discipline is more visible and undoubtedly curriculum planners are much more aware that statistics is a distinct subject like chemistry or physics. Such planners recognize that proposals for new duplicate statistics courses will automatically be challenged by the statistics unit with a request to explain how it is that in times of restraint, their department has the resource surplus needed to provide instruction in the subject of another department on campus.

There is a great trend in statistics toward diversification as nonstatistical researchers become increasingly involved in developing the new statistical methodology needed for their applications. This is not a new trend, of course. Factor analysis, kriging and pattern recognition were developed long ago in substantive areas. But the pace of diversification is quickening and many statistical areas are finding new homes on foreign soil. Although many of the pioneer decision analysts were statisticians, that subject now lives primarily outside of the statistical house in industrial engineering, operations research, the business school and other departments. Computer scientists are interested in smoothing and, through their work on artificial intelligence, in imaging, and so on. This trend is impacting on the statistical instruction offered in other disciplines and is an important current within the main stream. Only a few decades before Professor Hotelling's time, the subject of statistics did not exist at all. Lectures on this topic were simply incorporated as needed in existing disciplines, notably political economy. With increased specialization starting around the turn of this century new subjects like sociology were born and eventually, statistics itself. Unless statisticians diligently press to expand the boundaries of their subject it may well redissolve and be lost as a separate subject. As in earlier times, it would simply be incorporated as needed into other disciplines where it would be taught and developed in a piecemeal fashion.

Perhaps one should end on an optimistic note by giving Professor Hotelling the last word. Combining his conclusions it might be argued that "A thorough going reform of school mathematics is currently needed, including a change in the system of training and licensing teachers so as to ensure a better knowledge of mathematics on the part of teachers of the subject. Putting a sound program of statistical teaching into effect will take time partly because of the scarcity of suitable teachers of statistics. Nevertheless the process is well under way, and the prospects are good for substantial improvements in the teaching of statistics."

I would close by thanking my colleagues, Professors Ned Glick, Nancy Heckman and John Petkau, for their thoughtful comments on an earlier draft of these remarks.

# Comment

**Kenneth J. Arrow**

Harold Hotelling (1895–1973) was perhaps the most important single figure in the development and diffusion of mathematical statistics in the United States. His interests were in fact widely varied. He started in journalism, turned to study in mathematics to receive a PhD from Princeton (with a dissertation on topology) and became a junior researcher in the Food Research Institute at Stanford University, where his assignment to estimate crop yields and food requirements developed into research work on mathematical economics and mathematical statistics. His development as a statistician was powerfully reinforced by a period in which he worked with R. A. Fisher at Rothamstead, and he always put Fisher's work foremost in his lectures.

In 1931, he was appointed Professor of Economics at Columbia University; there was no institutionalization of the teaching of mathematical statistics at that time. He was to replace the now almost forgotten pioneer econometrician (the word had not yet been but was soon to be coined), H. L. Moore. His work had become more predominantly statistical, and his most famous papers in this field, which dealt primarily with multivariate analysis, date from the following decade: the generalization of Student's test to simultaneous tests of hypotheses about the means of several variables, the analysis of many statistical variables into their principal components and the general analysis of relations between two sets of variables. He continued his important series of papers on economics, culminating in his presidential address to the

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Econometric Society on the measurement of the general welfare in 1938.

During this period at Columbia, he organized a set of courses on mathematical statistics, which were eventually listed in the course catalogue of the graduate faculties under heading of their own, "statistics." It appeared parallel to "economics" or "mathematics." But there was no departmental structure, no degree-granting authority, no scholarships behind this listing. The only organizational assistance he had was a grant from the Carnegie Corporation to hire an "assistant." When it is noted that Abraham Wald and Henry B. Mann were among the assistants in different years, the reader can infer Hotelling's ability to recognize talent, his generosity in promoting it and the depressed state of the academic market, especially for European immigrants and especially for theoretical statisticians.

I enrolled as a graduate student at Columbia in 1940, with the purpose of studying mathematical statistics, the existence of which I had discovered while an undergraduate. I could read a catalogue intelligently enough and understood that one couldn't enroll in "statistics." I did what I assumed to be the next best thing; I entered the Department of Mathematics. Hotelling very generously encouraged me; I quickly discovered from him that mathematics departments had no use for statistics, and, on his advice, I switched to economics. Intellectually, the Economics Department was as remote from Hotelling's interests in either economics or statistics as was the Mathematics Department. But it was more tolerant. It recognized his stake in a little corner of his own and would not necessarily discriminate against his students.

Toward the end of this period, Hotelling did begin to attract first class students, indeed the bulk of the next generation of theoretical statisticians. His relations with his (and Wald's) students were extraordinary: the encouragement of the self-doubtful, the quick recognition of talent, the tactfully made research suggestion at crucial moments created a rare human and scholarly community. He was as proud of his students as he was modest about his own work.

But he was openly dissatisfied with the low status of statistics at Columbia or any other university. This was not a question of desire for power. It was a commitment to the practical and intellectual importance of statistics in the world. He saw it as a new way of thinking, the making of inferences in the presence of uncertainty. Statistics had applications, and no one was more zealous than Hotelling in stressing applications. But statistics could not simply be taught as part of economics or psychology or agronomy; it was a tool whose varied applications were based on a common discipline. It had to be given its own place in the university, and the teaching of statistics, even in applied fields, had to be done by those trained in the central core of the subject.

After World War II, a separate Department of Statistics, such as Hotelling had urged, was created at the University of North Carolina, headed by Professor Gertrude Cox, and Hotelling was invited to be among the first members. Columbia University did not immediately see fit to create a similar department to keep Hotelling and he left. Perhaps it was the shock of his going that induced Columbia to then create such a department, a step followed at several institutions elsewhere in the country.

When I first came to Stanford as a joint appointment in economics and in the year-old Department of Statistics, my two colleagues were M. A. Girschick and Albert Bowker, both Hotelling students. I immediately set to work creating a basic course in mathematical statistics to be common to all departments. I believe I was representative of a widespread trend in carrying out the Hotelling program.

How has it come out? My impression is that the Hotelling ideal, the widespread use of appropriate and sound statistical methods, has been fully fulfilled; but the educational structure is more like the one he deplored. Mathematically well-trained statisticians, capable of innovation in methodology and possessing the necessary abilities to prove results and to find approximations when needed, are present in adequate numbers in economics, psychology and electrical engineering departments. Statistics departments concentrate more on the basic theoretical insights and techniques of very widespread application, although the spread of exploratory data analysis has mitigated the tendencies to great abstraction. The unified basic statistics course is still partly with us but in a greatly eroded form. Many departments offer their own sequences. They have the intellectual resources, and they find the choice of topics even at the basic level to differ according to field of application. Statistics departments, for their part, sometimes find the basic service teaching a chore and do not resist separate courses in application departments.

Unity nevertheless remains at the level of training of teachers of statistics, because specialized econometricians and their counterparts in other fields must get their advanced training from statistics departments. In the deepest sense, Hotelling's vision of mathematical statistics as the central core of all applied statistics has been amply brought to fruition.