

techniques as current systems often prove costly, manpower intensive and just plain hostile to the user.

A number of NCIST efforts have drawn upon the invaluable assistance of Professor Herbert Solomon, currently Chairman of the Stanford University Department of Statistics. His long association with ONR includes a tour as the first head of their statistics branch (1948–1952) and another as Chief Scientist in the ONR London Office (1978–1979), where his interest in international science was heightened considerably.

Under his stimulus, distinguished academics, familiar with the Soviet scientific scene and with recent developments in fields related to Solomon's, were approached to prepare monographs under ONR spon-

sorship. This article is one such project. Another monograph, by Joseph Glaz, surveys statistical methods as they appear in recent Russian literature on remote sensing, and is available as a Stanford report under NR-042-267, dated October 6, 1986.

In the end, adequate comprehension of international science can come only from a much greater interest on an individual basis. Personal contact, participation at international meetings, much more extensive efforts by the professional societies to translate journals and texts on a regular basis are necessary if we are to achieve this goal. In the meanwhile, we welcome Professor DeGroot's recognition of the significance of the problem by publishing this article in his journal.

Comment

Ilya Gertsbakh

I would like to add some comments to this interesting and informative survey.

1. The "Sedyakin principle" could be a real help for estimating lifetimes under varying load and for extrapolating the results of accelerated life tests to normal operating conditions. Unfortunately, I have not seen any convincing empirical evidence that would justify a wide applicability of this principle. It is of some theoretical and practical interest to find out those models of damage accumulation for which the Sedyakin principle would be correct. In fact, it is a very strong assumption that the whole loading history influences the future behavior only via the integral of the failure rate. Let us consider, for example, the two following stressing patterns. A unit stress is applied on the time intervals of (0, 1), (2, 3), (4, 5), . . . , and a zero stress is applied on (1, 2), (3, 4) Assume that under the zero stress the failure rate is zero and that under a unit stress the failure rate is one. The second stress pattern is an application of a constant unit stress. Assume that item 1 (2) survived time $t_1 = 2n$ ($t_2 = n$) under the first and second stress patterns, respectively. The Sedyakin principle states then that for both items the future statistical prognoses are equivalent. This certainty is rather doubtful because an alternating load may have a strong adverse effect on the residual lifetime.

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2. A series of interesting works on the deviation from exponentiality for various classes of distributions have been published recently by Brown (1987) and Brown and Ge (1984), which considerably generalize and improve the results of Obretnev (1977) and Azlarov and Volodin (1981) mentioned in the survey. In particular, an upper bound $2(1 - \mu_2/\mu_1^2) = 2\rho$ has been obtained for the IFR family, and a bound of type $A\rho^{1/2}$, $1 \leq A \leq 4\sqrt{6}/\pi$, for a class of NBUE distributions.

3. The Pavlov-Ushakov nonparametric reliability estimator for multiply censored data in fact does coincide with the Kaplan-Meier product limit estimator. Like the authors of the survey, I was puzzled by the fact that both estimators produced exactly the same numerical results. A short note on this subject has been submitted by me to *Communications in Statistics*. The Pavlov-Ushakov result (1984) can be restated as follows: If the decision on withdrawal of an item from the test at time t is based only on the testing history during $[0, t)$, i.e., is of Markovian type, the corresponding product limit estimator is unbiased. Professor Willem R. van Zwet had indicated to me that this fact follows from more general results of Gill (1980) and Jacobsen (1986). The latter work characterizes the whole class of censoring schemes that provide unbiased product limit estimators. Recent work by Kordonsky and Rastrigin (1985) and by Kordonsky et al. (1986) present quite realistic examples of situations in which the Kaplan-Meier product limit estimator produces negative bias. This might be caused by the

fact that the items in the sample are subject to different environmental conditions.

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Comment

Asit P. Basu

In this survey the authors describe four major areas of reliability theory. The number of areas that could be discussed under reliability is quite broad, as can be seen from the list of topics at the research conferences on reliability at the University of Missouri-Columbia in 1984, 1986 and the forthcoming 1988 (see, for example, Basu, 1986). Here I shall limit my comments to some statistical problems that have not been addressed in detail in the survey. The journal *Teoriya Veroyatnostei i ee Primeneniya (Theory of Probability and Its Applications)* contains a number of useful articles in the field. Although Rukhin and Hsieh did not mention the journal in their list, they mentioned some of the articles from the journal.

An important problem has been to consider unbiased estimates of the reliability function $\bar{F}(t) = P(T > t)$. A survey of this is given in Basu (1985). The problem of unbiased estimates of reliability, however, is a special case of the general problem of unbiased estimation studied in depth by Kolmogorov (1950). Kolmogorov's work has inspired considerable research on unbiased estimates of reliability (see, for example, Lumel'skii and Sapozhnikov (1969), who also considered unbiased estimates for multivariate normal distributions and multivariate discrete distributions). The case of the multivariate normal distribution was also studied independently by Ghurye and Olkin (1969) in the USA, whereas Klein and Basu (1985) have considered bivariate exponential distributions. It may be of interest to note that, because of Kolmogorov's work, the Rao-Blackwell theorem is also referred to as the Rao-Blackwell-Kolmogorov theorem in the Russian literature.

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Similarly, because of the extensive contribution of Gnedenko, the Weibull distribution is also referred to as the Weibull-Gnedenko distribution in the Russian literature (see, for example, Savvushkina and Tyurin, 1984).

Besides Weibull and exponential distributions, other models have also been considered. For example, Volodin (1974) has considered the discrimination of gamma and Weibull distributions assuming a generalized gamma distribution as model.

This is an important survey of Soviet work on reliability, and I would like to thank the authors for their important contributions. I wish we had a more exhaustive survey of the area. I hope that additional Soviet books and papers on reliability will be translated into English.

ACKNOWLEDGMENT

This research was sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant AFOSR-87-0139. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation thereon.

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