Notre Dame Journal of Formal Logic
Volume 53, Number 1, 2012

# Thoroughly Relativistic Perspectives

Mark Ressler

**Abstract** This article formulates five relative systems to evaluate the charge of self-refutation with regard to global relativism. It is demonstrated that all five of these systems support models with at least one thoroughly relativistic perspective. However, when these systems are extended to include an operator expressing the valuation of statements in a perspective, only one relative system, based on a nonnormal modal logic, supports a thoroughly relativistic perspective.

## 1 Introduction

The question of whether global relativism is self-refuting has been explored by means of formal systems in two notable attempts. The system RL formulated by Hales models a relativity operator on strict analogy with the possibility operator of modal logic, with the peculiar consequence that what is absolute in this system is also relative, arguing that this is a way to model a consistent form of relativism, in Hales [5, 6]. By contrast, Bennigson models a relativity operator more properly on the notion of contingency rather than possibility, with less peculiar results, arguing that the usual problems of self-refutation that appear to arise from such a system reflect "principally the limitations of the proposed system for modeling, not any reason to conclude that global relativism is incoherent," Bennigson [1, p. 20].

The approach I take here is to explore five different relative systems, each of which will ultimately be extended to include an additional operator suggested by a key argument for the self-refutation of relativism. I consider the question of self-refutation in terms of whether there can be a thoroughly relativistic perspective within one of these relative systems, and I demonstrate that there is a system with a model that supports such a perspective, even with the extension of an operator that expresses the valuation of relative and absolute statements within a perspective.

Received April 15, 2010; accepted June 11, 2011; printed April 5, 2012
2010 Mathematics Subject Classification: Primary 03B60; Secondary 03H10
Keywords: relativism, nonnormal modal logic, self-refutation
 10.1215/00294527-1626545

## 2 Five Relative Systems

I suggest that the question of self-refutation with regard to relativism cannot properly be answered on the basis of a single formulation of a relative system. On the one hand, if that single system can be demonstrated to be self-refuting, that does not preclude the formulation of another system that is not self-refuting. On the other hand, if that single system can be demonstrated not to be self-refuting, there seems to be some need to explain why relativism has commonly been thought to be self-refuting, which might be explained on the basis of another system that represents a more common conception of relativism.[1]

Consequently, my strategy with regard to the question of self-refutation of relativism is to explore a wide range of relative systems, and my strategy in generating these systems is to start with a simple relative system and to see how various features of that simple system can be further relativized to yield increasingly more complex systems.

Consider, then, the following five relative systems.

**2.1 RL1: Simple Relativity Semantics** RL1 is a system that models a single kind of relativity.

*2.1.1 Syntax* The language of RL1 consists of

(i) an infinite number of sentence letters: $s_1, s_2, s_3, \ldots$;
(ii) grouping symbols: *(*, *)*; and
(iii) two relativity operators: REL() and ABS().

The well-formed formulas of RL1 are defined as follows:

(a) All sentence letters are well-formed formulas.
(b) If $\alpha$ is a well-formed formula, then so are $\text{REL}(\alpha)$ and $\text{ABS}(\alpha)$.

*2.1.2 Semantics* An interpretation of RL1 is a structure $\langle M, R, v\rangle$, where

(i) $M$ is a relativizing domain in the form of a nonempty set of relativizing factors,
(ii) $R$ is a binary relation on $M$, and
(iii) $v$ is a function that assigns truth values to statements relative to elements in $M$, with the relativity operators assigned values as follows:

$$v_m(\text{REL}(\alpha)) = 1 \text{ iff } v_{m'}(\alpha) \neq v_{m''}(\alpha) \quad \text{for some } m' \text{ and } m'' \text{ where } mRm' \text{ and } mRm'', \text{ and } = 0 \text{ otherwise.}$$
$$v_m(\text{ABS}(\alpha)) = 1 \text{ iff } v_{m'}(\alpha) = v_{m''}(\alpha) \quad \text{for all } m' \text{ and } m'' \text{ where } mRm' \text{ and } mRm'', \text{ and } = 0 \text{ otherwise.}$$

*2.1.3 Comments* This system provides fairly simple support for modeling the relativity operators, where the analogy with the semantics for the modality of possibility and necessity is clear. Note, however, that the semantics for the relativity operators do not correspond directly to the standard modal operators $\Diamond$ and $\Box$, as in Hales's system RL. The REL() and ABS() operators are modeled as contradictories here, since if $\text{ABS}(\alpha) = 0$, then there must be some $m'$ and $m''$ such that $mRm'$ and $mRm''$ where $v_{m'}(\alpha) \neq v_{m''}(\alpha)$, which is precisely the condition for $\text{REL}(\alpha) = 1$.

Notably absent from this system are the usual logical connectives ($\neg, \wedge, \vee, \supset, \equiv$), which may seem to be an unacceptable omission. However, the reason for this omission is that the conception of entailment may vary according to each *perspective* indicated by some $m \in M$. Since the semantics of the logical operators would therefore

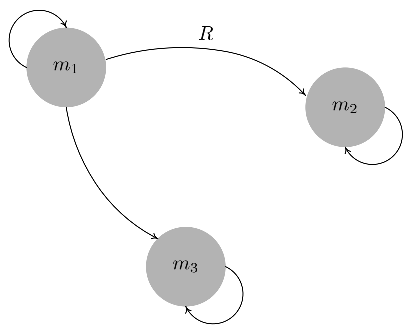


**Figure 1** Sample RL1 System.

effectively be relativized to the relativizing domain, there is no systemwide behavior of these operators to be modeled. Considered independently from the overall relative system, particular perspectives may have their own semantics for logical operators, but since these semantics may not be common for all perspectives, these logical features drop out of the overall system. Consequently, statements containing these logical operators are valued as propositional parameters, rather than analyzing them further into atomic statements and connectives.

RL1 and the following systems are formulated as sentence logics without predication or quantification, for the sake of simplicity. I suggest that the omission or addition of these features will not affect the behavior of the two relativity operators. Yet if the usual logical connectors are excluded from these systems because the semantics for these symbols may vary accordingly by perspective, then it would seem that the semantics for the quantification operators should likewise vary according to those perspectives. If there is variation in the semantics for quantification, then it seems that there would indeed be some obstacle in extending these relative systems to include predication and quantification, contrary to my previous suggestion. If the possibility of adding quantification is precluded from these systems, then there may be further concerns about the expressibility of the thesis of global relativism within the systems, for example, in the form of the statement '$(\forall\alpha)\textsc{rel}(\alpha)$'.

Yet I think these concerns may be circumvented if any difference in semantics for quantification and predication between perspectives is implemented according to the pattern of a nonnormal modal logic, as will be done with the relativity operators in system RL5 below. According to a nonnormal system of predication, elements of the relativizing domain would be divided into normal and nonnormal quantificational classes, where the valuations of sentences according to the normal elements would follow the standard semantics for predication and quantification, and the nonnormal elements would assign values to the quantificational symbols arbitrarily in accordance with their deviant conception of quantification. Of course, if the quantificational symbols could be modeled this way, then it would seem that the remaining logical operators could be modeled in the same way. Unfortunately, this implementation would result in an extremely complex formal system, even for the simplest relative system, RL1. Furthermore, the implementation of nonnormal semantics for the usual connectives and quantifiers would affect only the valuation of the base statements to which the relativity operators are applied. So for the purposes of modeling the relativity operators, statements containing these connectives and quantifiers could effectively be treated merely as propositional parameters, thereby avoiding needless complexity.

In this way, the thesis of global relativism could thereby be expressed in an extended system of RL1 that includes predication and quantification. Within the formulation of RL1 as a sentence logic as presented here, however, the thesis of global relativism would be expressed merely as one of the sentence letters, such as $s_r$, which would be valued true or false in various perspectives. While the representation of the thesis of global relativism as a mere sentence letter seems to leave the logical behavior of the instance of the relativity operator $\textsc{rel}()$ unanalyzed within $s_r$, possibly raising concerns about equivocation in the statement of relativism, I suggest that this representation be understood merely as an expedient that stands in place of a full implementation of nonnormal quantification.

Given the absence of logical connectors and quantifiers from RL1 as presented, no axioms including those symbols are included in the system. Even if RL1 were extended to include a nonnormal implementation of these symbols, the deviance in the semantics of these symbols suggests that no axioms would be acceptable universally across all perspectives. While it is tempting to think that modeling the two relativity operators as contradictories should warrant the inclusion of an axiom expressing that relationship, such an axiom would need to rely on some logical connector, most probably the biconditional, and a difference in the conception of that connector would preclude universal acceptability of such an axiom.

With regard to proof theory, consider two possible rules of inference employing the relativity operators, suggested on analogy with the Rule of Necessitation in modal logics of possibility and necessity:

$$\text{Rule of Absolution:} \quad \text{If } \vdash \alpha \text{ then } \vdash \textsc{abs}(\alpha) \tag{1}$$

$$\text{Rule of Relativization:} \quad \text{If } \vdash \alpha \text{ then } \vdash \textsc{rel}(\alpha) \tag{2}$$

Since there are no axioms in RL1, these rules hold vacuously. Consequently, nothing can be proved on the basis of these rules. Nor do any other rules of inference seem appropriate here. Even if the system were extended to include the usual logical connectors and quantifiers, the potential deviance in the conception of the connectors would cast doubt on the universality across perspectives of any rules of inference based on these connectors. For this reason, no proof theory will be provided for RL1 or any of the other systems presented below.[2]

**2.2 RL2: Multiple Relativities** RL2 is a system that models several kinds of relativity, with multiple relativizing domains and multiple corresponding accessibility relations.

*2.2.1 Syntax* The language of RL2 consists of

(i) an infinite number of sentence letters: $s_1, s_2, s_3, \ldots$;
(ii) grouping symbols: *(*, *)*;
(iii) a finite number $n$ of specific relativity operators: $\textsc{rel}_1()$, $\textsc{rel}_2()$, …, $\textsc{rel}_n()$; $\textsc{abs}_1()$, $\textsc{abs}_2()$, …, $\textsc{abs}_n()$; and
(iv) two general relativity operators: $\textsc{rel}()$ and $\textsc{abs}()$.

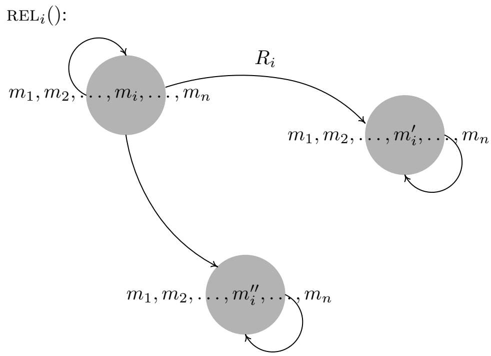


**Figure 2** Sample RL2 System.

The well-formed formulas of RL2 are defined as follows:

(a) All sentence letters are well-formed formulas.
(b) If $\alpha$ is a well-formed formula, then so are $\textsc{rel}(\alpha)$, $\textsc{abs}(\alpha)$, $\textsc{rel}_i(\alpha)$, and $\textsc{abs}_i(\alpha)$ for every $0 < i \leq n$.

*2.2.2 Semantics* An interpretation of RL2 is a structure $\langle \mathfrak{M}, \mathfrak{R}, v \rangle$, where

(i) $\mathfrak{M} = \langle M_1, M_2, \ldots, M_n \rangle$, where each $M_i$ is a relativizing domain in the form of a nonempty set of relativizing factors,
(ii) $\mathfrak{R} = \langle R_1, R_2, \ldots, R_n \rangle$, where each $R_i$ is a binary relation on $M_i$, and
(iii) $v$ is a function that assigns truth values to statements relative to combinations of elements from each $M_i$, such as $v_{m_1,m_2,\ldots,m_n}(s_1) = 1$ or 0, each $m_i \in M_i$. The relativity operators are assigned values as follows:

$$v_{m_1,m_2,\ldots,m_i,\ldots,m_n}(\textsc{rel}_i(\alpha)) = 1 \text{ iff } v_{m_1,m_2,\ldots,m_i',\ldots,m_n}(\alpha) \neq v_{m_1,m_2,\ldots,m_i'',\ldots,m_n}(\alpha)$$

for some $m_i'$ and $m_i''$, where $m_i R_i m_i'$ and $m_i R_i m_i''$, and $= 0$ otherwise.

$$v_{m_1,m_2,\ldots,m_i,\ldots,m_n}(\textsc{abs}_i(\alpha)) = 1 \text{ iff } v_{m_1,m_2,\ldots,m_i',\ldots,m_n}(\alpha) = v_{m_1,m_2,\ldots,m_i'',\ldots,m_n}(\alpha)$$

for all $m_i'$ and $m_i''$, where $m_i R_i m_i'$ and $m_i R_i m_i''$, and $= 0$ otherwise.

$$v_{m_1,m_2,\ldots,m_n}(\textsc{rel}(\alpha)) = 1 \text{ iff } v_{m_1,m_2,\ldots,m_n}(\textsc{rel}_i(\alpha)) = 1$$

for some $i$ and $= 0$ otherwise.

$$v_{m_1,m_2,\ldots,m_n}(\textsc{abs}(\alpha)) = 1 \text{ iff } v_{m_1,m_2,\ldots,m_n}(\textsc{abs}_i(\alpha)) = 1$$

for all $i$ and $= 0$ otherwise.

*2.2.3 Comments* This system concurrently models multiple varieties of relativity, such as moral relativism, conceptual relativism, and the special theory of relativity. Each kind of relativity is indexed numerically, with separate relativizing domains and separate corresponding accessibility relations. Even where the relativizing domains are the same for two different kinds of relativity, for instance, where two different domains of discourse are both relative to cultures, it may be that the accessibility relations still differ for some reason. Perhaps standards of theory evaluation are shared with regard to one domain of discourse but are not shared with regard to another, for example. However this situation may arise, relativizing domains and accessibility relations are kept separate in this system.

The evaluation of a particular kind of relativity is always made with regard to a *compound perspective*, namely, a perspective that is constructed from elements from each relativizing domain. So if there are three relativizing domains, one containing cultures, the second conceptual schemes, and the third inertial frameworks, a compound perspective would be that of a particular culture using a particular conceptual scheme within a particular inertial framework. When evaluating cultural relativism from a compound perspective, according to this example, the valuations of cultures accessible to the evaluating culture are considered, but only those valuations where the conceptual schemes and frameworks of inertial motion are the same as those of the evaluating compound perspective. Alternate conceptual schemes accessible to the compound perspective's conceptual scheme are irrelevant to the evaluation of cultural relativism, only to conceptual relativism. More generally, when a particular kind of relativity is evaluated, valuations of accessible elements of the corresponding relativizing domain are considered only where the elements of the other relativizing domains are held constant.

The rationale behind the semantics of the two general relativity operators is that a statement is generally relative if it is specifically relative in any way, and generally absolute if it is specifically absolute in every way and therefore relative in no way. This system therefore enables the formulation of a claim of global relativism in which everything is relative in the sense that every statement is relative according to some kind of relativity, though not necessarily that every statement is relative according the same kind of relativity.

RL1 is clearly a special case of RL2 in which there is only a single kind of relativity.[3]

**2.3 RL3: Relativized Accessibility Relations** RL3 is a system that models a single kind of relativity in which the accessibility relations are themselves relativized.

*2.3.1 Syntax* As in RL1.

*2.3.2 Semantics* An interpretation of RL3 is a structure $\langle M, \mathfrak{R}, v \rangle$, where

(i) $M$ is a relativizing domain in the form of a nonempty set of relativizing factors,

(ii) $\mathfrak{R} = \langle R_{m_1}, R_{m_2}, \ldots, R_{m_n} \rangle$, where
  (a) each $m_i \in M$,
  (b) each $R_{m_i}$ is a binary relation on $M$, and

(iii) $v$ is a function that assigns truth values to statements relative to ordered pairs of elements in $M$ such as $v_{m_1,m_2}(s_1) = 1$ or 0 with the relativity operators assigned values as follows:

$$v_{m_1,m_2}(\text{REL}(\alpha)) = 1 \text{ iff } v_{m_1,m'_2}(\alpha) \neq v_{m_1,m''_2}(\alpha)$$

for some $m'_2$ and $m''_2$, where $m_2 R_1 m'_2$ and $m_2 R_1 m''_2$, and $= 0$ otherwise.

$$v_{m_1,m_2}(\text{ABS}(\alpha)) = 1 \text{ iff } v_{m_1,m'_2}(\alpha) = v_{m_1,m''_2}(\alpha)$$

for all $m'_2$ and $m''_2$, where $m_2 R_1 m'_2$ and $m_2 R_1 m''_2$, and $= 0$ otherwise.

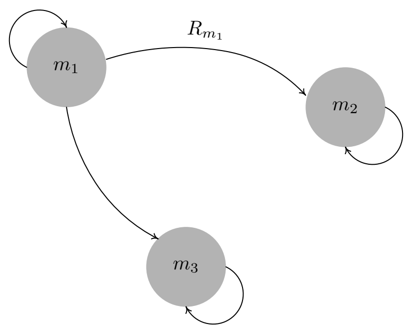


**Figure 3** Sample RL3 System from the perspective of $m_1$.

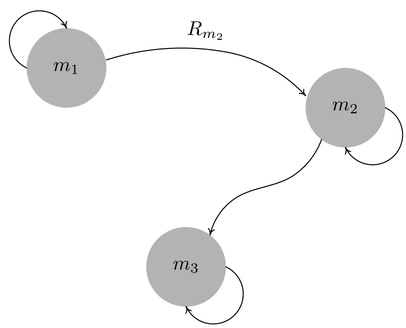


**Figure 4** Sample RL3 System from the perspective of $m_2$.

*2.3.3 Comments* The valuation relative to ordered pairs $\langle m, m' \rangle$ of elements in the relativizing domain should be understood as the valuation of the statement for $m'$ relative to $m$'s perspective. In this example, call $m$ the *evaluating perspective* and $m'$ the *evaluated perspective*. Where $m = m'$, the valuation is for a perspective from its own perspective. To make the relativization of accessibility relations in RL3 distinct from RL1, the valuations of statements in perspectives must likewise be relativized to these two perspectives. If valuations had been relativized simply to a single element of the relativizing domain, then the semantics for the relativity operators would have referenced only $mR_m m'$ for valuations according to $m$, with the consequence that the only elements of the relativized accessibility relations that were relevant to the semantics were elements where the first term was the element of the relativizing domain to which the relations were relativized, namely, $m$. In such a case, these relativized relations could easily be reduced to a single accessibility relation in an RL1 system without any variation in the valuations of the relativity operators.

This system permits the relativization of judgments of relativity itself to perspectives. The semantics permit the valuation of a statement to be relative for a perspective from its own perspective, but absolute for that perspective from another perspective. RL3 therefore represents a more radical kind of perspectivism than either RL1 or RL2. Since valuations are made for a perspective from a perspective, this system seems particularly well suited to model what has come to be known as assessor contextualism as opposed to agent contextualism, where judgments in agent contextualism are made relative to the agent's context, and judgments in assessor contextualism are made relative to an assessor's context.[4] Not only are the valuations of atomic statements made relative to an assessor's context here, but the valuations of the relativity operators are likewise relativized by means of the relativized accessibility relations. This relativization can be given some sense by noting that the conception of the meaning of these accessibility relations can vary according to perspectives. According to different conceptions of the nature and function of these accessibility relations, the relations themselves will differ by perspective.

Here again, it seems that RL1 can be understood as a special case of RL3, where the relativized accessibility relations are all identical and where the valuation of statements for $m$ from the perspective of $m'$ are identical to the valuations of statements for $m$ from $m$'s own perspective.

**2.4 RL4: Multiple Relativities and Relativized Accessibility Relations** Systems RL2 and RL3 introduced two different complicating factors to produce more advanced systems than RL1. Yet both of these complicating factors might likewise be modeled within a single relative system. RL4 therefore models both several different kinds of relativities and the relativization of accessibility relations.

*2.4.1 Syntax* As in RL2.

*2.4.2 Semantics* An interpretation of RL4 is a structure $\langle \mathfrak{M}, \mathfrak{R}, v \rangle$, where

(i) $\mathfrak{M} = \langle M_1, M_2, \ldots, M_n \rangle$, where each $M_1$ is a relativizing domain in the form of a nonempty set of relativizing factors;

(ii) $$\mathfrak{R} = \langle \begin{array}{llll} \langle R_{1,p_1}, & R_{1,p_2}, & \ldots, & R_{1,p_q} \rangle \\ \langle R_{2,p_1}, & R_{2,p_2}, & \ldots, & R_{2,p_q} \rangle \\ \langle \ldots, & \ldots, & \ldots, & \ldots \rangle \\ \langle R_{n,p_1}, & R_{n,p_2}, & \ldots, & R_{n,p_q} \rangle \end{array} \rangle, \text{ where}$$

(a) each $R_{i,p_j}$ is a binary relation on $M_i$,

(b) each $p_j$ is a combination of elements from each $M_i$ such as $p_1 = \langle m_1, m_2, \ldots, m_n \rangle$,

(c) $\langle p_1, p_2, \ldots, p_q \rangle$ is an enumeration of all compound perspectives; and

(iii) $v$ is a function that assigns truth values to statements relative to ordered pairs of $p_j$ such as $v_{p_1,p_2}(s_1) = 1$ or 0, with the relativity operators assigned values as follows:

(a) $v_{p_1,p_2}(\text{REL}_i(\alpha)) = 1$ iff $v_{p_1,p'_2}(\alpha) \neq v_{p_1,p''_2}(\alpha)$ for some $p'_2$ and $p''_2$, where

- $m_j \in p_2,\ p'_2,\ p''_2$ for all $j \neq i$, and
- $m_i R_{i,p_1} m'_i$ and $m_i R_{i,p_1} m''_i$ where $m_i \in p_2, m'_i \in p'_2$, and $m''_i \in p''_2$;

and = 0 otherwise.

(b) $v_{p_1,p_2}(\text{ABS}_i(\alpha)) = 1$ iff $v_{p_1,p'_2}(\alpha) = v_{p_1,p''_2}(\alpha)$ for all $p'_2$ and $p''_2$, where

- $m_j \in p_2,\ p'_2,\ p''_2$ for all $j \neq i$, and

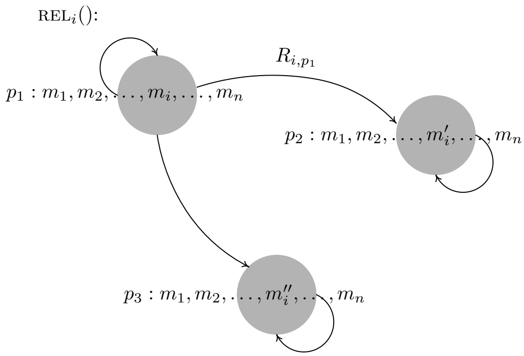


**Figure 5** Sample RL4 System from the perspective of $p_1$.

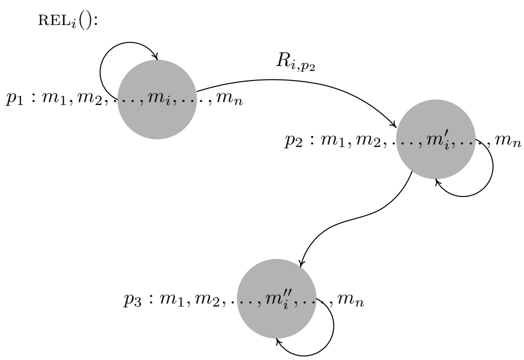


**Figure 6** Sample RL4 System from the perspective of $p_2$.

- $m_i R_{i,p_1} m_i'$ and $m_i R_{i,p_1} m_i''$ where $m_i \in p_2$, $m_i' \in p_2'$, and $m_i'' \in p_2''$;

and = 0 otherwise.

(c) $v_{p_1,p_2}(\text{REL}(\alpha)) = 1$ iff $v_{p_1,p_2}(\text{REL}_i(\alpha)) = 1$ for some $i$, and = 0 otherwise.

(d) $v_{p_1,p_2}(\text{ABS}(\alpha)) = 1$ iff $v_{p_1,p_2}(\text{ABS}_i(\alpha)) = 1$ for all $i$, and = 0 otherwise.

*2.4.3 Comments* The use of $p_j$ as a symbol for a compound perspective $\langle m_1, m_2, \ldots, m_n \rangle$ is simply an attempt to increase the readability of this very complex semantic system, where the cross-relativizations can be difficult to follow. As in RL3, valuations are always made to ordered pairs of perspectives, but as in RL2, these perspectives are compound perspectives compounded out of one element from each relativizing domain associated with a specific kind of relativity. Call the first compound perspective in this ordered pair an *evaluating context* and the second an *evaluated context*. So a particular kind of relativity or absoluteness for a statement is evaluated for an evaluated context from the perspective of an evaluating context.

A specific kind of relativity ($\text{REL}_i()$) is evaluated true in this way when there are two evaluated contexts ($p_2'$ and $p_2''$) that give different valuations when evaluated according to the evaluating context ($p_1$), with the following conditions: (1) the two evaluated contexts and the initial evaluated context ($p_2$) must all share elements ($m_j$) from every other relativizing domain besides the domain associated with the specific kind of relativity being evaluated, and (2) the elements ($m_i'$ and $m_i''$) of the two evaluated contexts from the relativizing domain associated with the specific kind of relativity being evaluated must both be accessible from the element ($m_i$) of that relativizing domain belonging to the initial evaluated context, according to the accessibility relation ($R_{i,p_1}$) for that kind of relativity according to the evaluating context. Similarly for a specific kind of absoluteness. The general relativity operators follow straightforwardly according to the pattern in RL2, but relativized to combinations of evaluating and evaluated contexts.

RL1, RL2, and RL3 all form special cases of RL4. If $n$ is 1, then there is only a single kind of modality, and RL4 reduces to RL3. If each compound perspective values sentences for other perspectives the same way that each perspective values itself, and the accessibility relations are all identical, then the relativization of accessibility relations becomes trivial, and RL4 effectively reduces to RL2. If both of these situations obtain, then RL4 effectively reduces to RL1. Given the complexity of this system, however, it seems preferable for expository purposes to articulate the three earlier systems prior to the presentation of this system.

**2.5 RL5: Nonnormal Relativity** Systems RL1 through RL4 above are all normal modal systems. RL5 is a system that embodies a nonnormal modal logic, while modeling a single kind of relativity with a single nonrelativized accessibility relation.

*2.5.1 Syntax* As in RL1.

*2.5.2 Semantics* An interpretation of RL5 is a structure $\langle M, N, R, V\rangle$, where

- (i) $M$ is a relativizing domain in the form of a nonempty set of relativizing factors,
- (ii) $N$ is a subset of $M$,
- (iii) $R$ is a binary relation on $M$, and
- (iv) $v$ is a function that assigns truth values to statements relative to elements in $M$, with the relativity operators assigned values as follows:
  - (a) If $m \in N$, then
    - $v_m(\text{REL}(\alpha)) = 1$ iff $v_{m'}(\alpha) \neq v_{m''}(\alpha)$ for some $m'$ and $m''$, where $mRm'$ and $mRm''$; and = 0 otherwise;
    - $v_m(\text{ABS}(\alpha)) = 1$ iff $v_{m'}(\alpha) = v_{m''}(\alpha)$ for all $m'$ and $m''$, where $mRm'$ and $mRm''$; and = 0 otherwise.
  - (b) If $m \notin N$, then
    - $v_m(\text{REL}(\alpha))$ is arbitrary;
    - $v_m(\text{ABS}(\alpha))$ is arbitrary.

*2.5.3 Comments* Following the pattern of nonnormal modal logics, this system divides the perspectives contained within $M$ into two classes: normal perspectives

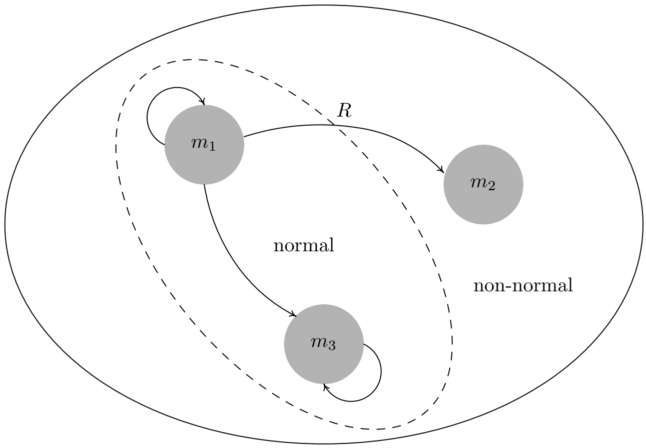


**Figure 7** Sample RL5 System.

where $m \in N$, and nonnormal perspectives where $m \notin N$. The semantics for the relativity operators are the same as in RL1 for normal perspectives, but for nonnormal perspectives, REL$(\alpha)$ and ABS$(\alpha)$ are valued arbitrarily. Consequently, RL5 would be the relative analogue of the modal system S0.5.

This system is motivated by a consideration of the proposed Rule of Absolution above in Section 2.1.3, formulated on analogy with the Rule of Necessitation in modal logics of possibility and necessity, where modal systems accepting the Rule of Necessitation are known as normal modal logics. The Rule of Absolution is not a rule that is likely to be adopted within a thoroughly relativistic perspective, so it would seem that any self-refutation argument that employed a logic presupposing the Rule of Absolution would beg the question against relativism. Although none of the systems RL1 through RL4 specify any proof theory for the reasons discussed in Section 2.1.3, the semantics for those four earlier systems were analogous to the semantics for normal modal logics. RL5 is therefore formulated explicitly to provide the semantics of a nonnormal relative system.

RL1 is clearly a special case of RL5, where $N = M$. The same kinds of complications as in RL2 through RL4, namely, multiple relativities and relativized accessibility relations, might likewise be incorporated into nonnormal relative systems, thus yielding three further systems, RL6, RL7, and RL8. Consequently, it would seem that system RL8, featuring multiple relativities, relativized accessibility relations, and nonnormal semantics, should properly be considered the logic of relative systems, since every other system would form a special case of RL8. However, for the purposes of this article, it will be clearer to consider systems RL1 through RL5 separately, to focus first on the effects of each complicating factor in isolation, then in combination in the case of RL4. Since the nonnormal features of RL5 appear as exceptions to the normal semantics that hold only for nonnormal perspectives, it does not seem that the combination of such nonnormal features with the other two complicating factors would pose any significant issues beyond the consideration of those factors in isolation.

## 3 Self-Refutation

Not all charges of self-refutation against relativism can be addressed by means of an analysis of particular logical systems such as self-refutation charges based upon the nature of language. With regard to those self-refutation charges for which logical analysis is feasible, an appropriate conception of self-refutation is required. For example, Passmore's analysis of absolute self-refutation depends upon the contradictory nature of a particular thesis, Passmore [12], but it is not clear that this conception of self-refutation would properly apply to a position such as dialethism that explicitly countenances contradictions. Rather than a case of self-refutation, the contradiction in question might represent an instance of self-consistency in dialethism. Since relativism likewise seems to countenance contradictions in a certain way, special care must be taken with regard to the operative conception of self-refutation deployed against relativism, else the purported contradiction might merely represent an instance of relativization consistent with the thesis of relativism.

Given these considerations, I propose that global relativism be understood as a perspective that adopts a certain relative logic and locates itself as a particular perspective within that logic, with the further claim that all statements according to its perspective are relative. I propose further that self-refutation with regard to such relativism be understood as a demonstration that no perspective of global relativism can be located within that logic, since there is no perspective within that logical system according to which all statements are relative. Since there are multiple relative systems that could be adopted, self-refutation with regard to relativism would need to show further that there is no logical system in which a perspective of global relativism could be located.

Suppose then that a relativist were to adopt one of the five proposed relative systems as representing the logic of a claim of global relativism. Is that claim self-refuting? This question depends upon whether that system can support a thoroughly relativist perspective, namely, one in which for every sentence the relativity operator REL() holds true when applied to that sentence. For RL1, this evaluation proceeds fairly easily. RL1 is not self-refuting in this regard, which can be proved as follows.

**Theorem 3.1** *There is an* RL1 *model in which* $v_m(\text{REL}(\alpha)) = 1$ *for every* $\alpha$ *for some* $m$.

Let $\langle M, R, v\rangle$ be an RL1 model as follows:

$$M = \{m_1, m_2\}.$$
$$R = \{\langle m_1, m_1\rangle, \langle m_1, m_2\rangle, \langle m_2, m_2\rangle\}.$$

Let $v_{m_1}(s_i)$ assign values arbitrarily for every simple sentence $s_i$. Let $v_{m_2}(s_i) = 1$ where $v_{m_1}(s_i) = 0$ and $v_{m_2}(s_i) = 0$ where $v_{m_1}(s_i) = 1$. According to the theorem, $v_{m_1}(\text{REL}(\alpha)) = 1$ for all $\alpha$.

**Proof** The proof proceeds by induction on the level of nesting of the relativity operators, since by the formation rules for RL1, $\alpha$ is either a simple sentence $s_i$ or a sentence of the form $\text{REL}(\beta)$ or $\text{ABS}(\beta)$.

*Base case*: $\text{REL}(\alpha)$, where $\alpha$ is one of $s_1, s_2, s_3, \ldots$

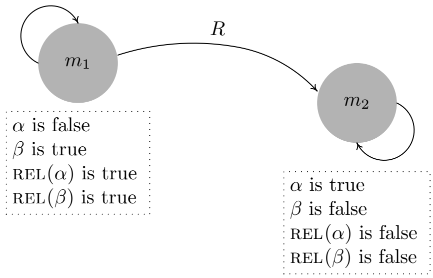


**Figure 8** Model for Theorem 3.1.

According to the definition of $v$ in the model:

$$\text{If } v_{m_1}(s_i) = 1, \text{ then } v_{m_2}(s_i) = 0;$$
$$\text{If } v_{m_1}(s_i) = 0, \text{ then } v_{m_2}(s_i) = 1;$$
$$m_1 \text{ can access both } m_1 \text{ and } m_2.$$

Therefore, for every basic sentence $s_i$, there are two perspectives, $m_1$ and $m_2$ accessible to $m_1$ where $v_{m_1}(s_i) \neq v_{m_2}(s_i)$, so $v_{m_1}(\text{REL}(s_i)) = 1$. This establishes the base case for the first level, where $\alpha$ is a basic sentence with no nested relativity operators.

*Inductive step*: $\text{REL}(\alpha)$, where $\alpha$ is either $\text{REL}(\beta)$ or $\text{ABS}(\beta)$

Suppose that $\alpha$ is $\text{REL}(\beta)$, with some level of nested relativity operators in $\beta$. Then $v_{m_1}(\text{REL}(\beta)) = 1$ for all $\beta$, by the inductive hypothesis. Since $m_2$ can access only itself, in general, $v_{m_2}(\text{REL}(\gamma)) = 0$ for all $\gamma$, so $v_{m_2}(\text{REL}(\beta)) = 0$. Therefore, there are two perspectives, $m_1$ and $m_2$ accessible to $m_1$ where $v_{m_1}(\text{REL}(\beta)) \neq v_{m_2}(\text{REL}(\beta))$, so $v_{m_1}(\text{REL}(\text{REL}(\beta))) = 1$.

Suppose now that $\alpha$ is $\text{ABS}(\beta)$. $v_{m_1}(\text{REL}(\beta)) = 1$ for all $\beta$, by the inductive hypothesis, so $v_{m_1}(\text{ABS}(\beta)) = 0$. Since $m_2$ can access only itself, in general, $v_{m_2}(\text{ABS}(\gamma)) = 1$ for all $\gamma$, so $v_{m_2}(\text{ABS}(\beta)) = 1$. Therefore, there are two perspectives, $m_1$ and $m_2$ accessible to $m_1$ where $v_{m_1}(\text{ABS}(\beta)) \neq v_{m_2}(\text{ABS}(\beta))$, so $v_{m_1}(\text{REL}(\text{ABS}(\beta))) = 1$.

So if $v_{m_1}(\text{REL}(\beta)) = 1$, where $\beta$ contains some level of nesting of relativity operators, then $v_{m_1}(\text{REL}(\text{REL}(\beta))) = 1$ and $v_{m_1}(\text{REL}(\text{ABS}(\beta))) = 1$ proving the inductive step. Therefore, according to the model, $v_{m_1}(\text{REL}(\alpha)) = 1$ for all $\alpha$. □

So RL1 is not self-refuting, since there is a model in which there is a thoroughly relativistic perspective, namely, $m_1$ in the proof. However, since RL1 is a special case of each of RL2 through RL5, as noted earlier, models structurally equivalent to this RL1 model can also be used to demonstrate that these other relative systems are also not self-refuting. So the following theorems can be proved using the same essential model and the same technique as in Theorem 3.1.

**Theorem 3.2** *There is an* RL2 *model in which* $v_p(\text{REL}(\alpha)) = 1$ *for every* $\alpha$ *for some compound perspective* $p$.

**Theorem 3.3** *There is an* RL3 *model in which* $v_{m,m'}(\text{REL}(\alpha)) = 1$ *for every* $\alpha$ *for some evaluating perspective* $m$ *and some evaluated perspective* $m'$.

**Theorem 3.4** *There is an* RL4 *model in which* $v_{p,p'}(\text{REL}(\alpha)) = 1$ *for every* $\alpha$ *for some evaluating context* $p$ *and some evaluated context* $p'$.

**Theorem 3.5** *There is an* RL5 *model in which* $v_m(\text{REL}(\alpha)) = 1$ *for every* $\alpha$ *for some* $m$.

While the RL1 model from Theorem 3.1 will suffice for the proofs of these theorems, more complex models reflecting the advanced relativistic features of each system can be devised to prove these theorems as well. The strategy for devising such models can be generalized from the RL1 model, as follows. First, allow $v$ to assign values to simple sentences such that two perspectives directly contradict each other. Second, allow the designated relativistic perspective full access to all perspectives, including itself, but give the remaining nonrelativistic perspectives reflexive access only. Consequently, nonrelativistic perspectives will hold all sentences to be absolute, which will therefore allow all sentences in the relativistic perspective to be relative at any level of nesting of the relativity operators.[5]

However, it would be premature to conclude that relativism in general is therefore not self-refuting, since there is a more sophisticated pattern of self-refutation argument that can be addressed by an analysis of these logical systems, a pattern that I will call the *For-x objection*. The argument of this objection is that if the language of each system is extended to allow it to express the valuations of sentences according to particular perspectives, sentences concerning what is true or false for a perspective turn out to be absolute for all perspectives.[6] None of the five proposed relative systems has a language sufficiently powerful to express the valuation of sentences according to perspective. So the languages of each of these systems will need to be extended in order to evaluate this objection.

Accordingly, let the language of RL1 be extended to include an unlimited number of names $a_1, a_2, a_3, \ldots$, and a corresponding number of operators $\text{FOR}_x()$ where $x$ is one of the names, such as $\text{FOR}_{a_1}()$. The rules for well-formed sentences are extended to include the following:

If $\alpha$ is a sentence, then so is $\text{FOR}_x(\alpha)$ for all operators $\text{FOR}_x()$.

The model structure for RL1 will likewise need to be extended to $\langle M, I, R, v\rangle$ where $I$ is an index function mapping each name $a_n$ either to some member of the relativizing domain $M$ or to the empty set (if $M$ does not contain the empty set) in the event that the name does not refer to anything in the relativizing domain. The semantics of the $\text{FOR}_x()$ operators will be as follows:

$v_m(\text{FOR}_x(\alpha)) = 1$ if and only if

(a) $I$ maps $x$ to some element $m'$ in the relativizing domain,
(b) $m'$ is accessible to $m$ according to $R$, and
(c) $v_{m'}(\alpha) = 1$;
and = 0 otherwise.

Clause (a) ensures that some perspective is successfully referenced in the $\text{FOR}_x()$ operator. Call clause (a) the *reference condition*. Clause (b) is needed since if one entire perspective is not accessible to another in the valuation of the relativity operators, then it does not seem proper to allow the second perspective to be able to

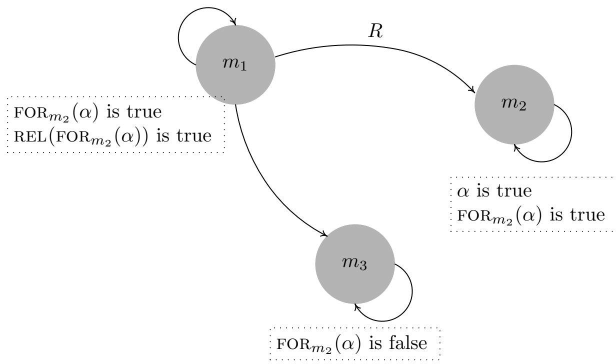


**Figure 9** Failure of For-x objection for true sentences in RL1+F.

discuss the values of sentences in the first perspective. Call clause (b) the *accessibility condition*. Clause (c) simply transmits the valuations of true sentences through the accessibility relation. Call clause (c) the *valuation condition*.

Call this extended system RL1+F. Given this language extension, the For-x objection claims the following:

$$v_m(\text{ABS}(\text{FOR}_x(\alpha))) = 1 \text{ for all } \alpha, \text{ all } m, \text{ and all } x. \tag{3}$$

I think this claim is not justified for all sentences. I think that RL1+F models can easily be devised according to which all true sentences to which the FOR$_x$() operator is applied will be relative. The strategy is similar to the general strategy for finding thoroughly relativistic models mentioned earlier. Designate one perspective to be relativistic. Let a different, nonrelativistic perspective value each sentence such that it and the relativistic perspective contradict each other. Let a different nonrelativistic perspective value each sentence arbitrarily. Arrange the accessibility relations such that the relativistic perspective can access all perspectives, including itself, but the nonrelativistic perspectives can access only themselves. Consequently, since the accessibility condition fails in each nonrelativistic perspective with regard to the FOR$_x$() operator with regard to other perspectives, the FOR$_x$() operator can be valued true in one perspective but false in another, allowing the relativistic perspective to hold the FOR$_x$() operator to be relative. This strategy is illustrated in Figure 9, where $m_1$ is the designated relativistic perspective.

Yet with regard to false sentences in RL1+F, the situation is quite different. Here the valuation condition of the semantics for the FOR$_x$() operator fails for all false sentences, so it turns out that the application of the FOR$_x$() operator to false sentences becomes absolute. This can be proved as follows.

**Theorem 3.6** *In every model of* RL1+F, $v_m(\text{REL}(\text{FOR}_x(\alpha))) = 0$ *for every m, where I maps x to m′ and* $v_{m'}(\alpha) = 0$.

**Proof** Consider an arbitrary RL1+F model. Consider an arbitrary sentence $\alpha$ and an arbitrary $m' \in M$ where $v_{m'}(\alpha) = 0$. Let $I$ map $a'$ to $m'$.

Since the valuation condition (c) of the semantics for the $\text{FOR}_x()$ operator in RL1+F fails, given that $v_{m'}(\alpha) = 0$ by hypothesis, $v_m(\text{FOR}_{a'}(\alpha)) = 0$ for every $m$. Therefore, for every $m$, there are no $m''$ and $m'''$ where $v_{m''}(\text{FOR}_{a'}(\alpha)) \neq v_{m'''}(\text{FOR}_{a'}(\alpha))$. So $v_m(\text{REL}(\text{FOR}_{a'}(\alpha))) = 0$ for every $m$. □

So the For-x objection succeeds against system RL1+F. Whereas the demonstrations in Theorems 3.1 through 3.5 did not critically require the additional relativistic features of RL2 through RL5, it seems that these features might be required to evaluate the For-x objection. Perhaps one of these systems will have sufficient relativistic resources to refute the For-x objection. Consequently, those systems must likewise be extended with suitable $\text{FOR}_x()$ operators.

I will discuss RL5 later, since it poses special problems with regard to this objection. Let the languages of RL2 through RL4 be extended as in RL1+F. Let the semantics for the $\text{FOR}_x()$ operators in these systems be as follows:

(i) RL2+F: $v_{m_1,m_2,\ldots,m_n}(\text{FOR}_x(\alpha)) = 1$ iff
  (a) $I$ maps $x$ to some compound perspective $\langle m'_1, m'_2, \ldots, m'_n\rangle$ where each $m'_i$ is an element from the corresponding relativizing domain $M_i$,
  (b) $m'_i$ is accessible to $m_i$ according to $R_i$ for every $i$, and
  (c) $v_{m'_1,m'_2,\ldots,m'_n} = 1$, and = 0 otherwise.

(ii) RL3+F: $v_{m_1,m_2}(\text{FOR}_x(\alpha)) = 1$ iff
  (a) $I$ maps $x$ to some element $m'_2$ in the relativizing domain,
  (b) $m'_2$ is accessible to $m_2$ according to $R_{m_1}$, and
  (c) $v_{m_1,m'_2}(\alpha) = 1$, and = 0 otherwise.

(iii) RL4+F: $v_{p_1,p_2}(\text{FOR}_x(\alpha)) = 1$ iff
  (a) $I$ maps $x$ to some compound perspective $p'_2$,
  (b) for every position $i$, where $m_i \in p_2$ and $m'_i \in p'_2$, $m'_i$ is accessible to $m_i$ according to $R_{i,p_1}$, and
  (c) $v_{p_1,p'_2}(\text{FOR}_x(\alpha)) = 1$, and = 0 otherwise.

The semantics for each of the $\text{FOR}_x()$ operators contain three clauses similar to the conditions for the $\text{FOR}_x()$ operator in RL1+F, namely, a reference condition, an accessibility condition, and a valuation condition, as appropriate to the particular system. Since each system also contains a valuation condition that does not differ significantly from the valuation condition of RL1+F, and Theorem 3.6 depends solely upon the valuation condition of RL1+F, the same general proof for self-refutation against RL1+F in Theorem 3.6 will also apply with regard to RL2+F through RL4+F, demonstrating that thoroughly relativistic perspectives cannot be modeled in these systems, given the absoluteness of the application of the $\text{FOR}_x()$ operator to false sentences. Specifically, the following theorems can be proved.

**Theorem 3.7** *In every model of* RL2+F, $v_{m_1,m_2,\ldots,m_n}(\text{REL}(\text{FOR}_x(\alpha))) = 0$ *for every compound perspective* $\langle m_1, m_2, \ldots, m_n\rangle$*, where* $I$ *maps* $x$ *to a compound perspective* $\langle m'_1, m'_2, \ldots, m'_n\rangle$*, and* $v_{m'_1,m'_2,\ldots,m'_n}(\alpha) = 0$.

**Theorem 3.8** *In every model of* RL3+F, $v_{m,m'}(\text{REL}(\text{FOR}_x(\alpha))) = 0$ *for every* $m$ *and* $m'$*, where* $I$ *maps* $x$ *to* $m''$*, and* $v_{m,m''}(\alpha) = 0$.

**Theorem 3.9** *In every model of* RL4+F, $v_{p,p'}(\text{REL}(\text{FOR}_x(\alpha))) = 0$ *for every* $p$ *and* $p'$*, where* $I$ *maps* $x$ *to* $p''$*, and* $v_{p,p''}(\alpha) = 0$.

So whereas the accessibility condition of the semantics for the $\text{FOR}_x()$ operators can be used to generate models that avoid the For-x objection for true sentences, the valuation condition of those semantics enables the For-x objection to succeed against false sentences. Furthermore, it would seem that the reference condition could likewise be used to prove the For-x objection against any model in these systems, if there are names in the language that are not mapped to any perspective in the system. The failure of the reference condition would force the $\text{FOR}_x()$ operators for unmapped names to be valued false for every perspective in the system, thereby forcing those $\text{FOR}_x()$ operators to be absolute. Yet since the use of the valuation condition for false sentences is sufficient to prove the For-x objection for RL1+F through RL4+F, I will not pursue any proofs on the basis of a failure of the reference condition. Therefore, it would seem that the $\text{FOR}_x()$ operator is fatal to global relativism. The novel result here is that it is not the truths in a perspective that cause problems for global relativism, but rather the falsehoods. The additional relativistic features of multiple relativities and relativized accessibility relations were insufficient to counter this charge of self-refutation.

Perhaps, though, it may be thought that the addition of the $\text{FOR}_x()$ operator begs the question against relativism, on the grounds of interpretive charity. If the proposed relative systems turn out to be self-refuting only with the addition of the $\text{FOR}_x()$ operator, then perhaps the $\text{FOR}_x()$ operator is not properly part of the claim of relativism and therefore cannot be used to show that the relative systems are self-refuting. While I think it is important to consider the countercharge of begging the question against relativism, I do not think this particular countercharge succeeds. I would argue that it is precisely because of the claims of relativism that anyone would need to speak of something being true for a perspective. An absolutist would have no reason to introduce a $\text{FOR}_x()$ operator, because it would seem to introduce a vacuous contrast, since everything true is true for every perspective under absolutism, and likewise for what is false. Consequently, not only does the introduction of the $\text{FOR}_x()$ operator not beg the question against relativism, it seems to be an important part of the claim of relativism, since the relativist perspective in which the doctrine of relativism is enunciated must be able to talk about the valuations of sentences according to different perspectives in order to articulate how the relativity operators work. If the original formulations of RL1 through RL5 resisted the charge of self-refutation, it seems they did so only by suppressing certain features of relativism whose consequences needed to be evaluated. It is the extensions of RL1 through RL5 including the $\text{FOR}_x()$ operator that should properly be proposed as candidate logics of relativism. Unfortunately, the complications introduced in the normal systems RL2, RL3, and RL4 seem to have been insufficient to resist the charge of self-refutation.

However, I have delayed discussing system RL5 and its extension including a $\text{FOR}_x()$ operator. The problem with this system is that there are at least two viable ways to formulate the semantics of the $\text{FOR}_x()$ operator in RL5+F. According to the first option, since RL5 is similar to RL1 insofar as it lacks multiple relativities and relativized accessibility relations, the semantics for the $\text{FOR}_x()$ operator in RL5+F could be the same as in RL1+F. If the semantics are formulated in this way, there would still be a way in which a thoroughly relativistic perspective could appear within RL5+F. Since nonnormal perspectives value the $\text{REL}()$ operator arbitrarily, then let there be a nonnormal perspective that always values $\text{REL}(\text{FOR}_x(\alpha)) = 1$ for every $\alpha$. Then this nonnormal perspective would be a thoroughly relativistic perspective that could refute the For-x objection.

This alternative might seem to be a fairly unartful dodge to escape the self-refutation charge, but I think there is a more serious objection to be made against it. While this alternative would appear to provide the logical model needed to counter the charge of self-refutation, there is an argumentative problem with this strategy. The five relative systems have been proposed as systems that a relativist might endorse as representing the logic of global relativism. As such, each system articulates the behavior of the relativity operators within perspectives, one of which is understood to be a relativistic perspective. The problem is that if the relativistic perspective were identified with a nonnormal perspective in which every sentence is arbitrarily taken to be relative, then the putatively relativistic perspective would seem to belie the very relativistic principles that it is articulating.[7] Nonnormal perspectives are anarchistic in their treatment of the relativity operators, so if global relativism can only be articulated from within a nonnormal perspective, then global relativism would seem not to be relativism at all but anarchy or even blatant irrationality, as some have thought. This would seem to be a straightforward kind of self-refutation, whereby the claim of relativism ultimately undermines itself. Consequently, the global relativist cannot properly claim that the thoroughly relativistic perspective is a nonnormal one. It must be a normal perspective in which the behavior of the relativity operators is exactly as relativism says it should be.

According to the second option, though, the presence of nonnormal perspectives in RL5+F demands different treatment with regard to the semantics of the $\textsc{for}_x()$ operator from the treatment according to RL1+F. This differential treatment should follow analogously to the treatment of the relativity operators. Let the semantics for the $\textsc{for}_x()$ operator in RL5+F be as follows:

RL5+F: If $m \in N$, then $v_m(\textsc{for}_x(\alpha)) = 1$ iff
(a) $I$ maps $x$ to some element $m'$ in the relativizing domain,
(b) $m'$ is accessible to $m$ according to $R$, and
(c) $v_{m'}(\alpha) = 1$, and = 0 otherwise.
If $m \notin N$, then $v_m(\textsc{for}_x(\alpha))$ is assigned arbitrarily.

If $m$ is a normal perspective, then it adopts the semantics for the $\textsc{for}_x()$ operator as in RL1+F, but if it is a nonnormal perspective, then it assigns values to the operator arbitrarily, as it does for the relativity operators. Under this alternative, the For-x objection fails in RL5+F, which can be proved as follows.

**Theorem 3.10** *There is an* RL5+F *model in which* $v_m(\textsc{rel}(\alpha)) = 1$ *for every* $\alpha$ *for some normal perspective* $m$.

Let $\langle M, N, I, R, v\rangle$ be an RL5+F model as follows:

$$\begin{aligned} M &= \{m_1, m_2\},\\ N &= \{m_1\},\\ I &= \{\langle a_1, m_1\rangle, \langle a_2, m_2\rangle, \langle a_3, \emptyset\rangle, \ldots\},\\ R &= \{\langle m_1, m_1\rangle, \langle m_1, m_2\rangle, \langle m_2, m_2\rangle\}. \end{aligned}$$

Let $v_{m_1}(s_i)$ assign variables arbitrarily for every simple sentence $s_i$. Let $v_{m_2}(s_i) = 1$ where $v_{m_1}(s_i) = 0$, and $v_{m_2}(s_i) = 0$ where $v_{m_1}(s_i) = 1$.

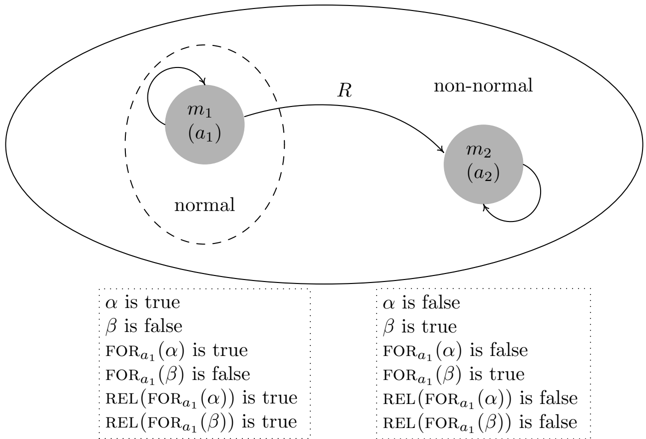


**Figure 10** Model for Theorem 3.10.

The normal perspective $m_1$ will value the REL(), ABS(), and $\text{FOR}_x()$ operators according to the semantics for normal perspectives in RL5+F, but the semantics for nonnormal perspectives allow $m_2$ to value those operators arbitrarily. Let the valuation for $m_2$ be as follows:

Let $v_{m_2}(\text{REL}(\alpha)) = 1$ where $v_{m_1}(\text{REL}(\alpha)) = 0$, and $v_{m_2}(\text{REL}(\alpha)) = 0$ where $v_{m_1}(\text{REL}(\alpha)) = 1$.
Let $v_{m_2}(\text{ABS}(\alpha)) = 1$ where $v_{m_1}(\text{ABS}(\alpha)) = 0$, and $v_{m_2}(\text{ABS}(\alpha)) = 0$ where $v_{m_1}(\text{ABS}(\alpha)) = 1$.
Let $v_{m_2}(\text{FOR}_x(\alpha)) = 1$ where $v_{m_1}(\text{FOR}_x(\alpha)) = 0$, and $v_{m_2}(\text{FOR}_x(\alpha)) = 0$ where $v_{m_1}(\text{FOR}_x(\alpha)) = 1$.[8]

Consequently, the valuation of every sentence is contradictory between the two perspectives, even for the relativity and $\text{FOR}_x()$ operators. Then $v_{m_1}(\text{REL}(\alpha)) = 1$ for every $\alpha$ in the normal perspective $m_1$.

**Proof** According to the definition of $v$ for simple sentences $s$,

$$v_{m_1}(s) \neq v_{m_2}(s).$$

According to the definition of $v$ for the nonnormal perspective $m_2$ with regard to the REL(), ABS(), and $\text{FOR}_x()$ operators,

$$v_{m_1}(\text{REL}(\beta)) \neq v_{m_2}(\text{REL}(\beta)),$$
$$v_{m_1}(\text{ABS}(\beta)) \neq v_{m_2}(\text{ABS}(\beta)),$$
$$v_{m_1}(\text{FOR}_x(\beta)) \neq v_{m_2}(\text{FOR}_x(\beta)),$$

for every $\beta$.

So whether $\alpha$ is a simple sentence or an application of the REL(), ABS(), and $\text{FOR}_x()$ operators, there are two perspectives, $m_1$ and $m_2$ accessible to $m_1$ where $v_{m_1}(\alpha) \neq v_{m_2}(\alpha)$, so $v_{m_1}(\text{REL}(\alpha)) = 1$ for every $\alpha$. □

Note that since the valuation of the nonnormal perspective is always contradictory to the normal perspective, this model can overcome the For-x objection based not only on failure of the valuation condition, but also on the failure of the reference condition. If there is some $x$ that does not name any perspective in the model, the normal perspective will value the corresponding $\text{FOR}_x()$ operator false for every sentence, but the nonnormal perspective will always value the $\text{FOR}_x()$ operator true in accordance with the model, thereby allowing the normal perspective to value that $\text{FOR}_x()$ operator to be relative, even on failure of reference.

This second alternative likewise might seem to be a dodge, an ad hoc maneuver that is completely unmotivated except as a way to escape the self-refutation charge. To the contrary, I would argue that this alternative is precisely what global relativism requires. For something to be relative, there must be a disagreement between perspectives. Under global relativism, this disagreement should extend also to the nature of relativism and to the behavior of the relativity operators. Given such a disagreement, some perspectives cannot be expected to assign values to the relativity operators according to the semantic rules dictated by relativism. Consequently, from a normal relativistic perspective, those deviant perspectives would seem to assign values to the relativity operators arbitrarily as nonnormal perspectives, though there may be perfectly comprehensible rules for these valuations from within their own perspectives.[9] In this way, the use of a nonnormal system would seem to be required by global relativism. Systems RL1 through RL4 might be suitable for more restricted forms of relativism, and therefore the formulations of these systems was not completely pointless, but RL5 seems conceptually more consonant with the radical nature of global relativism. If nonnormal perspectives can be justified in their deviant valuations of the relativity operators in this way, then the same justification can extend to the valuation of the $\text{FOR}_x()$ operator to counter the For-x objection. The deviance in the semantics of these operators within nonnormal perspectives merely represents the disagreement that relativism requires as a claim of radical difference between perspectives applied reflexively to its own logic. The nonnormal semantics simply provide a fairly elegant way to incorporate situations into a logical system where that logic does not apply.[10]

This point may be pushed further by exploring an additional line of objection. What about statements concerning whether relativism is self-refuting? If there is some relative system that allows global relativism not to be self-refuting, then it may seem that relativism is absolutely not self-refuting, and thereby self-refuting after all. However, since there are four other systems in which a claim of global relativism would be self-refuting, there seems to be some grounds for claiming that relativism is only relatively self-refuting, relative to the system in which it is modeled. Yet what about statements concerning whether relativism is self-refuting according to system RL5+F? It would seem that these statements cannot themselves be relative to a particular logical system.[11]

To address this latter objection, the question of expressibility must first be addressed. Just as the expressibility of the $\text{FOR}_x()$ operator in the languages of the proposed relative systems affected the proper evaluation of the self-refutation charge against the claims of global relativism, so will the expressibility of whether a particular system is self-refuting or not. If sentences concerning self-refutation are evaluated merely as propositional parameters with no internal logical structure, the valuation of such sentences would be treated arbitrarily in the various relative systems without correlating such sentences to the conditions under which a system is self-refuting. So the languages of the proposed relative systems must be extended to include terms, operators, and predicates sufficient to express whether a particular system is self-refuting or not, and the semantics of those systems must be specified. Suppose this to be done. In RL1 through RL4 as well as in the normal perspectives in RL5, the semantics for these new terms and operators can be assumed to be uniform across perspectives. However, for nonnormal perspectives in RL5, those semantics would be expected to be different, for precisely the same reasons that the relativity operators and the $\text{FOR}_x()$ operator had different semantics, namely, that global relativism posits a disagreement even on the behavior of these terms and operators, and this disagreement is represented within nonnormal perspectives. Consequently, there may be nonnormal perspectives within an RL5 model that consider RL5 to be self-refuting even though that system is not self-refuting with regard to thoroughly relativistic normal perspectives in that same model, thus justifying the claim that RL5 is only relatively non-self-refuting within an extension of RL5.

It might seem incoherent or at least bizarre that a perspective within a system would hold a position on that system contrary to that system's metatheory, but this supposed incoherence seems not to take the perspectival nature of these systems seriously. Within any of the proposed relative systems, there can be perspectives that are thoroughly absolutist. From the absolutist perspective, that perspective is not part of a relative system at all, but standing proudly and absolutely alone. Yet that perspective's judgment on itself does not preclude it from being included as a subtheory within a relative system.[12] Likewise, a nonnormal perspective holding RL5 to be self-refuting would not consider itself to be part of RL5 at all, but would consider itself to be standing proudly and deviantly separate from such self-refuting nonsense. Yet this deviant attitude, deviant from a normal, thoroughly relativistic RL5 perspective, does not preclude it from being included within an RL5 system. What would seem to be incoherent is a perspective that held RL5 to be self-refuting, while locating itself within an RL5 system, just as it would seem incoherent if a perspective claimed itself to be thoroughly relativistic in a system that could not support such a perspective, which is the effective nature of self-refutation with regard to these relative systems. Yet whatever semantics are outlined for the extension of the system for the terms and operators allowing the expressibility of self-refutation in the language, these semantics would fall under the same kinds of arguments I have outlined above showing that RL5 is not self-refuting even with the addition of the $\text{FOR}_x()$ operator. So if semantics for a language expressing self-refutation could be devised to extend these relative systems, the deviance of nonnormal perspectives in an extension of RL5 should enable the construction of a model according to which a sentence claiming that this very system is not self-refuting could be valued by a normal perspective to be both true and relative.

## 4 Conclusion

Each of the five relative systems that were formulated to evaluate the charge of self-refutation against relativism were shown to support thoroughly relativistic perspectives, but only with the omission of an operator expressing the valuation of sentences within perspectives. With the addition of such an operator, only a nonnormal relative system could support a thoroughly relativistic perspective. Yet the claim of global relativism seems precisely to require a nonnormal relative system for its formulation, one in which the nature of relativity itself is allowed to be relative. From a formal perspective, then, it seems that global relativism is not self-refuting after all. It is simply not normal.

## Notes

1. Indeed, Hales appreciates the need for such an explanation, but thinks it can be addressed from within the single system he presents [5, pp. 37–39].

2. Therefore, the logic of relative systems does not seem to be a logic at all in a traditional sense, since nothing can be proved using the logic. Rather, relative systems consist solely of systems of semantics on which arguments can be based in some metalanguage.

3. There is a further question whether RL2 can be reduced to RL1, at least with regard to the general relativity operators, since compound perspectives can be taken to form elements of a single relativizing domain. The key question is whether there is a transformation of the multiple accessibility relations in an RL2 system into a single accessibility relation in an RL1 system that preserves the valuations of the two general relativity operators. I suspect that there are RL2 models where such a transformation is not available, particularly with regard to reflexive elements among compound perspectives, since the valuation of some specific relativity operators may rely critically on the inclusion of such reflexive elements in the relation, while others may rely critically on the exclusion of such elements from the relation. A proof would be needed to substantiate this suspicion. Still, since there clearly seems to be a need to model multiple kinds of relativity separate from the general relativity operators, even if a reduction of RL2 to RL1 for the general relativity operators were possible on these grounds, that would not eliminate the theoretical interest in RL2 systems.

4. See, for example, Lyons [7], MacFarlane [8] and [9].

5. I should note that this strategy is not completely satisfactory with regard to RL2 and RL4, which model several kinds of relativity. The problem is that in order for a designated relativistic perspective in these systems to value every statement to be thoroughly relative, it would seem that all the nonrelativistic perspectives must be identical. The evaluation of statements according to compound perspectives across the various accessibility relations seems to undermine the attempt to make one perspective relativistic with all other perspectives absolute. At some level of nesting of the relativity operators, it seems that the valuation of the REL() operator becomes false for all perspectives unless all absolute perspectives have identical subtheories. I have noticed this situation in attempting to devise models for RL2 and RL4, though I have not formulated a proof. There may be a relation between the number of kinds of relativity, the number of elements in the relativizing domains, and the level of nesting at which statements start to become absolute, though I have not established such a relation yet. While the formal requirements for relativity do not require a one-to-one relation between the relativizing domain and the range of relativized subtheories, I have some suspicions concerning relativistic claims in which many perspectives share the same subtheory. Although this situation does not affect the argument here, it may prove to be significant for someone who might make a claim of global relativism. Since there would indeed appear to be multiple kinds of relativity that could and perhaps should be modeled together in a global system, the way

that these multiple relativities seem to undermine global claims of relativity at certain levels of nesting may prove problematic. In such a case, the relativistic claim would need to argue either that the relativizing domain could be redefined to reduce the degree of agreement between perspectives, or that there is nothing suspicious about large numbers of perspectives sharing the same theory.

6. This style of argument appears in Passmore [12, p. 68], Burnyeat [3, pp. 192–94], Putnam [14, p. 121], Pinto [13], Mosteller [11, p. 11], and Boghossian [2, pp. 54–57].

7. Indeed validity in a nonnormal system, which I have not considered in the formulation of the proposed relative systems, is typically defined in terms of normal perspectives.

8. Note that these valuation conditions for $m_2$ are well-defined, since the valuation of the REL(), ABS(), and FOR$_x$() operators for $m_1$ are stipulated to occur according to the semantics for normal perspectives prior to the valuation of these operators for $m_2$.

9. Perhaps Hales, with his alternative account of the behavior of the relativity operators as duals, would count as inhabiting such a nonnormal perspective, as well as Margolis with his antirealist conception of relativism, in Margolis [10].

10. Similarly, with regard to nonnormal logics of the modality of possibility, such logics might be understood to incorporate situations where possibly the behavior of possibility and necessity differs from the system being articulated, in other words, that the nature of possibility is possibly different.

11. A comparable objection might be raised with regard to the contradictory nature of the two relativity operators. Should not sentences expressing that contradictory nature be absolute across all perspectives? The response is that they should not, because such sentences require the expression of biconditional and negation operators, but the semantics for such operators were allowed to vary across perspectives. The argument concerning nonnormal perspectives that follows will apply likewise in this case. While I have adopted a particular normal perspective in outlining the semantics for the relativity operators in this study, nonnormal perspectives cannot be expected to agree with my perspective.

12. For a comparable point, see van Haaften [4].

## References

[1] Bennigson, T., "Is relativism really self-refuting?" *Philosophical Studies*, vol. 94 (1999), pp. 211–36. 89

[2] Boghossian, P. A., *Fear of Knowledge: Against Relativism and Constructivism*, Oxford University Press, Oxford, 2006. 111

[3] Burnyeat, M. F., "Protagoras and self-refutation in Plato's *Theaetetus*," *The Philosophical Review*, vol. 85 (1976), pp. 172–95. 111

[4] van Haaften, W., "Relativism and absolutism: How can both be right?" *Metaphilosophy*, vol. 27 (1996), pp. 324–26. 111

[5] Hales, S. D., "A consistent relativism," *Mind*, vol. 106 (1997), pp. 33–52. MR 1438989. 89, 110

[6] Hales, S. D., *Relativism and the Foundations of Philosophy*, The MIT Press, Cambridge, 2006. 89

[7] Lyons, D., "Ethical relativism and the problem of incoherence," *Ethics*, vol. 86 (1976), pp. 107–21. 110

[8] MacFarlane, J., "Future contingents and relative truth," *Philosophical Quarterly*, vol. 53 (2003), pp. 321–36. 110

[9] MacFarlane, J., "Making sense of relative truth," *Proceedings of the Aristotelian Society*, vol. 105 (2005), pp. 321–39. 110

[10] Margolis, J., *The Truth about Relativism*, Blackwell, Oxford, 1991. 111

[11] Mosteller, T., *Relativism in Contemporary American Philosophy*, Continuum Studies in American Philosophy. Continuum, London, 2006. 111

[12] Passmore, J. A., *Philosophical Reasoning*, Basic Books, New York, 1961. 100, 111

[13] Pinto, R. C., "Inconsistency, rationality and relativism," *Informal Logic*, vol. 17 (1995), pp. 279–88. 111

[14] Putnam, H., *Reason, Truth and History*, Cambridge University Press, Cambridge, 1981. 111

**Acknowledgments**

This article has been extracted from a monograph tentatively entitled *The Logic of Relativism*, to be published later, in which the nature of relativism and the charges of self-refutation against relativism are explored in greater detail. My thanks to Graham Priest for his assistance in the course of this research.

mark@markressler.com