(Astrologers were interested in this phenomenon!) All of these are shape theoretic features, and give examples of the way in which such presentations of the shape space can be instructive.

I hope that these remarks will help to fill out the fascinating paper by Bookstein, and will at the same time indicate the additional advantages which can be gained from global geometrical studies, in supplementation of the less exacting but equally relevant local ones.

ADDITIONAL REFERENCES

BOWER, F. O. (1930). *Size and Form in Plants.* Macmillan, London.

KENDALL, D. G. (1984). Shape manifolds, procrustean metrics, and complex projective spaces. *Bull. London Math. Soc.* **16** 81–121.

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Comment

Noel Cressie

Professor Bookstein has ably presented a synthesis of the "size or shape variables" approach and the "deformation" approach within the field of morphometrics. He has developed a number of results which can be used to test for association between shape and size, and for shape differences between groups. These results exploit the relative spatial locations of landmarks (well defined "summary" points that are biologically apparent in each specimen under study), whence one is able to summarize the geometry of the specimen. This is clearly a desirable direction to take. Professor Bookstein has shown us the inadequacy of the first approach, which uses very little spatial information, and has given us a way of making statistical inferences within the second approach, which up to now has been essentially descriptive.

There are other ways to analyze size and shape; the approach of the so-called "mathematical morphology" school of Matheron and Serra (Serra, 1982) in France, is trying to solve a different problem. Automated cytological examinations of a smear enable rapid objective detection of deformed cells, whose frequency of occurrence is typically only a minute fraction of the total number of cells examined. The use of landmark techniques here is clearly not appropriate, although one is still interested in (perhaps more gross) deformations of cells. Here the geometry of the specimen (represented as a set) is of paramount importance; morphological transformations that take set into set do not destroy the spatial nature of the problem. After a suitable series of transformations (e.g., smoothing, convexifying, etc.) on the set, the *final* stage is to calculate real valued summaries (e.g., volume, surface area, connectivity number, etc.).

The great advantage of mathematical morphology is that it is applicable in $\mathbb{R}^n$, $n \geqq 1$, whereas Bookstein is not yet able to make the jump from $\mathbb{R}^2$ to $\mathbb{R}^3$. With this in mind, there is the notion of "the skeleton" (see Serra, 1982, Chapter XI) of an object, which may be useful in the analysis of landmark data. It can essentially be obtained (in $\mathbb{R}^2$) in the following graphic way. Imagine the object to be a bounded field of grass, and a fire is started at the same instant at every point on the boundary. The fire burns inward at constant speed, so that there is a line in the field where the fire is reached simultaneously from at least two different points on the boundary. When applying this idea to jawbones for example, it may suggest landmarks (the various "junctions" of the skeleton) and curvatures (the part of the skeleton linking the "junctions"). This may be an answer to the question of the statistical sufficiency of landmarks to describe the specimen, and at the same time of the generalizability to $\mathbb{R}^n$.

There is another possibility for generalization to higher dimensions contained in the article by Kendall (1984). He considers k-ads in m dimensions (e.g., here we have $k = 3$, $m = 2$), and constructs shape spaces and shape measures on these spaces. The mathematics is formidable, but there is the formalism for dealing with higher dimensional landmark data.

I would like to join Professor Bookstein in his concern that the null statistical model of identical uncorrelated normal perturbations at each landmark, is unrealistic. Would he comment on what happens when the bivariate Gaussian distribution is correlated, perhaps differently from landmark to landmark, and also when another distribution is controlling the perturbations? (Kendall's shape measures would generate a very general type of model.) I am worried about the robustness of the approach, and I think finding exploratory ways of checking the assumptions made about these spatial data is an open and interesting problem.

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