origins of statistics are closer to these disciplines than to mathematics; statistics has been advanced by both past (factor analysis, multidimensional scaling) and present (pattern recognition) research in these disciplines; faculty in these disciplines are often close to the problems of data and inference. A fully trained and experienced statistician is certainly best prepared to teach statistics, but such persons are still in short supply. In many institutions the mathematics or statistics department will offer instructors lacking any experience with data. A psychologist is preferable. To scorn the psychologist because he cannot read the latest papers in the *Annals of Statistics* is a sign of allegiance to mathematics rather than to the understanding of data.

Although I have been deliberately extreme in arguing for the dominance of data over theorems in the teaching of introductory statistics, the principle is now widely accepted. Reports on the preparation of industrial statisticians (ASA, 1980), on teaching statistics to engineers (Hogg et al., 1985) and on teaching statistics in schools of business (Chicago, 1987) have in common the recommendation of increased experience with data and broader coverage at the expense of mathematical depth. The increasing availability of easy interactive computing is rapidly relieving the drudgery of data analysis, although not the need for thought. Younger faculty are usually well trained in computing, and often have an interest in, and experience with, applied problems. There is every reason to hope that the teaching of statistics will rapidly improve wherever trained statisticians are in charge, and some hope that small institutions will recognize the need to employ a statistician to direct the teaching of statistics.

Our disciplinary infirmity is not necessarily terminal. Statistics is rediscovering itself. The fragmentation of teaching that Hotelling lamented may remain, because campus politics demands that it be so. But we shall at least have a clearer case for control over the introductory teaching of statistics when what we teach is in fact statistics.

## REFERENCES

ASA COMMITTEE ON TRAINING OF STATISTICIANS FOR INDUSTRY (1980). Preparing statisticians for careers in industry (with discussion). *Amer. Statist.* **34** 65–80.

HOGG, R. V., ET AL. (1985). Statistical education for engineers: An initial task force report. *Amer. Statist.* **39** 168–175.

MOORE, D. S. (1988). Should mathematicians teach statistics? (with discussion). *College Math. J.* **19** 3–7.

NISBETT, R. E., FONG, G. T., LEHMAN, D. R. and CHENG, P. W. (1987). Teaching reasoning. *Science* **238** 625–631.

STIGLER, S. M. (1986). *The History of Statistics: The Measurement of Uncertainty Before 1900.* Belknap-Harvard, Cambridge, Mass.

UNIVERSITY OF CHICAGO GRADUATE SCHOOL OF BUSINESS (1987). Making statistics more effective in schools of business. Report of June 1986 conference.

# Comment

**James V. Zidek**

The republication of Professor Hotelling's papers is timely. For one thing almost all statistics curriculum planners at academic institutions have taken copies of copies of copies . . . of these ageless classics so that their messages, although immortal, are fading. I was reminded just the other day of the freshness of these articles when I received a memorandum from Department $X$ about the service course that the Department of Statistics had been providing for their students for some years. To preserve confidentiality I would paraphrase the substance of this memo as saying "There is a strong preference for recreating our own course so that it would more directly prepare students for what they will actually do in field $X$. There is a disjuncture between the current course and the rest of the student's curriculum and they find it difficult to bridge this gap. Members of Department $X$ feel that a different approach is required, which shows how statistics as a tool for conducting research in field $X$ can illuminate the problems of interest in that field." Compare this with Professor Hotelling's 1940 comment that ". . . most students of statistics enter upon the subject not for its intrinsic interest but for the idea of applying statistical methods as a tool . . ." This is followed in 1949 by "The major evil is that those teaching statistical methods are all too often not

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specialists in the subject. Their original selection was seldom on the basis of scholarship in this field, they are not encouraged to make advanced studies in it (statistics: ed insert), and their environment is such as to draw their attention in every direction except to the central truths and problems of ... (statistics: ed insert). Frequently they lack the knowledge of mathematics necessary to begin to read the more serious literature of the subject they are teaching. Many have been utterly unable to keep up with the rapid progress which has been taking place in statistical methods and theory, progress which affects even the most elementary things to be taught. There results a widespread teaching of wrong theories and inefficient methods."

The degree of similarity between the problems of teaching statistics then and now is remarkable. To deal with some of these problems the course/program catalogue of the University of British Columbia has, thanks to a committee of yesteryear described below, a page devoted just to courses on "probability and statistics." There are no less than 83 such courses and even a complicated table of three columns with 22 introductory courses in various departments and a solemn message: "students may obtain credit for only one course in any column and at most only 3 units from all of these introductory courses." Can there be students with such an appetite for elementary statistics they would want to violate these restrictions?

In fact, Professor Hotelling would probably be amazed at how much his subject has grown in the more or less 40 years since the publication of his famous articles. Among the 25,000 undergraduate and graduate students at the University of British Columbia, for example, about 3,400 (ignoring a small degree of overlap between courses) took elementary courses in probability and statistics last year. About 800 took more advanced courses (excluding probability) with an elementary statistics prerequisite. In other words, something like 15% of the student population was enrolled in statistics courses during the past year. I suppose the percentage of students who take a course on statistics at some time during their studies must be very high.

Clearly statistics is one of our major industries. For comparison the three largest faculties, arts, graduate studies and science with 7,147, 3,949 and 3,745 registered students, respectively, do not exactly dwarf the crop of budding statisticians. There are several subjects with higher annual enrollments to be sure. Biology enrolled about 4,200 students at various levels, for example. Chemistry drew 5,000, economics 5,500, English 9,000, mathematics 10,000 and physics 4,400. Clearly, if all instruction in statistics were concentrated in a single department as is the case for most other subjects, that department would be among the largest in the University. As matters stand, of the students in elementary probability/statistics courses and advanced nonprobability courses, only 25% are enrolled in the courses offered by the Departments of Mathematics or Statistics.

The University has been quite concerned over the years about the existence of such a large, unstructured, nonformalized, largely unmonitored and unregulated activity in its midst. This may have been a factor that led to the recent establishment of the Department of Statistics. One of the concerns, no doubt is, what Hotelling calls "The obvious inefficiency of overlapping and duplicating courses given independently in numerous departments by persons who are not really specialists in the subject ..." A more fundamental concern is about quality and quality control. Who is to judge the competence of the prospective statistics instructor in Department $X$? Is his or her selection of material reasonable? What about the textbook? Is this individual keeping up with developments in the field? What happens if this individual goes on leave—will there be a satisfactory replacement? Is this individual reading the reviews of new textbooks that are published in the statistical journals? Is this individual helping to develop the skills of basic statistical reasoning or is he/she merely transferring, possibly with the assistance of a computer motivator, a set of loosely related facts and formulas? Does he/she know the subject well enough to instill in his/her students a sense of respect for the basic principles of statistics and an understanding of how statistics ought *not* to be used? Of course, this just summarizes some of the concerns of Professor Hotelling.

These concerns did lead in 1971 to the establishment at the University of British Columbia of a Presidential Committee on the Proliferation of Statistics Courses. Because "proliferation" in this context always had a negative connotation for me, I was quite surprised that what might potentially have been one of the Committee's most controversial and significant findings was already engendered in its very title at the time of its formation. No doubt some of the Committee's members who, like me, were opposed a priori to proliferation, were pleased at this bit of good fortune. On reading the report, however, it became clear that there must have been as many pro-proliferators on the Committee who were equally gleeful but for entirely the opposite reason. So the Committee had a good title. And the title did contain one of the actual findings, although predictably there was no reported consensus on whether this was a bad or good thing. The Committee recommended the establishment of the "probability and statistics" page described above. This was one of the rather weak recommendations that was acted upon. Another suggested the establishment of a standing, interdisciplinary watchdog committee and this was done some years later. That no substantial or sudden reorganization of existing statistics courses should take place was not hard for the Committee to swallow, but the idea of assigning small course numbers to elementary and large numbers to advanced courses bit the dust. Overall, the pro-proliferators won and Professor Hotelling lost another round.

The Committee reported that there were as many opinions of how statistics should be taught on campus as there were individuals. But whereas it might be expected that, say, psychologists would be given some special role in deciding how psychology should be taught at the University, no special weight was accorded to statisticians in the Committee's deliberations. The issue was too important, it seems to be left to the statisticians. In particular, it was noted that there is a transfer problem that students have to overcome in bringing statistical techniques to bear on substantive issues, a problem with which statisticians would not necessarily be equipped to deal. The Committee did not report testing this position by applying it to other subjects like English, mathematics or computer science where instruction is still largely confined to the associated departments.

Since the publication of this report, proliferation has remained alive and well. Whereas the course catalogue's first list of probability and statistics courses contained 57 entries, last year's contained 83 courses as noted above. The number of students has increased too. The Committee found merely 1882 students in elementary courses and 259 in advanced courses back in 1970–1971. The increase from 1970–1971 to 1986–1987 was 150% in elementary courses and 200% in advanced courses (excluding probability). Only some of these increases would be explained by the 129% increase in enrollment. Statistics instruction is a growing industry. Incidentally, in 1970–1971 like 1986–1987, about 25% of statistics instruction was given in the mathematics (and statistics) department so the "proliferation index" has remained constant, at least.

I would conclude by noting a few modern features of statistics instruction that are not embodied in Hotelling's articles. The first of these is the emergence of statistical consulting in the academic context. There are courses in this area and usually, an associated consulting unit. Although these units are thought of as part of the research infrastructure, their role in statistics instruction is vital. Their activities yield data and projects for consulting (and other) courses. Graduate students are trained in the application of statistical methodology through their active participation in the solution of consulting problems; this vital training cannot be achieved through conventional course work. Instruction also takes place at another largely unrecognized level. Graduate students and faculty from other units on campus learn in a painless osmotic fashion of current developments in statistical science. Undoubtedly this has improved the quality of statistics instruction in courses offered by other units on campus. This example of the important pedagogical function of research emphasizes the need for *all* statistical instructors to be involved in research with a significant statistical component.

The computer is reshaping statistics instruction. In particular, instructors are grappling with the problems of finding out how the computer can be used to enhance understanding statistical principles and the related problem of software development. It is all too easy to misuse computers and divert students rather than help them. Even worse, the great user friendliness of modern packages can give students a dangerous illusion of understanding. Students in even the most elementary of statistics courses can perform remarkably complex statistical calculations, with only the thinnest veneer of understanding to help them interpret their results. All in all, statistical educators face major challenges from these new modes of instruction.

I am indebted to my colleague Ned Glick, who pointed out some of the indirect impacts on the teaching of statistics that computer-oriented uses or misuses of statistics can have. The increasing accessibility of packages for statistical computing may make faculty in Department $X$ feel less need for theory. Consideration of underlying assumptions or even the logic of hypothesis testing may be regarded as too theoretical and somehow irrelevant. And the apparent reduction of statistical analysis to a chore for the computer-aided technical assistant may suggest that the statistics course itself should be taught by a such an assistant. Worse yet, there has been a suggestion in some departments that statistics instruction should be relegated to the computer itself, equipped with a series of instruction modules!

There has been some sign that the trend toward proliferation may be reversing. The rapid growth of universities in the 1960s guaranteed in every department a supply of eager young graduates keen to test their new statistical skills in the classroom. Some have been saddled with their department's elementary statistics course ever since. This diet, unrelieved by excursions into more advanced levels of statistics instruction, gradually began to pall. Staffing statistics courses became increasingly difficult, the quality of instruction declined, and increasingly more creative solutions had to be developed. By now it is not uncommon to find departments willing to turn over elementary instruction to statisticians.

There are a great many more statistics departments now than in Professor Hotelling's day. Thus, the discipline is more visible and undoubtedly curriculum planners are much more aware that statistics is a distinct subject like chemistry or physics. Such planners recognize that proposals for new duplicate statistics courses will automatically be challenged by the statistics unit with a request to explain how it is that in times of restraint, their department has the resource surplus needed to provide instruction in the subject of another department on campus.

There is a great trend in statistics toward diversification as nonstatistical researchers become increasingly involved in developing the new statistical methodology needed for their applications. This is not a new trend, of course. Factor analysis, kriging and pattern recognition were developed long ago in substantive areas. But the pace of diversification is quickening and many statistical areas are finding new homes on foreign soil. Although many of the pioneer decision analysts were statisticians, that subject now lives primarily outside of the statistical house in industrial engineering, operations research, the business school and other departments. Computer scientists are interested in smoothing and, through their work on artificial intelligence, in imaging, and so on. This trend is impacting on the statistical instruction offered in other disciplines and is an important current within the main stream. Only a few decades before Professor Hotelling's time, the subject of statistics did not exist at all. Lectures on this topic were simply incorporated as needed in existing disciplines, notably political economy. With increased specialization starting around the turn of this century new subjects like sociology were born and eventually, statistics itself. Unless statisticians diligently press to expand the boundaries of their subject it may well redissolve and be lost as a separate subject. As in earlier times, it would simply be incorporated as needed into other disciplines where it would be taught and developed in a piecemeal fashion.

Perhaps one should end on an optimistic note by giving Professor Hotelling the last word. Combining his conclusions it might be argued that "A thorough going reform of school mathematics is currently needed, including a change in the system of training and licensing teachers so as to ensure a better knowledge of mathematics on the part of teachers of the subject. Putting a sound program of statistical teaching into effect will take time partly because of the scarcity of suitable teachers of statistics. Nevertheless the process is well under way, and the prospects are good for substantial improvements in the teaching of statistics."

I would close by thanking my colleagues, Professors Ned Glick, Nancy Heckman and John Petkau, for their thoughtful comments on an earlier draft of these remarks.

# Comment

**Kenneth J. Arrow**

Harold Hotelling (1895–1973) was perhaps the most important single figure in the development and diffusion of mathematical statistics in the United States. His interests were in fact widely varied. He started in journalism, turned to study in mathematics to receive a PhD from Princeton (with a dissertation on topology) and became a junior researcher in the Food Research Institute at Stanford University, where his assignment to estimate crop yields and food requirements developed into research work on mathematical economics and mathematical statistics. His development as a statistician was powerfully reinforced by a period in which he worked with R. A. Fisher at Rothamstead, and he always put Fisher's work foremost in his lectures.

In 1931, he was appointed Professor of Economics at Columbia University; there was no institutionalization of the teaching of mathematical statistics at that time. He was to replace the now almost forgotten pioneer econometrician (the word had not yet been but was soon to be coined), H. L. Moore. His work had become more predominantly statistical, and his most famous papers in this field, which dealt primarily with multivariate analysis, date from the following decade: the generalization of Student's test to simultaneous tests of hypotheses about the means of several variables, the analysis of many statistical variables into their principal components and the general analysis of relations between two sets of variables. He continued his important series of papers on economics, culminating in his presidential address to the

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