

fact that the items in the sample are subject to different environmental conditions.

#### ADDITIONAL REFERENCES

- BROWN, M. (1984). Inequalities for distributions with increasing failure rate. In *Contributions to the Theory and Applications of Statistics* (A. E. Gelfand, ed.). Academic, Orlando, Fla.
- BROWN, M. and GE, G. (1984). Exponential approximations for two classes of aging distributions. *Ann. Probab.* **12** 869-875.

- JACOBSEN, M. (1986). Right censoring and the Kaplan-Meier and Nielson-Aalen estimators. Preprint No. 6, Institute of Math. Statistics, Univ. Copenhagen.
- KORDONSKY, KH. B., et al. (1986). Estimation of reliability parameters for several competing failure causes (in Russian). *Izv. Akad. Nauk SSSR Tekhn. Kibernet.* **24** (6).
- KORDONSKY, KH. B. and RASTRIGIN, V. L. (1985). Random censoring on the phase space trajectories (in Russian). *Izv. Akad. Nauk SSSR Tekhn. Kibernet.* **23** (6).

## Comment

Asit P. Basu

In this survey the authors describe four major areas of reliability theory. The number of areas that could be discussed under reliability is quite broad, as can be seen from the list of topics at the research conferences on reliability at the University of Missouri-Columbia in 1984, 1986 and the forthcoming 1988 (see, for example, Basu, 1986). Here I shall limit my comments to some statistical problems that have not been addressed in detail in the survey. The journal *Teoriya Veroyatnostei i ee Primeneniya (Theory of Probability and Its Applications)* contains a number of useful articles in the field. Although Rukhin and Hsieh did not mention the journal in their list, they mentioned some of the articles from the journal.

An important problem has been to consider unbiased estimates of the reliability function  $\bar{F}(t) = P(T > t)$ . A survey of this is given in Basu (1985). The problem of unbiased estimates of reliability, however, is a special case of the general problem of unbiased estimation studied in depth by Kolmogorov (1950). Kolmogorov's work has inspired considerable research on unbiased estimates of reliability (see, for example, Lumel'skii and Sapozhnikov (1969), who also considered unbiased estimates for multivariate normal distributions and multivariate discrete distributions). The case of the multivariate normal distribution was also studied independently by Ghurye and Olkin (1969) in the USA, whereas Klein and Basu (1985) have considered bivariate exponential distributions. It may be of interest to note that, because of Kolmogorov's work, the Rao-Blackwell theorem is also referred to as the Rao-Blackwell-Kolmogorov theorem in the Russian literature.

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Similarly, because of the extensive contribution of Gnedenko, the Weibull distribution is also referred to as the Weibull-Gnedenko distribution in the Russian literature (see, for example, Savvushkina and Tyurin, 1984).

Besides Weibull and exponential distributions, other models have also been considered. For example, Volodin (1974) has considered the discrimination of gamma and Weibull distributions assuming a generalized gamma distribution as model.

This is an important survey of Soviet work on reliability, and I would like to thank the authors for their important contributions. I wish we had a more exhaustive survey of the area. I hope that additional Soviet books and papers on reliability will be translated into English.

#### ACKNOWLEDGMENT

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#### ADDITIONAL REFERENCES

- BASU, A. P. (1985). Estimating the reliability of complex systems—a survey. In *The Frontiers of Modern Statistical Inference Procedures* (E. J. Dudewicz, ed.) 271-287. American Sciences Press, Columbus, Ohio.
- BASU, A. P., ed. (1986). *Reliability and Quality Control*. North-Holland, Amsterdam.
- GHURYE, S. G. and OLKIN, I. (1969). Unbiased estimation of some multivariate probability densities and related functions. *Ann. Math. Statist.* **40** 1261-1271.
- KLEIN, J. P. and BASU, A. P. (1985). Estimating reliability for bivariate exponential distribution. *Sankhyā Ser. B* **47** 346-353.
- KOLMOGOROV, A. N. (1950). Unbiased estimators. *Izv. Akad. Nauk SSSR Ser. Mat.* **14** 303-326.

- LUMEL'SKII, Y. P. and SAPOZHNIKOV, P. N. (1969). Unbiased estimates of density functions. *Theory Probab. Appl.* **14** 357-364.
- SAVVUSHKINA, N. Y. and TYURIN, Y. N. (1984). An agreement

- criticon for the Weibull-Gnedenko distribution. *Engrg. Cybernetics* **22** 113-117.
- VOLODIN, I. N. (1974). On the discrimination of gamma and Weibull distributions. *Theory Probab. Appl.* **19** 383-390.

# Rejoinder

Andrew L. Rukhin and H. K. Hsieh

We would like to thank all the discussants for their encouraging comments. Their additional accounts on Soviet research in reliability are informative and certainly will be welcomed by readers.

Professors Barlow and Khalil's firsthand historic review of the activities at Gnedenko's School of Reliability at Moscow University should give readers a fresh look at Gnedenko's contribution to reliability research. Professor Singpurwalla's and Professor Gertsbakh's additional explanations of the "Sedyakin Principle" give readers more insight about the principle. Other remarks and additional references mentioned by the discussants are also valuable for those who are interested in reliability studies.

Professor Gertsbakh mentioned in his discussion that he had sent a note for publication on the equivalence between Ushakov's (1980) and Kaplan-Meier's (1958) nonparametric reliability estimators. This remark relieves us from pondering further about this problem. We are looking forward to seeing his paper.

Professors Barlow and Khalil indicated that the references to Soviet and non-Soviet authors given in the survey is very limited. Professor Basu also wished that we had a more exhaustive survey of the area. We share their concerns. Due to the broad scope of reliability theory and diversified resources, some important work of Soviet authors was indeed omitted or overlooked. As to the work of non-Soviet authors, we included only those directly related to the work of Soviet authors as we emphasized the latter. An excellent review of statistical methods in reliability, based mainly on English language publications, has been given by Lawless (1983) and discussants.

As a link between the comments of Professors Singpurwalla and Basu we would like to note that the best unbiased estimator of the reliability function for many models (exponential, normal) has a Bayes nature. However, the corresponding (generalized) priors are rather peculiar (cf. Rukhin and Ananda, 1987).

We agree with Professor Basu on the theoretical importance of the translated journal *Theory of Prob-*

*ability and Its Applications (Teoriya Veroyatnostei i ee Primeneniya)*. In fact there are some other journals translated from Russian that also contain reliability related papers, for example, *Theory of Probability and Mathematical Statistics (Teoriya Veroyatnostei i Matematicheskaya Statistika)* and *Automatic Control and Computer Sciences (Automatika i Vychislitel'naya Tekhnika)*. There are at least fifty Soviet scientific and technical journals that have been translated into English. Readers who are interested in journals in translation may refer to the following three resources:

1. *Guide to Soviet Scientific-Technical Research Journals*, the 1984 issue, published by Allerton Press, Inc., New York.
2. *Journals in Translation*, 1982, published jointly by British Library Lending Division, United Kingdom, and International Translations Centre, The Netherlands.
3. *A Guide to Scientific and Technical Journals in Translation*, 2nd ed., 1972, compiled by C. J. Himmelsbach and G. E. Brociner and published by Special Libraries Association, New York.

We feel encouraged after reading Dr. Weinberg's discussion on the need for obtaining knowledge published in non-English languages to benefit American society. We believe that our efforts in writing the survey are worthwhile, and that the survey, although limited, can stimulate Western scholars to pay more attention to Soviet research work as well as work published in non-English languages. Finally, we hope, as does Professor Basu, that additional Soviet books and papers on reliability will be translated into English.

## ADDITIONAL REFERENCES

- LAWLESS, J. F. (1983). Statistical methods in reliability. *Technometrics* **25** 205-235.
- RUKHIN, A. and ANANDA, M. (1987). Estimating exponential reliability function and exponential density. Technical Report, Dept. of Mathematics and Statistics, Univ. Massachusetts.