal{L}$, $\Gamma \approx_{\{\mathcal{L}\}} \Gamma'$ if and only if $\Gamma \precsim \Gamma'$ and $\Gamma' \precsim \Gamma$.

Just as in the blurred subset case, we have that $\Gamma \approx_{\{\mathcal{L}\}} \Gamma'$ if and only if: Γ would have been identical to Γ' if the things actually related to each other by $\approx_{\mathcal{L}}$ had been identical; or, if and only if someone engaging in the conflation on $\mathcal{L}$ described by $\approx_{\mathcal{L}}$ would thereby conflate Γ with Γ'.

Finally, we lift $\approx$ to a relation on sequents. This is the blurring relation that will be applied in the next step. The idea is simple: two sequents are blurred if and only if their antecedents are blurred and their succedents are blurred.

Definition 3 ($\approx$ on sequents) $[\Gamma \rhd \Delta] \approx_{[\rhd]} [\Gamma' \rhd \Delta']$ if and only if $\Gamma \approx_{\{\mathcal{L}\}} \Gamma'$ and $\Delta \approx_{\{\mathcal{L}\}} \Delta'$.

There are a few facts that will come in handy later in the paper, and I record them here without proof (which is straightforward).

Fact 1 *If* $\Gamma \approx_{\{\mathcal{L}\}} \Gamma'$ *and* $\Sigma \approx_{\{\mathcal{L}\}} \Sigma'$, *then* $\Gamma \cup \Sigma \approx_{\{\mathcal{L}\}} \Gamma' \cup \Sigma'$.

Fact 2 *We have that* $\precsim$ *is a preorder, and that* $\approx_{\{\mathcal{L}\}}$ *and* $\approx_{[\rhd]}$ *are equivalence relations.*

2.2 From $\vdash$ to $\vdash^{\approx}$ Just as $\vdash$ is a set of sequents, so too is $\vdash^{\approx}$; it is the set of sequents that are blurred with some $\vdash$-valid sequent.

Definition 4 ($\vdash^{\approx}$) We have $\vdash^{\approx} = \{s : \exists s' \in \vdash (s \approx s')\}$.

(Just as with $\vdash$, I will write $\Gamma \vdash^{\approx} \Delta$ for $[\Gamma \rhd \Delta] \in \vdash^{\approx}$ and abbreviate in other usual ways.) This is the approach I recommend; this new consequence relation $\vdash^{\approx}$ satisfies all of the desiderata argued for in Section 1.3.

Theorem 1 (Desiderata)

Intersubstitutivity: *For* $A, B \in \mathcal{L}$, *if* $A \approx B$, *then:*
- $\Gamma, A \vdash^{\approx} \Delta$ *iff* $\Gamma, B \vdash^{\approx} \Delta$, *and*
- $\Gamma \vdash^{\approx} A, \Delta$ *iff* $\Gamma \vdash^{\approx} B, \Delta$.

Validity preservation: *We have* $\vdash \subseteq \vdash^{\approx}$.

Conservativity: *If* $\Gamma \vdash^{\approx} \Delta$ *but* $\Gamma \nvdash \Delta$, *then some member of either* Γ *or* Δ *is properly conflated.*

Proof

Intersubstitutivity: Suppose $A \approx B$. By Fact 2, $\Gamma \approx \Gamma$ and $\Delta \approx \Delta$; by Fact 1, then, $\Gamma, A \approx \Gamma, B$ and $A, \Delta \approx B, \Delta$. The claim is immediate from these.

Validity preservation: This is immediate from the reflexivity of $\approx_{[\rhd]}$.

Conservativity: Suppose $\Gamma \vdash^{\approx} \Delta$ and $\Gamma \nvdash \Delta$. Then $[\Gamma \rhd \Delta] \approx_{[\rhd]} [\Gamma' \rhd \Delta']$ for some Γ', Δ' such that $\Gamma' \vdash \Delta'$; so $\Gamma \approx \Gamma'$ and $\Delta \approx \Delta'$, while either $\Gamma \neq \Gamma'$ or $\Delta \neq \Delta'$.

Without loss of generality, suppose $\Gamma \neq \Gamma'$. Then either there is some $A \in \Gamma$ but $A \notin \Gamma'$, or else there is some $A \in \Gamma'$ but $A \notin \Gamma$. Without loss of generality again, suppose there is some $A \in \Gamma$ but $A \notin \Gamma'$. Since $\Gamma \precsim \Gamma'$ (since $\Gamma \approx \Gamma'$), there is some $A' \in \Gamma'$ such that $A \approx A'$. But since $A \notin \Gamma'$, $A \neq A'$; that is, A is properly conflated (as is A'). □

So $\vdash^{\approx}$ gives a reasonable picture of the conflated consequence relation based on $\vdash$: it allows for intersubstitution, preserves unconflated validities, and adds validities only in cases of proper conflation. The three desiderata of Section 1.3 are achieved.[9]

2.3 What is and is not preserved Blurring is quite general; it applies to any consequence relation (that uses sets of premises and conclusions) on any language, with any conflation relation. As a result, there is not much that can be shown about $\vdash^{\approx}$ in total generality: most of its interesting properties depend on particular properties of $\mathscr{L}$, $\vdash$, and $\approx$. It is more fruitful to investigate which properties of $\vdash$ are *preserved* by blurring. If blurring provides a good understanding of conflation's effects on validity, as I have argued, then this will show us what effects conflation can and cannot have. This can be explored without having to first decide what *unconflated* consequence is like.

Definition 5 (Properties of consequence relations) A set X of sequents is:

reflexive	iff	$[A \rhd A] \in X$ for all $A \in \mathscr{L}$;
monotonic	iff	$[\Gamma \rhd \Delta] \in X$ implies $[\Gamma, \Gamma' \rhd \Delta, \Delta'] \in X$ for all $\Gamma, \Gamma', \Delta, \Delta' \subseteq \mathscr{L}$;
simply transitive	iff	$[A \rhd B] \in X$ and $[B \rhd C] \in X$ together imply $[A \rhd C] \in X$ for all $A, B, C \in \mathscr{L}$;
completely transitive	iff	if there is a $\Sigma \subseteq \mathscr{L}$ such that $[\Gamma, \Sigma_1 \rhd \Sigma_2, \Delta] \in X$ for every partition $\langle \Sigma_1, \Sigma_2 \rangle$ of Σ, then $[\Gamma \rhd \Delta] \in X$;[10]
compact	iff	for every $[\Gamma \rhd \Delta] \in X$, there are finite $\Gamma_{\text{fin}} \subseteq \Gamma$ and $\Delta_{\text{fin}} \subseteq \Delta$ such that $[\Gamma_{\text{fin}} \rhd \Delta_{\text{fin}}] \in X$.

2.3.1 *What is preserved*

Theorem 2 *If $\vdash$ is reflexive, monotonic, or compact, then so is $\vdash^{\approx}$.*

Proof

Reflexive: Since $\vdash \subseteq \vdash^{\approx}$, this is immediate.

Monotonic: Suppose $\vdash$ is monotonic, and suppose $\Gamma \vdash^{\approx} \Delta$, to show that $\Gamma, \Gamma' \vdash^{\approx} \Delta, \Delta'$. Since $\Gamma \vdash^{\approx} \Delta$, there are $\Gamma'' \approx \Gamma$ and $\Delta'' \approx \Delta$ such that $\Gamma'' \vdash \Delta''$. Since $\vdash$ is monotonic, $\Gamma'', \Gamma' \vdash \Delta'', \Delta'$. By Facts 2 and 1, $\Gamma, \Gamma' \approx \Gamma'', \Gamma'$ and $\Delta, \Delta' \approx \Delta'', \Delta'$. So $\Gamma, \Gamma' \vdash^{\approx} \Delta, \Delta'$.

Compact: Suppose $\vdash$ is compact, and suppose $\Gamma \vdash^{\approx} \Delta$, to show that there are finite $\Gamma_{\text{fin}} \subseteq \Gamma$ and $\Delta_{\text{fin}} \subseteq \Delta$ such that $\Gamma_{\text{fin}} \vdash^{\approx} \Delta_{\text{fin}}$. Since $\Gamma \vdash^{\approx} \Delta$, there are $\Gamma' \approx \Gamma$ and $\Delta' \approx \Delta$ such that $\Gamma' \vdash \Delta'$; and since $\vdash$ is compact, there are finite $\Gamma'_{\text{fin}} \subseteq \Gamma'$ and $\Delta'_{\text{fin}} \subseteq \Delta'$ such that $\Gamma'_{\text{fin}} \vdash \Delta'_{\text{fin}}$.

Now, for every member A' of Γ'_{fin}, choose some member A of Γ such that $A' \approx A$, and collect them into the set Γ_{fin}. There will always be some such member of Γ, since there is some such for every $A' \in \Gamma'$ (since $\Gamma' \gtrsim \Gamma$) and $\Gamma'_{\text{fin}} \subseteq \Gamma'$. Moreover, since Γ'_{fin} is finite, so will Γ_{fin} be; and $\Gamma_{\text{fin}} \subseteq \Gamma$. Finally, note that $\Gamma_{\text{fin}} \approx \Gamma'_{\text{fin}}$. Do the same to generate Δ_{fin} from Δ'_{fin} and Δ.

Since $\Gamma'_{\text{fin}} \vdash \Delta'_{\text{fin}}$ and $[\Gamma'_{\text{fin}} \rhd \Delta'_{\text{fin}}] \approx [\Gamma_{\text{fin}} \rhd \Delta_{\text{fin}}]$, we have $\Gamma_{\text{fin}} \vdash^{\approx} \Delta_{\text{fin}}$; but we have already seen that Γ_{fin} and Δ_{fin} are finite subsets of Γ and Δ, respectively. □

Some of the familiar properties of consequence relations, then, are preserved by blurring. If the initial consequence relation exhibits them, blurring will not change the situation.

2.3.2 Transitivity and equivocation But this is not at all so for transitivity. In fact, there are consequence relations $\vdash$ and blurring relations $\approx$ such that $\vdash$ exhibits both kinds of transitivity defined above and $\vdash^{\approx}$ does not exhibit either of them.

For an example, take $\mathcal{L}$ to be the usual language of classical propositional logic, let $\vdash$ be the usual consequence relation of classical propositional logic, and let $\approx$ be the smallest equivalence relation on $\mathcal{L}$ such that $A \wedge B \approx A \vee B$ for all $A, B \in \mathcal{L}$. (Note that no atomic propositions are properly conflated by this $\approx$.) Classical propositional logic exhibits both forms of transitivity, so it remains only to show that $\vdash^{\approx}$ exhibits neither.

But our three desiderata combine to guarantee this. Since $\vdash^{\approx}$ preserves all the $\vdash$ validities, we have $p \vdash^{\approx} p \vee q$ and $p \wedge q \vdash^{\approx} q$. By intersubstitution on the latter, we have $p \vee q \vdash^{\approx} q$. Finally, since neither p nor q is properly conflated and $p \nvdash q$, conservativity gives $p \nvdash^{\approx} q$.[11] This shows that $\vdash^{\approx}$ does not exhibit simple transitivity. Since $\vdash$ is monotonic, so is $\vdash^{\approx}$, by Theorem 2; in the presence of monotonicity, complete transitivity suffices for simple transitivity, so $\vdash^{\approx}$ is not completely transitive either. (For the relations between these and other notions of transitivity, see Humberstone [17], Ripley [25], and Shoesmith and Smiley [29].)

Moreover, since classical propositional logic is reflexive, monotonic, and compact, this case suffices to show that blurring can fail to preserve transitivity even in the presence of these other properties. (Furthermore, all of $\vdash$, $\approx$, and $\vdash^{\approx}$ in this case are substitution-invariant, so even insisting on this—which would be ill-motivated, as it would rule out the intended applications to material consequence—would not change the situation.)

Note as well that the failure of transitivity in this case depends directly, and only, on the three desiderata argued for in Section 1.3. If these desiderata are well-motivated, then, *any* good understanding of conflation will be subject to an analogue of this situation. There is real tension between conflation and transitivity, even if unconflated validity is as transitive as you could ask for.

This tension is precisely what we usually understand as *equivocation*. How can it be that transitivity of entailment fails, that A entails B and B entails C without A entailing C? One (all too) familiar way is for B to be hiding an equivocation. This feature is exactly what leads to the failure of transitivity that appears in the above proof. So we have another sign that blurring provides a good model of conflation: we already knew that conflation gives rise to equivocation, and this is now seen to be derivable from the blurring-based approach, because of its respect for our desiderata.

This is not at all to say, however, that equivocation is the *only* possible source of nontransitivity in consequence. Nontransitive consequence relations have been argued to arise from at least three phenomena other than conflation (and other than tonk; see footnote 11): relevance (see Lewy, Watling, and Geach [19], Smiley [30], Tennent [31]), vagueness (see Cobreros, Egré, Ripley, and van Rooij [6], Ripley [23], Zardini [33]), and liar/Curry/Russell-style paradoxes (see Cobreros, Egré, Ripley, and van Rooij [7], Hallnäs [15], Schroeder-Heister [27], Weir [32]). For each of these cases, it would be at least contentious to understand it as a form of conflation. (I do think that understanding vagueness as conflation is plausible, but will not argue that here.)

2.4 Generating $\mathcal{L}^{\approx}$ The above presentation of blurring works with a single language throughout, and this language is assumed at the outset to be *unconflated*. For example, in the case of Fred, we want the language to be able to talk about Ant A and Ant B.

Once the consequence relation is blurred, anything said about Ant A can be interchanged salva validity for the same thing said about Ant B, even if this was not the case beforehand. This might be a sensible picture of our common conflation of weight with mass, or of the weight-loss industry's conflation of health with slenderness. But it is probably not the best way to capture what is happening with poor Fred. After all, Fred's utterances do not use two distinct terms interchangeably; Fred just uses the one name "Charley." This is an important aspect of some cases of conflation, and blurring on its own does nothing to capture this.

But it does put us in a good position to do so. Intersubstitutivity guarantees that $\approx$ is a *congruence* for $\vdash^{\approx}$, and so we can divide by it without trouble. For $A \in \mathcal{L}$, let $[A] = \{A' : A \approx A'\}$, and for $\Gamma \subseteq \mathcal{L}$, let $[\Gamma] = \{[A] : A \in \Gamma\}$. Finally, let $\mathcal{L}^{\approx}$ be $[\mathcal{L}]$. Propositions that are *blurred* in $\mathcal{L}$ are taken to *identical* members of $\mathcal{L}^{\approx}$. Blurred consequence applies to $\mathcal{L}^{\approx}$ in the obvious way: $[\Gamma] \vdash^{\approx} [\Delta]$ iff $\Gamma \vdash^{\approx} \Delta$. (This is well defined because of intersubstitutivity.)

$\mathcal{L}^{\approx}$ gives us a more faithful representation of Fred's conflated talk and thought; in the Fred case, instead of having two distinct members "Ant A is happy" and "Ant B is happy" like $\mathcal{L}$ does, $\mathcal{L}^{\approx}$ has a single member ["Ant A is happy"], which is identical to ["Ant B is happy"]. It is this member that provides the best picture of "Charley is happy" in Fred's mouth. (Now is not the time to complain that one is a set and the other is an utterance; we are already knee-deep in this kind of abstraction!)

In some cases, it might be important to preserve original bits of $\mathcal{L}$ unblurred while generating new members. For example, something like this is probably called for in the weight/mass example: we can talk separately of weight and mass, or we can conflate them. We seem to have all three options available to us.

This too can be achieved via blurring. Start from a language $\mathcal{L}$ with *two copies* of each of the sentences in question. For example, start with all of W_1: "Charley has more weight than a paper clip"$_1$, W_2: "Charley has more weight than a paper clip"$_2$, M_1: "Charley has more mass than a paper clip"$_1$, and M_2: "Charley has more mass than a paper clip"$_2$. Now, set $\approx$ to (properly) conflate one copy of each, while leaving the other copy alone. That is, let $W_1 \approx M_1$, but keep W_2 and M_2 improperly conflated. Finally, blur as usual. On dividing by $\approx$ as recommended above, three equivalence classes result: $[M_2]$ and $[W_2]$, which are not properly conflated, giving us a picture of our unconflated uses of "mass" and "weight"; and $[M_1] = [W_1]$, which is properly conflated, giving us a picture of our conflated uses.

3 Tetravaluations

There is an intuitive model-theoretic treatment available for a restricted version of the above approach, based on *tetravaluations*.

Definition 6 A *tetravaluation for $\mathcal{L}$* (henceforth usually a *valuation*) is a function from $\mathcal{L}$ to $\{1, 0, \odot, \rrbracket\}$.

As with valuational approaches in general, what matters is not what these values *are*, but rather how *many* of them there are, and how they are deployed. In particular, I need to say when a valuation is a counterexample to a sequent.

Definition 7 A valuation v is a *counterexample* to a sequent $\Gamma \rhd \Delta$ (written $v \wr [\Gamma \rhd \Delta]$) if and only if $v(\gamma) = 1$ or $\odot$ for every $\gamma \in \Gamma$ and $v(\delta) = 0$ or $\odot$ for every $\delta \in \Delta$.

Definition 7 ensures that the values 1 and 0 behave in counterexamples just as they ordinarily do: 1 is a value that a premise (of an argument) can take in a counterexample (to that argument), and 0 is a value that a conclusion can take in a counterexample. The additional values $\odot$ and ☽ fill in the other two possibilities: $\odot$ is a value that *either* a premise or a conclusion can take in a counterexample, and ☽ is a value that *neither* a premise or a conclusion can take in a counterexample.

Sets of valuations determine consequence relations in the usual way.

Definition 8 Given a set V of valuations, its associated *consequence relation* $\mathcal{C}(V)$ is $\{s : \neg\exists v \in V(v \wr s)\}$.

Conversely, consequence relations determine sets of valuations, again in the usual way.

Definition 9 Given a set S of sequents, its associated *set of valuations* $\mathcal{V}(X)$ is $\{v : \neg\exists s \in S(v \wr s)\}$.

Fact 3 (Galois connection) *$\mathcal{C}$ and $\mathcal{V}$ form a Galois connection between sets of valuations and consequence relations; that is, $X \subseteq \mathcal{V}(\vdash)$ if and only if $\vdash \subseteq \mathcal{C}(X)$, for any set X of valuations and any consequence relation $\vdash$.*

Proof This is immediate from the definitions: $X \subseteq \mathcal{V}(\vdash)$ if and only if no valuation in X counterexamples any sequent in $\vdash$, which holds in turn if and only if $\vdash \subseteq \mathcal{C}(X)$. □

Fact 4 *For any sets X, Y of valuations and any consequence relations $\vdash, \vdash'$:*

(i) *if $X \subseteq Y$, then $\mathcal{C}(Y) \subseteq \mathcal{C}(X)$;*
(ii) *if $\vdash \subseteq \vdash'$, then $\mathcal{V}(\vdash') \subseteq \mathcal{V}(\vdash)$;*
(iii) *$\mathcal{C} \circ \mathcal{V}$ and $\mathcal{V} \circ \mathcal{C}$ are closure operations;*[12]
(iv) *$\mathcal{C} \circ \mathcal{V} \circ \mathcal{C}(X) = \mathcal{C}(X)$;*
(v) *$\mathcal{V} \circ \mathcal{C} \circ \mathcal{V}(\vdash) = \mathcal{V}(\vdash)$.*

Proof All are immediate from Fact 3 (see, e.g., Galatos, Jipsen, Kowalski, and Ono [13, Lemma 3.7]). □

These facts will be exploited in what follows.

3.1 Closures The closure operations $\mathcal{C} \circ \mathcal{V}$ and $\mathcal{V} \circ \mathcal{C}$ are worthy of study in their own right, as are the closed consequence relations and sets of valuations they give rise to. (I will omit the $\circ$ in what follows, calling these closures simply $\mathcal{CV}$ and $\mathcal{VC}$, respectively.)

First, the closure $\mathcal{CV}$, and the closed consequence relations.

Fact 5 *For any set X of valuations, $\mathcal{C}(X)$ is monotonic.*

Fact 6 *If $\vdash$ is a monotonic consequence relation, then $\vdash = \mathcal{CV}(\vdash)$.*[13]

Things very like tetravaluations are studied in [16], and a proof of Fact 6 can be found there, mutatis mutandis (see [16, Proposition 2, p. 407]). It follows from these facts that the *closed* consequence relations are exactly the *monotonic* ones, and so the closure of a consequence relation is the smallest monotonic consequence relation containing it.

The tetravaluational approach to blurring works best with *closed* consequence relations, and I will restrict my attention in the remainder of the article to monotonic consequence relations for this reason. As we have seen (in Theorem 2), when monotonic consequence relations are blurred, the result is also a monotonic consequence relation, so there is no worry that this restriction will interfere with blurring. (It does, however, narrow the scope of the treatment.)

Now to the closure $\mathcal{VC}$ on sets of valuations. To explore this closure, it helps to define an *information order* on valuations.

Definition 10 Let $\sqsubseteq$ be the smallest partial order on $\{1, 0, \odot, \rightmoon\}$ such that $\rightmoon \sqsubseteq 1, 0 \sqsubseteq \odot$. Extend it to valuations pointwise: $v \sqsubseteq v'$ if and only if $v(A) \sqsubseteq v'(A)$ for all $A \in \mathscr{L}$.

The counterexample relation and the information order interact in a pleasant way, recorded in Fact 7. (This is why the order is defined as it is.)

Fact 7 *We have that $\wr$ is monotonic on the left with regard to $\sqsubseteq$; that is, if $v \wr s$ and $v \sqsubseteq v'$, then $v' \wr s$.*

Proof Let s be $\Gamma \rhd \Delta$. Since $v \wr s$, we know that $v[\Gamma] \subseteq \{1, \odot\}$ and $v[\Delta] \subseteq \{0, \odot\}$. Because $v \sqsubseteq v'$, it follows that $v'[\Gamma] \subseteq \{1, \odot\}$ and $v'[\Delta] \subseteq \{0, \odot\}$; so $v' \wr s$. □

It is now straightforward to characterize the closure of a set of valuations.

Fact 8 *For any set of valuations X, $\mathcal{VC}(X) = \{v : v \sqsubseteq v'$ for some $v' \in X\}$.*

Proof First, to show that $\mathcal{VC}(X) \subseteq \{v : v \sqsubseteq v'$ for some $v' \in X\}$. Suppose $v \in \mathcal{VC}(X)$. Then there is no sequent $s \in \mathcal{C}(X)$ such that $v \wr s$. But consider the sequent $s_v = [\{A : v(A) \in \{1, \odot\}\} \rhd \{A : v(A) \in \{0, \odot\}\}]$. We have $v \wr s_v$, so $s_v \notin \mathcal{C}(X)$. That is, there is some $v' \in X$ such that $v' \wr s_v$. Such a v' must assign 1 or $\odot$ to everything v assigns 1 or $\odot$ to, and it must assign 0 or $\odot$ to everything v assigns 0 or $\odot$ to; that is, $v \sqsubseteq v'$.

Second, to show that $\mathcal{VC}(X) \supseteq \{v : v \sqsubseteq v'$ for some $v' \in X\}$. Suppose $v \sqsubseteq v'$ for some $v' \in X$. Since $v' \in X$, it is not a counterexample to anything in $\mathcal{C}(X)$; so by Fact 7 neither is v. But then $v \in \mathcal{VC}(X)$. □

In what follows, I restrict my attention to closed sets of valuations: those sets X such that $\mathcal{VC}(X) = X$. Restricting both to closed (monotonic) consequence relations and closed sets of valuations in this way gives a tight connection, which will have useful results.

3.2 Blurring With this machinery in hand, it is time to move to blurring, again governed by an equivalence relation $\approx$ on $\mathscr{L}$.

Definition 11 Given a valuation v, $v^{\approx}$ is the valuation such that $v^{\approx}(A) = \min_{\sqsubseteq}\{v(B) : B \approx A\}$. Given a set X of valuations, $X^{\approx} = \mathcal{VC}(\{v^{\approx} : v \in X\})$.

The remainder of the article will show that this is indeed a way of capturing the blurring of Section 2. It is worth noting, however, that the closure $\mathcal{VC}$ inserted in the definition of $X^{\approx}$ does real work: even if the set X is closed, $\{v^{\approx} : v \in X\}$ need not be; $X^{\approx}$, on the other hand, is defined so as to always be closed. In some cases, this would not create much difference, mainly because of Fact 4(iv). However, it will turn out to matter in at least one case (see Theorem 3(iv)); I will return to this issue there.

3.2.1 Preserving relations To show that this valuational blurring is in important respects the same as the purely consequence-theoretic blurring of Section 2, I will explore four relations between consequence relations and sets of valuations, showing that these two ways of blurring preserve these relations. That is, given a consequence relation $\vdash$ related in way R to a set X of valuations, together with a relation $\approx$, the consequence relation $\vdash^{\approx}$ generated by blurring $\vdash$ via the methods of Section 2 is related in way R to $X^{\approx}$.

As usual, I will say that a consequence relation $\vdash$ is *sound* for a set X of valuations if and only if $\vdash \subseteq \mathcal{C}(X)$, and *complete* for X if and only if $\mathcal{C}(X) \subseteq \vdash$. I will also say that X is *plenty big* for $\vdash$ if and only if $\mathcal{V}(\vdash) \subseteq X$, and *small enough* for $\vdash$ if and only if $X \subseteq \mathcal{V}(\vdash)$. Plenty bigness and small enoughness appear not to have standard names; they are the two directions of *absoluteness*, in the sense of [10]. Intuitively, a set of valuations is plenty big for a consequence relation if and only if it includes all the valuations consistent with the relation, and a set of valuations is small enough for a consequence relation if and only if it includes only valuations consistent with the relation.

Two preliminary facts will help. (The proofs are straightforward.)

Fact 9 *For all valuations v, $v^{\approx} \sqsubseteq v$.*

Fact 10 *If $A \approx B$, then $v^{\approx}(A) = v^{\approx}(B)$.*

Now, on to preservation.

Theorem 3

(i) *If $\vdash$ is sound for X, then $\vdash^{\approx}$ is sound for $X^{\approx}$.*
(ii) *If $\vdash$ is complete for X, then $\vdash^{\approx}$ is complete for $X^{\approx}$.*
(iii) *If X is small enough for $\vdash$, then $X^{\approx}$ is small enough for $\vdash^{\approx}$.*
(iv) *If X is plenty big for $\vdash$, then $X^{\approx}$ is plenty big for $\vdash^{\approx}$.*

Proof

Sound: Suppose that $\vdash$ is sound for X, and suppose there is some $w \in X^{\approx}$ such that $w \wr [\Gamma \rhd \Delta]$, to show that $[\Gamma \rhd \Delta] \notin \vdash^{\approx}$. $w \sqsubseteq v^{\approx}$ for some $v \in X$. By Fact 7, $v^{\approx} \wr [\Gamma \rhd \Delta]$, and so by Fact 10, $v^{\approx} \wr [\Gamma' \rhd \Delta']$ for every $[\Gamma' \rhd \Delta'] \approx [\Gamma \rhd \Delta]$. So by Facts 9 and 7, v itself is a counterexample to every such $\Gamma' \rhd \Delta'$. Since $\vdash$ is sound for X and $v \in X$, no such $\Gamma' \rhd \Delta'$ is in $\vdash$. But then $\Gamma \rhd \Delta$ is not in $\vdash^{\approx}$ either.

Complete: Suppose that $\vdash$ is complete for X, and take any sequent $[\Gamma \rhd \Delta] \notin \vdash^{\approx}$, to show that $X^{\approx}$ contains a counterexample to the sequent.

Since $[\Gamma \rhd \Delta] \notin \vdash^{\approx}$, there must be no $[\Gamma' \rhd \Delta'] \in \vdash$ such that $[\Gamma' \rhd \Delta'] \approx [\Gamma \rhd \Delta]$. Let $\Gamma^{\approx} = \{A : A \approx \gamma \text{ for some } \gamma \in \Gamma\}$ and a similar statement hold for $\Delta^{\approx}$; since $[\Gamma^{\approx} \rhd \Delta^{\approx}] \approx [\Gamma \rhd \Delta]$, we know that $\Gamma^{\approx} \nvdash \Delta^{\approx}$.

Since $\vdash$ is complete for X, there is a valuation $v \in X$ such that $v \wr [\Gamma^{\approx} \rhd \Delta^{\approx}]$. This gives $v(\gamma) \in \{1, \odot\}$ for all $\gamma \in \Gamma^{\approx}$ and $v(\delta) \in \{0, \odot\}$ for all $\delta \in \Delta^{\approx}$. Since these sets contain everything blurred with any member of Γ and Δ, respectively, this gives $v^{\approx}(\gamma) \in \{1, \odot\}$ for all $\gamma \in \Gamma$ and $v^{\approx}(\delta) \in \{0, \odot\}$ for all $\delta \in \Delta$. That is, $v^{\approx} \wr [\Gamma \rhd \Delta]$. Since $v \in X$, $v^{\approx} \in X^{\approx}$.

Small enough: Suppose $X \subseteq \mathcal{V}(\vdash)$. By Fact 4(i), $\mathcal{CV}(\vdash) \subseteq \mathcal{C}(X)$. By Fact 4(iii), this gives $\vdash \subseteq \mathcal{C}(X)$. Applying Theorem 3(i), $\vdash^{\approx} \subseteq \mathcal{C}(X^{\approx})$. By Fact 4(ii), $\mathcal{VC}(X^{\approx}) \subseteq \mathcal{V}(\vdash^{\approx})$. But $X^{\approx} = \mathcal{VC}(X^{\approx})$, so $X^{\approx} \subseteq \mathcal{V}(\vdash^{\approx})$.

Plenty big: Suppose $\mathcal{V}(\vdash) \subseteq X$. By Fact 4(i), $\mathcal{C}(X) \subseteq \mathcal{CV}(\vdash)$. By Fact 6 (since $\vdash$ is assumed monotonic), this gives $\mathcal{C}(X) \subseteq \vdash$. Applying Theorem 3(ii), $\mathcal{C}(X^{\approx}) \subseteq \vdash^{\approx}$. By Fact 4(ii), $\mathcal{V}(\vdash^{\approx}) \subseteq \mathcal{VC}(X^{\approx})$. But $\mathcal{VC}(X^{\approx}) = X^{\approx}$, so $\mathcal{V}(\vdash^{\approx}) \subseteq X^{\approx}$. □

Thus, soundness, completeness, small-enoughness, and plenty-bigness are all preserved when the valuational approach is paired with the consequence-theoretic approach of Section 2.

Recall that $X^{\approx}$ is defined as $\mathcal{VC}(\{v^{\approx} : v \in X\})$. If we had left out the closure, defining it instead simply as $\{v^{\approx} : v \in X\}$, Theorems 3(i)–3(iii) would still hold, but Theorem 3(iv) would not. (The last step of the proof is where the trouble would arise. That step needs $\mathcal{VC}(X^{\approx}) \subseteq X^{\approx}$, but this only holds because $X^{\approx}$ is closed.)

3.2.2 Desiderata The desiderata of Section 1.3 were phrased entirely in consequence-theoretic terms, so they do not apply as directly to the valuational approaches I have considered here as they do to the consequence-theoretic approach of Section 2. Nonetheless, we can use the map $\mathcal{C}$ to translate them into the valuational setting.

Suppose we start with a language $\mathcal{L}$, a set X of valuations for $\mathcal{L}$, and an equivalence relation $\approx$ on $\mathcal{L}$, and use the valuational approaches considered above to generate a new set $X^{\approx}$ of valuations. Then our desiderata phrased in terms of $\vdash$ and $\vdash^{\approx}$ can be understood directly as desiderata for $\mathcal{C}(X)$ and $\mathcal{C}(X^{\approx})$. Unsurprisingly, they are satisfied.

Theorem 4 (Desiderata) *The desiderata of Section 2 for $\vdash$ and $\vdash^{\approx}$ are satisfied by $\mathcal{C}(X)$ and $\mathcal{C}(X^{\approx})$, for any set X of valuations. That is, we have the following.*

Intersubstitutivity: *If $[\Gamma, A \rhd \Delta] \in \mathcal{C}(X^{\approx})$ and $A \approx B$, then $[\Gamma, B \rhd \Delta] \in \mathcal{C}(X^{\approx})$, and if $[\Gamma \rhd A, \Delta] \in \mathcal{C}(X^{\approx})$ and $A \approx B$, then $[\Gamma \rhd B, \Delta] \in \mathcal{C}(X^{\approx})$.*

Validity preservation: *We have $\mathcal{C}(X) \subseteq \mathcal{C}(X^{\approx})$.*

Conservativity: *If $[\Gamma \rhd \Delta] \in \mathcal{C}(X^{\approx})$ and $[\Gamma \rhd \Delta] \notin \mathcal{C}(X)$, then there is some $A \in \Gamma \cup \Delta$ that is properly conflated.*

Proof By Theorems 3(i) and 3(ii), $\mathcal{C}(X)^{\approx} = \mathcal{C}(X^{\approx})$. Substituting $\mathcal{C}(X^{\approx})$ for $\mathcal{C}(X)^{\approx}$ in Theorem 1 gives all three desiderata. □

3.3 Comparison with Camp's approach Camp [4], like in this section, gives a way to understand conflation (there called "confusion") that involves tetravaluations. I will wind down by pointing to an important difference between the ways Camp uses tetravaluations and the way I have used them here.[14]

The core difference is captured in Definition 7; as I have used tetravaluations, validity of a sequent is not about *preservation* of any particular status. On the other hand, [4, p. 145] understands validity as preservation of two distinct statuses. As a result, Camp's recommended consequence relation is reflexive, monotonic, and completely transitive, in the sense of Definition 5. (See [29, Theorem 2.1] for the connection between preservation, reflexivity, monotonicity, and complete transitivity.) This means that Camp's approach can be simulated within the tetravaluational approach I have presented without ever using the values $\rrbracket, \odot$ (see footnote 13). But the present approach cannot be simulated within Camp's, as his framework has no room for failures of reflexivity or complete transitivity. Failures of reflexivity might not be too important for understanding conflation; I have allowed for them here mainly for generality's sake, since they are easy to accommodate. However, failures of transitivity are central to the treatment I have recommended; this is the formal reflection of equivocation.

Unfortunately, Camp's logical approach violates one of his own key desiderata (and, in turn, one of mine). In Section 1.3, I discussed Camp's requirement of "inferential charity" in arguing for validity preservation as a desideratum. However, Camp's own approach is not inferentially charitable. It has the result that disjunctive syllogism—the argument form that moves from "A or B" and "Not A" to B—is not valid, owing entirely to the possibility of conflation. At least if disjunctive syllogism is valid in an unconflated language, Camp's recommended approach fails to secure validity preservation.[15]

3.4 Final comments The tetravaluational approach spelled out in this section is not as flexible as the purely consequence-theoretic approach of Section 2, as it requires monotonic consequence relations in order to work at all. But in the presence of monotonic consequence relations, the tetravaluational approach is a convenient model-theoretic match to the more flexible consequence-theoretic approach.

In addition, the connection between monotonic consequence relations and tetravaluations, to my knowledge first explored in [16] but developed further here, provides a convenient tool for exploring nonreflexive and nontransitive relations more generally. Since conflation naturally gives rise to nontransitive consequence relations, it provides one application for this tool, but it is a tool that is likely to be of broader use, just as the connection between "Tarskian" consequence relations and bivaluations has proved to be.[16]

More generally, both the consequence-theoretic approach to blurring presented in Section 2 and the valuational approaches presented in this section satisfy the desiderata defended in Section 1.3 for a logical approach to conflation. Because these approaches satisfy the desiderata, they do not preserve *transitivity* (in any sense) of consequence: they show us how conflation can give rise to equivocation. So I reckon we have here a promising logical approach for understanding the (so far) undertheorized phenomenon of conflation.

Notes

1. There are, of course, existing explorations of conflation and related phenomena. See, for example, Camp [4], Field [11], Frost-Arnold [12], Gupta [14], Lawlor [18], Millikan [20], and Sharp [26]. (Many of these refer to the phenomenon in question as "confusion.")

2. I will not here engage in debate about the nature of propositions; I think what I have to say here can remain largely neutral there.

3. Thanks to Charles Pigden for this example.

4. This is not because I think that conflation of individuals or properties *just is* a certain kind of propositional conflation; I have argued above only that they *result in* propositional

conflation, and this is all I mean to claim. Nonetheless, I will not explore here any ways in which conflation of individuals or properties might go beyond propositional conflation, leaving that instead for other work. Thanks to an anonymous referee for pushing me here.

5. Just as we have mostly moved away from a "logical-truths" conception of logic to a "consequence-relation" conception, so too I reckon we should move from a "truths" conception of meaning to a "consequence-relation" conception. This is obviously too big a point to be argued for here (but see Brandom [2], Dummett [9], Ripley [22]); my use of a consequence relation here is meant simply to leave room for such an approach.

6. The core ideas to follow extend without major modification to substructural logics of all sorts, including those that cannot make do with sets of premises and conclusions in this way; but I will stick with sets here for simplicity. Note, however, that I do not require $\vdash$ to be reflexive, monotonic, transitive, substitution-invariant, compact, or cetera. "Consequence relation," in the sense to be used here, is quite general: *any* set of sequents.

7. An anonymous referee objects, arguing that since "$\exists x(x = c)$ is false when c is a confused name," and $c = c$ is a logical truth, conflation can in fact invalidate otherwise-valid arguments, like introduction of the particular quantifier. Suppose, however, that $\vdash$ is the usual consequence relation of first-order classical logic with equality, which surely validates this argument. Then any approach meeting this desideratum will in fact ensure that $\vdash^{\approx} \exists x(x = c)$; it is the claim that this must be false that is mistaken. As the referee points out, [12] endorses this claim; but I side with Camp in rejecting it. Whether this is a plausible response to the referee's objection turns on issues about the relation between conflation and reference; these are issues I do not have space to explore here.

8. Because of this, the approach as it stands will not interact with any constituent structure $\mathcal{L}$ happens to exhibit. This is perhaps not the best eventual approach, but it is a good place to start, and it is all I will consider here. (It is also part of what allows me to remain neutral between various theories of propositions.)

9. An anonymous referee suggests a different treatment: adding axioms "A iff B" whenever $A \approx B$. Such a treatment can immediately be seen to be more restrictive than blurring, as it requires an object-language biconditional to be present. To evaluate such a treatment with respect to the desiderata, some theory of this biconditional would have to be assumed. In many usual settings, however—say, working over classical logic or intuitionistic logic with their respective biconditionals—the situation is clear: the suggested treatment would achieve intersubstitutivity and validity preservation, but not conservativity. (Section 2.3.2 gives some relevant discussion.)

10. Here, "every partition" should be understood to *include* $\langle \emptyset, \Sigma \rangle$ and $\langle \Sigma, \emptyset \rangle$. (Sometimes these are called *quasipartitions*.)

11. In this example, the conflation of $A \wedge B$ with $A \vee B$ behaves similarly to A tonk B. (See Prior [21] for the original presentation of tonk, and Belnap [1], Cook [8], and Ripley [24] for discussions relevant to the present one.)

12. That is, each of these operations $\mathcal{O}$ is *increasing* ($S \subseteq \mathcal{O}(S)$); *monotonic* (if $S \subseteq T$, then $\mathcal{O}(S) \subseteq \mathcal{O}(T)$); and *idempotent* ($\mathcal{O}(\mathcal{O}(S)) = \mathcal{O}(S)$).

13. Fact 6 is the tetravaluational version of the *abstract soundness and completeness theorem* discussed in Dunn and Hardegree [10] and Humberstone [17]. That theorem imposes reflexivity and complete transitivity as additional conditions, and works with bivaluations (tetravaluations that do not use the values $\odot$ or $☽$); the move to tetravaluations allows us to remove these conditions. Analogous facts hold for *trivaluations*, using either $\{1, ☽, 0\}$ or $\{1, \odot, 0\}$; for the former the needed restriction is to *reflexive* and monotonic consequence relations, and for the latter it is to *completely transitive* and monotonic ones.

14. I focus here on differences between the structures Camp invokes and the structures I invoke; there are also notable differences in the intended interpretations of these structures. I do not go into these differences here, for space reasons. (Very briefly: Camp interprets tetravaluations epistemically, as recording hypothetical advice from "authoritative observers" [5, p. 694].)

15. Camp [4, pp. 158–159] discusses a related objection, which features modus ponens rather than disjunctive syllogism. Camp's response to that objection turns on his having offered no theory of conditionals; but as he *has* offered a theory of disjunction and negation, that reply does not generalize to this version of the objection.

16. For lots of examples of this latter usefulness, see [17].

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