

# A Conversation with Ildar Ibragimov

Peter J. Bickel and Lucien Le Cam

Ildar Abdulovich Ibragimov was born in 1932 in Leningrad. In 1956, he obtained his first degree in mathematics from Leningrad University. In 1967 he was awarded the Doctorate in Physical and Mathematical Sciences, also at Leningrad University. Within two years, he was named to a chair in Probability at Leningrad. In 1970 he was awarded the Lenin Prize jointly with Linnik, Prokhorov and Rozanov. After Linnik's death in 1972, he became Director of the Laboratory of Statistical Methods of the Leningrad Branch of the Steklov Mathematical Institute, a position he continues to hold. He was named Wald Lecturer for 1989 by the Institute of Mathematical Statistics and delivered his lectures in early August 1989 at the Annual Meeting of the Institute in Washington. The following interview took place in late August 1989 at the Department of Statistics of the University of California in Berkeley. We have chosen to meddle as little as possible with Professor Ibragimov's vigorous but at times unpolished English. Nevertheless, we have taken the liberty of occasionally clarifying Professor Ibragimov's remarks by adding in brackets some explanatory phrases.

## 1. THE EARLY YEARS

**Le Cam:** I hope you don't mind, but we wanted to ask you some personal questions, and one of the questions that intrigues me is your name, Ibragimov: it's not typical Russian.

**Ibragimov:** It's not a typical Russian name. Actually I'm not Russian because my mother is a Kazan Tartar and my father is Bashkir and so it is the name of my father. Ibragimov means the son of Ibragim. But I was born in Leningrad. You know, in Russia, we have many different people mixed together.

**Le Cam:** But how did that happen to come about? Your parents must have moved to Leningrad.

**Ibragimov:** I don't know precisely how it was. My mother had been graduated from Kazan University; she was a doctor. My father studied in the Forest Academy at Leningrad. He finished there. I don't know precisely where they met, but then they lived together in Leningrad. My father gave lectures in the Forest Academy. My mother worked as a doctor. And so I was born in Leningrad in '32.

**Le Cam:** Where did you study?

**Ibragimov:** You mean at school or university?

**Le Cam:** Both.

**Ibragimov:** By "school" it is difficult to say because in my life I changed a lot of schools. You know it was war time and my parents moved from one place to another. I moved with them, so it seems in my life

I changed from six schools, but it was not bad. Then I came to Leningrad University, and I was graduated from Leningrad University.

**Bickel:** How did you come to end up as a probabilist and statistician?

**Ibragimov:** Well, I think it is mostly because of Linnik. We had in our University some kind of contest for students in the solution of problems. They set up a special committee consisting of a few professors and a few senior students and they posed problems for junior students, for the first and second course at university. It happened that in one such contest I was first. Linnik at this time was the chairman of the committee. He invited me to his home. We just talked, and he gave me a few books to read, one of which was Hardy's book about number theory, the second was Bohr's book about almost periodic functions, and it seems to me there may have been some other book. But he talked with me about probability. I began to do probability, and so it was a beginning.

**Le Cam:** The first paper that I have seen of yours was in 1959.

**Ibragimov:** No. The first paper was '56. It was about strong unimodality. So I was a student at this time.

**Le Cam:** How did you come to write that book with Linnik on sums of independent random variables?

**Ibragimov:** It was Linnik's way to do things. Once he invited me and said "Well you know Ildar, I wrote a part of our common book. So now it's your task to finish the book." Well, I was a little bit surprised, but you know I never have enough courage to

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argue with Linnik. It was a little bit interesting, so I began to think how to do it with Linnik and he agreed.

**Le Cam:** It's quite a good book. It must have taken a lot of work.

**Ibragimov:** Thank you. Well, it was not so difficult as that. I can do everything very quickly. In fact it was interesting for me, and I even found something new to think about, the necessary and sufficient condition for a given rate of convergence to the normal law.

## 2. HAS'MINSKII COLLABORATION

**Le Cam:** There is another thing that puzzles me. You and Has'minskii have published a lot of papers and books together. There is such a distance between Leningrad and Moscow. How can that work?

**Ibragimov:** Well, it was really some story and I may tell it. I knew Has'minskii before and we both liked one another, but it was not a scientific friendship. It was just a friendship. I knew many people more or less the same. I do not remember precisely the beginning year. Maybe it was '69. And it was a school in information theory in some very nice place in Armenia, a place called Dilijan in the Armenian mountains. It is a very, very nice place, with forests. It was September, October; everything was in different colors. There were different kinds of lectures, and one of the lecturers was Abram Kagan. You know him. He talked about Pitman estimators. And he set us some very difficult problem about how to prove that Pitman estimators were asymptotically normal. He said this would be quite a difficult problem and that he tried and he failed, and moreover Linnik had tried, and so on.

And the reaction of Has'minskii and myself was: Of course Linnik never tried it very much, but it wasn't very interesting because he wasn't trying very hard.

We began to think about the problem and we both found slightly different solutions. I don't know who was the first [to write a] letter but we began to write. We never thought that we would work more on the problem, but just to prove something about mathematics. By then we had the next invitation—also from Armenia—all our statistics conventions are in Armenia. It was an invitation from Dr. Ambartzumian. She's a sister of a well-known astronomer, and at that time she had the chair of Probability at the Armenian University. She invited us both to give lectures. So we came. For us it was just very convenient to finish the work on the paper. We lived in the same room in the hotel. It was really wonderful because all these months we worked very much and very interestingly. In fact almost all our book was done in this one month.

**Le Cam:** That was very fast.

**Ibragimov:** Well, I don't mean that the book was written in one month. No, the work continued rather

long, but just our ideas were formed in this month. We knew what to do. There was a lot of technical work, of course, but anyway, almost all (except maybe the end) was done that year. It worked because we were both rather ignorant in statistics, Has'minskii and myself. We were rather amateurs and nobody was first, so we could work together. Any time in the process of the work when we try to solve some problem that arises which one of us knows better, for example, in an equation which Has'minskii knows much better than me, or some theory which I know better, then it was impossible to work together because one was much stronger. But here it was just the same. I don't know why we really began to think in unison. I think it was very exciting. We were both very happy and we never can forget this time. We were a little bit immature, so we didn't know very much about statistics. Maybe that was helpful. If some strong man tried to do something and failed, we would not try. After this we decided to write a few papers because we really had material for a few papers. We began to do it and then we had some idea to write a book. This time we didn't know so much and maybe we exaggerated our chances. Well, if you decide to write a book you need to be not so ignorant, so we decided to read some different papers by different people before us. We knew that there were a lot of clever men before us so our first thought was maybe it was not reasonable to write such a book because people knew it without us. But we continued.

In '74 I was in the United States at Chapel Hill. Maybe it was in the Autumn of '73. I don't remember precisely. I had a letter from Has'minskii, a rather excited letter, where he understood a theory he had just found in a Hájek paper. It was about what local asymptotic normality might mean for this kind of problem and we should reconsider our views on many things. Again we decided that we write the book. We used our ideas, we put it together with this idea [of local asymptotic normality] and that's how it was. But then again it was a long procedure. Sometimes one, sometimes the other was a little bit tired of the book and would say let's not write it. But it was very important to us to be together. Then we worked quickly. When we were not together we had other things to do. It was different. It seems to me it was a summer in the mid-'70s we lived together with my son. He was small at this time. We were near Leningrad in a small town, Zelenogorsk. Has'minskii visited me and this time we wrote a great part of the book.

**Le Cam:** So you actually worked together when it was written.

**Ibragimov:** No, we wrote, one of us in Leningrad, one in Moscow, and then we put it together. But the quickest work was when we were together. Sometimes we wrote different things. But usually our common

work doesn't sound very good. We do such work together and then we use a coin just to decide who is the writer of the paper.

**Le Cam:** Good statistics.

**Ibragimov:** Yes, that's good. But then when we are writing together, then always Has'minskii writes because somewhere in the beginning he said that his handwriting was better, and I agreed.

**Bickel:** And you said also that your English is better.

**Ibragimov:** Not me. Has'minskii said it, not me. I am rather modest. Has'minskii said that my English is better, so he always forced me to speak English.

### 3. LENINGRAD'S LABORATORY OF STATISTICAL METHODS

**Bickel:** Was that the main way you got into statistics? Were there other influences which made you move into statistics?

**Ibragimov:** It was one of the main reasons, but not the only one. Of course I was always a little interested in statistics because in Leningrad we had seminars in statistics and I had the opportunity to listen to different people. Charles Stein was in Leningrad and gave his lectures, and Rao was in Leningrad and gave a lecture. Neyman visited and gave a lecture, and Linnik had a seminar. So I had the opportunity to listen to really great statisticians, rather than mathematicians working in statistics. I never considered myself a real statistician because I didn't have such a deep understanding of statistics really. I was more interested in solving mathematical problems arising in statistics. Of course it's better if you can do both sides. But I had creative ability [in mathematics] in lieu of statistics. If someone finds the problem, I can do something in mathematics around this problem.

**Le Cam:** Linnik died when?

**Ibragimov:** '72.

**Le Cam:** And you were named Director of the Laboratory there?

**Ibragimov:** Yes.

**Le Cam:** I never understood exactly what the structure of the thing was.

**Ibragimov:** Well, we have the Steklov Mathematical Institute. Its center is in Moscow. In Leningrad we have a branch of the Steklov Institute. We are rather independent, but [administratively] not absolutely. But in our science, [we are] absolutely independent.

In Leningrad we have a [special] wise idea, an idea of Fadeev. Our Institute is divided into small groups of about 10 people. We have a group on number theory or algebra or theory of functions or mathematical physics. [Our group is] called the Laboratory of Statistical Methods. But it doesn't mean that you really

have to work in statistics. It means that you work in statistical methods, at least probability and statistics.

**Le Cam:** I was also told that in many instances you have groups of students and you give them some material to study together and then report in seminar. Is that true?

**Ibragimov:** Well, I don't know. It depends. Not always. No. Sometimes we ask some to study something for seminar, but it's not such a well organized procedure.

**Bickel:** Didn't Linnik have that kind of procedure though in the writing of his books? Didn't he have a group working on a subject and then a book would come out?

**Ibragimov:** No. About writing books—Linnik always wrote it himself. But in my case, his part was always written before he invited me and my part was different from his part. He agreed with me to just put it together. Sometimes he organized people to write books, but it doesn't mean that he was a supervisor. In the book with Rao and Kagan, I think they discussed together what to have in the book and then they divided the work and that's all. Linnik was not such a figure to be the chief. It was just some collaboration of independent people.

**Le Cam:** As Director there, do you have to do a lot of administrative work?

**Ibragimov:** No. Our group is small: nine people. It's not so much administrative work.

**Le Cam:** So it's different from our places here.

**Ibragimov:** Some people in our institutes have to do administrative work. I'm just happy (that I don't have much.)

**Bickel:** You mentioned earlier that you did not consider yourself a real statistician because you work on mathematical problems arising out of statistics. What is the general situation of applied statistics in the Soviet Union?

**Ibragimov:** Well, I'm not ready to analyze it very precisely. But I may say that of course applied statistics is behind in our country than it is in the United States and even very behind. There is a group of really very good people like Meshalkin, Ayvazian, Yaglom. We have a few people of very high quality. But I think we have not enough people really working in statistics and maybe not enough understanding from industry and so forth that statistics really is needed. Statistics really is some kind of culture, so we need to move farther in this direction.

**Bickel:** Do these people work in groups of scientists at other institutes helping them analyze the data?

**Ibragimov:** Certainly, of course. But Ayvazian, his work is in Mathematical Economics Institute of Academy of Sciences, and Meshalkin works with people in medicine. There are some other people who do

very good work in applied statistics or probability who work in some industrial institutes and so on.

#### 4. KOLMOGOROV AND OTHER SOVIETS

**Le Cam:** In Moscow, Kolmogorov was the head of the Laboratory there for a long time.

**Ibragimov:** I don't know very precisely. He was of course the organizer for the Laboratory and of course de facto he was head of the Laboratory. I don't know what administrative position he had. For example, he organized at the University of Moscow the Chair of Statistics and sometime was the holder of the Chair. But then he became the holder of the Chair of Mathematical Logic. And instead Rozanov became the holder of the Chair of Statistics. And his last year he worked in the Steklov Institute. He was the head of some laboratory of statistics.

**Le Cam:** Who has taken direction of that after him?

**Ibragimov:** Where?

**Le Cam:** In Moscow, do you know?

**Ibragimov:** In the Steklov Institute? It seems to me they just united everything and Prokhorov's the head. I don't know precisely, but it seems to me Prokhorov is the common chief of the probability and statistics group. We don't so much divide probability and statistics. In our country it is the same. If you speak about probability, it means statistics also. Anyway, for Kolmogorov it always was the same, and he always began his lectures in probability with describing some statistical problems and then explained that the real mathematician should solve this problem with probability.

**Le Cam:** That's good.

**Bickel:** Would you say that Kolmogorov was the main former of the tradition of statistics and probability in the Soviet Union?

**Ibragimov:** But certainly, of course. Everything came from Kolmogorov. And I think not only in our country.

**Bickel:** No.

**Le Cam:** But there was already a long tradition of probability before: Lyapunov, Markov, Bernstein. . .

**Ibragimov:** Of course there was a very good tradition, but I speak about our time. In probability Kolmogorov was well known. He was active here, he had a lot of ideas. . .

**Bickel:** He also seems from what you said to have been really interested not only in the mathematical development of the subject but in the subject as a whole.

**Ibragimov:** Yes. Always, always.

**Le Cam:** I met him once in Paris and he told me he was working on sunspot numbers. He was trying

to figure out whether there is a real cycle in there or whether it is just some sort of mildly periodic phenomena.

**Ibragimov:** Yes, this was some common paper coauthored by Kolmogorov, Arato and Sinai. It was a statistical paper about estimation of parameters in a very simple stochastic equation. But it seems that in the beginning he was interested in the movement of the axis of the Earth.

**Le Cam:** You have in Leningrad some young people whom I have never met, but I know their names: Arak, Zaitsev. Are they still there?

**Ibragimov:** No. Arak now is not in Leningrad. He is in Tallin. They both were my students, but Arak was my student at the University. He prepared his Ph.D. After that he moved to Estonia. He came from Estonia to Leningrad. And Zaitsev also was my student and he did very good work. But again probably it was coming from Kolmogorov. And he also worked in this area. It is a very difficult kind of problem and they did some very good jobs. They wrote a book about it. It has been translated into English now.

**Le Cam:** Yes.

**Ibragimov:** But it's really very good mathematics, very complicated, very difficult, very interesting, very beautiful. But after this Arak stopped working in this kind of area. Now he works mostly in such new probability problems arising from statistical fields, some kinds of random fields, some very interesting examples of random fields. And Zaitsev continued to work here and found some other very interesting theorems for the multidimensional case.

**Le Cam:** That's the paper on the Prokhorov distance?

**Ibragimov:** That's one of the papers.

**Le Cam:** It is very difficult to read.

**Ibragimov:** It is very clear, but it is just difficult. It is a very difficult problem.

**Bickel:** What is the area?

**Ibragimov:** I may explain it to you. It is very difficult if you don't know. It began with a paper of Kolmogorov of 1956. There are two theorems in that paper. [If] you have iid random variables and the sum of  $n$  of them then [the sum's] distribution may converge (for instance), to some stable law. However it might not converge to anything, but it will be close to some infinitely divisible distribution. [An old] question of Kolmogorov's was: is it possible for each  $n$  and each convolution power  $F^n$  to construct an infinitely divisible distribution  $J_n$  so that the distance between  $F^n$  and  $J_n$  will tend to zero.

The first [positive] answer was Prokhorov's. But then in 1956 Kolmogorov proved that if you take the uniform distance between the convolution  $F^n$  and the set of all infinitely divisible distributions, the maximum of that distance as  $F$  varies tends to zero,

and moreover the distance is less than a constant  $C \times n^{-1/5}$ . It is the first theorem. The second theorem considers the sum of independent variables, any of them, not iid, any random variable at Lévy distance [at most] epsilon from zero. Then he proved that in this case the distribution of the sum is close to an infinitely divisible distribution up to  $C\epsilon^{1/5}$ . So it was  $n^{-1/5}$  and  $\epsilon^{1/5}$ .

Kolmogorov's [new] question was: what is the proper rate of convergence? Then Prokhorov proved that it may be  $n^{-1/3}(\log n)^2$  and Kolmogorov threw away the  $\log n$ . Then Lucien Le Cam showed that you can use the Poisson exponential. It is a special infinitely divisible distribution. Then Meshalkin proved that it is not possible to do better than  $n^{-2/3}$ . Then everything stopped because you needed some absolutely new method.

So everything stopped. Then Arak used a new technique and proved that the proper order was  $n^{-2/3}$ . Then Zaitsev looked at the independent nonidentically distributed case. Everybody expected the proper rate to be  $\epsilon^{2/3}$ , because of the  $n^{-2/3}$ , but Zaitsev proved it is  $\epsilon \log(1/\epsilon)$ . It is precise, it is from both sides. It's very hard. Many, many pages, and some very ingenious very beautiful work.

**Le Cam:** Do you have any other people that are that good around Leningrad?

**Ibragimov:** Well, we have other young persons. Zaitsev is about 30 now. We have Brodin, maybe a few years older than Zaitsev. Just now we have finished a book about limit theorems. We have good people. We have Davidov, who is rather old, not so old as me, who from my point of view did some very good papers. And he has a very good student, Lifschitz, who is the same age as Zaitsev because he came in at the same time. We have Soley. He's very good, but he's very strange. He was my student. He works in the Institute sort of. He is really very good just in solving problems, but it is impossible to make him to write. But it is enough to solve, to tell, and stop. He promised many times to write, but he never does. It's a very difficult case.

**Le Cam:** One might do like the French did with Grothendieck. When Grothendieck talked, Dieudonné took down his lectures from the blackboard and published them.

**Ibragimov:** Well, we need someone to do it.

## 5. FACULTY APPOINTMENT POLICIES, VISITS, CROSS FERTILIZATION

**Le Cam:** You know of course that our department is looking for bright young people. Could some of these people apply?

**Ibragimov:** You mean, apply for work here?

**Le Cam:** Yes.

**Ibragimov:** To come for some time, or to come forever?

**Le Cam:** They would have to decide.

**Ibragimov:** I don't like for them to come forever. I prefer to have them in Leningrad. But to come for some short time, invite them. I think it's possible for them to come. The problem is to pay here because we do not have hard currency. They can pay for their own ticket in rubles, but here they need some, not very big, amount of money to pay for eating.

**Le Cam:** We have very little money for research and such things, but could some of them come and do some teaching?

**Ibragimov:** I think they could. Why not? They are bright men but of course their English is not very perfect. But you told me that when you came here you could say nothing, so I believe that after two weeks maybe they could speak English like I speak it. They are younger, so I think after two weeks they would speak much better. They can teach.

**Bickel:** This is changing the subject a little bit, but you mentioned that you want them to come back to Leningrad. Is that a general pattern in the Soviet Union, that you want to keep your best students rather than having them move to another university?

**Ibragimov:** No, I do not mean to keep all the best students, no. We just mentioned some special kind of people. But about these people, I prefer them to work in our Institute. I think you should always try to keep the best people. We try to keep them here. Isn't that right? Just now I talked about our Institute. It is very small. Leningrad is a city of 5 million people. We have about 80 researchers. It's not so much.

**Bickel:** So these people would be like members of the faculty at Berkeley, and it's true we want to keep such people. . . .

**Ibragimov:** I meant the people like Zaitsev and Brodin. I'm sure they would be good professors at Berkeley. We try to keep professors here. But about students it is different thing of course. I hope it would be possible to have more exchange between our countries. I think for young people to come for some time to different surroundings, just to know new ideas, new ways of thinking, it's very useful. And in this way I agree with you that it might be better, when you always push your students to some other place. When they are young they pick up new ideas much more quickly. In some sense I have been that way. Because of Linnik I knew Kolmogorov, and because of it I often visited Moscow when I was young. So I knew people not only from Leningrad but also from Moscow. For me it was very useful.

**Le Cam:** Did you tell Ildar that we'll have a year of statistics at the Mathematical Sciences Research Institute in 1991-92?

**Bickel:** Yes I did.

**Ibragimov:** I forgot about it.

**Bickel:** I told you that we hoped to invite you.

**Ibragimov:** Yes, I remember. It might be that this time . . . Who knows?

**Le Cam:** That might be an occasion when we could get some people to come. Normally we have so little research money, so little free money . . .

**Ibragimov:** Always we have not enough money.

**Le Cam:** So people who come have to teach.

**Bickel:** But we might have money for postdocs, particularly for. . . .

**Le Cam:** A year.

## 6. COMPUTATION AND ITS IMPACT ON STATISTICS

**Bickel:** Can I just change the subject?

**Ibragimov:** You can, because I told you I would obey you.

**Bickel:** Just to ask some general questions about statistics in the Soviet Union. To what extent has the computer influenced the development of statistics in the Soviet Union?

**Ibragimov:** Before I answer your questions . . . You, Peter: you work much with computers?

**Bickel:** No.

**Ibragimov:** Well so. I thought you worked much with computers. But some people do, and they like it very much. And if they are young, people are just excited with this work. They like it very much and certainly the future of statistics is connected with computers. Now our net of computers is not so good as in the United States, but everyone understands that it's very important and I told you that young people like to work with computers. It's a part of university studies, so the situation is the same. The difference is only that we have not such much available computer hardware.

**Bickel:** Do you see it already influencing the direction that research in probability and statistics takes?

**Ibragimov:** Well, not me personally, but in the country of course. I think it is more or less in the same direction as here. Mostly you know the people who work with the applications work with real data. Certainly these people now always work with computers. Maybe now we have not so much software as you have, but we have it and people work with it.

**Bickel:** In this country also there has been to some extent a link-up between numerical analysts and statisticians, because data analysis now involves very complicated numerical routines which may be quite sensitive. Has that happened in the Soviet Union, where numerical analysts, let's say, and statisticians interact in research?

**Ibragimov:** Certainly, certainly. I don't know everything about it. I have a very good friend of

mine—Romanovsky—who began as a probabilist and statistician and who now works mostly with computers. And he does it very much. So it's just one link, but a very good link. I know of some young mathematicians, not in probability, but in some other area of mathematics, who work a lot with computers, because it's really changed the face of mathematics—what the setup of the problem is, what it means that the problem is solved. It gives some kind of possibility to have some experiments, and so on. Especially these personal computers which are very easy to operate.

**Le Cam:** What kind of computers do you have? Personal ones?

**Ibragimov:** Well, we have such kind in our Institute, but I'm afraid I will say something wrong about it, but they all are from abroad. They're not Soviet-made computers. Maybe one is IBM, I'm not sure. There are some Taiwan-made computers but it seems that they are mostly American.

**Le Cam:** I was asking that because I remember reading in the newspapers just a few weeks ago that the State Department here is going to relax the rules and allow more American computers to be sold in Russia.

**Ibragimov:** Then maybe we will have better computers. I hope that maybe we will have our own computers.

**Le Cam:** Many are made in Japan anyway.

**Bickel:** Many of the components of the American machines are made in Japan.

**Ibragimov:** I know.

## 7. BAYESIAN STATISTICS

**Bickel:** I had one last question, about Bayesian statistics. To what extent does the Bayesian point of view appear in Soviet statistics?

**Ibragimov:** I don't know a real Bayesian in the Soviet Union. But for me, it's absolutely a foreign point of view. I can't understand it. So I don't know how it's possible to discuss it with me. I don't know any seriously working people in the Soviet Union who believe Bayes. For me, I can't take this point of view.

**Le Cam:** You mean the philosophy. The idea that you must get a prior distribution and follow it through.

**Ibragimov:** It's very far from me. Maybe it just means that it's something I don't know. It just means that I studied some different kind of probability. I just thought about it one year when I was young. Maybe now if you bring it to me I just mostly see it as some kind of statistics.

**Le Cam:** Yes.

**Ibragimov:** Of course this kind of probability exists. But to try to cast out the whole of statistical variation, that's very far from me.

**Le Cam:** I find that very hard to understand too.



**Ibragimov:** Thank you very much. I though I was not sufficiently clever. So it's not only me.

**Le Cam:** I can see that you could get some sort of theory based on that, but I can't see how you could apply it in practice when you don't know the prior distribution.

**Ibragimov:** That's my viewpoint. But sometimes you have very interesting results, for example [those] which were done by de Finetti. Some results are just startling. It's very interesting.

**Le Cam:** Yes, that's true.

**Bickel:** In this country, Berkeley is viewed as a stronghold of anti-Bayesian sentiment, I don't think quite justly.

**Ibragimov:** I'm not anti-Bayesian, just non-Bayesian.

**Le Cam:** We have quite a few Bayesian people around: Blackwell in particular. But I think that I would qualify myself as anti-Bayesian, even though David Freedman and Persi Diaconis are somewhat Bayesian. Do you know what they do? Persi and David write papers showing that the Bayes procedures are not good. And I, being anti-Bayesian, write papers saying it's good.

**Bickel:** I think David Freedman used to be a Bayesian, but now he has moved to your position because he has worked on different consulting problems where they abuse Bayesian ideas. You also hit the difficulty that clients are not so willing to accept your views. There is, however, a strong school of Bayesian statistics. In fact, the former Editor of this journal, Morris DeGroot, was a Bayesian.

**Ibragimov:** Yes, I know. I met him in Leningrad and his book was translated into Russian.

**Bickel:** So Russians have been exposed to the Bayesian point of view also.

**Ibragimov:** You know many Russian people can read not only Russian but English also.

**Bickel:** They can even speak it well.

**Ibragimov:** It's a great difference to speak and to read. Anyway, it seems that many know the American scientific [writing] better than you know Russian.

## 8. NEW DIRECTIONS IN STATISTICS AND PROBABILITY

**Le Cam:** Perhaps you might tell us what you see coming up in Russia for the main directions in probability and statistics. Here there is quite a bit of effort in the trajectories of processes and the infinite particle systems, on random permutations. Many different things. Are there any such subjects in Russia?

**Ibragimov:** Yes, of course. I think it is really very interesting, and maybe in some sense the future of probability. It seems there are many really new problems which we are studying in probability which are in use in some kind of statistical theory. And we have

some new kind of problems which we didn't know how to treat. They are really very interesting. We have some new kinds of processes which have some very long dependence, and you need some new technique on how to treat them. It's not the same as how you know what to do with independence or with mixture conditions. You can see it really exists. You can see it in many examples. But how to treat some general theory? I don't know if it is possible or not.

**Le Cam:** And in statistics itself, what directions are booming?

**Ibragimov:** I'm afraid to say. It's better to trust people like Fisher, Kolmogorov. What can I say. What about me? I'm just interested now in some kind of problems in probability and statistics. But what really of the future? I think it is difficult to say, because now much depends on this tie between statistics and computer science. So maybe you have some new kind of problems. What kind of problems, it's difficult to say.

**Le Cam:** I would not want to say either.

**Bickel:** In the United States, a very fashionable topic is the bootstrap.

**Ibragimov:** Yes, maybe too fashionable. From my point of view, the bootstrap is really a very good example of what you can have if you have good computers. In fact, it was nothing very new, but what is new is that Bradley Efron understood that just now it's time to use these things, and he disposes of them very well. Without good computers you really can't do it. But now he understood that sometimes it is not necessary to calculate everything, you just use a good computer. You may use the Monte Carlo technique and you have a very good answer. It's a really good example of how computers may change statistics.

**Le Cam:** About the bootstrap. I should say that one of our colleagues who is usually very cynical told me not long ago that it was passé.

**Ibragimov:** What does that mean?

**Le Cam:** It is in the past. There are no new things to do.

**Ibragimov:** I think it is not absolutely true. Maybe no new theoretical things. Of course, all of us understand the empirical distribution and all of us knew the Monte Carlo technique. But just understand that you might solve a statistical problem combining these methods and using computers. No, there was something in it. It's not so bad.

**Le Cam:** But there are no new tractable mathematical problems in it.

**Ibragimov:** No. But anyway it's a new kind of approach. Maybe not for a theoretical, but for a practical problem. If you consider our mathematics and statistics as a way to solve practical problems it was something new. It was good.

**Le Cam:** It was a good advance.

**Ibragimov:** Yes, it was some new idea.

**Bickel:** The theory somehow seems to involve proving some sort of uniformity in convergence of objects, and once you have proved that then the theory tells you that the bootstrap should work. But it doesn't give you too much more guidance beyond that.

**Le Cam:** Warry [Millar] tells me that in order to prove some theorems not only that you need that but you need second- and third-order Edgeworth expansions for the sup norm. You measure distance by the sup norm. And those are difficult to find.

**Bickel:** I thought there was an expansion for the sup on the line—it's not an Edgeworth expansion, of course—I thought there was work which was done in the Soviet Union on that.

**Le Cam:** I think Warry is after the Vapnik and Červonenkis class.

**Bickel:** I see.

**Le Cam:** Where are those people, Vapnik and Červonenkis?

**Ibragimov:** In Moscow.

**Le Cam:** At the Institute for Information Transmission?

**Ibragimov:** In some kind of institute in Moscow, I don't know very precisely. You can check it on the paper where they are working. But they work in Moscow. They are just two examples of people working in applied probability and statistics—very good people.

**Bickel:** Is their background essentially in probability and statistics? Or in engineering?

**Ibragimov:** I don't know why they began to work in this area. He [Vapnik] promised to come to Leningrad to give a talk at our seminar, so I may ask him why he began to work here, and I will write to you.

## 9. INTERNATIONAL CONFERENCES AND COMMUNICATION

**Bickel:** Is that fairly uncommon to invite people from Moscow to speak in Leningrad and vice versa. Is that very difficult?

**Ibragimov:** No, it's rather common. It may be more simple than in the United States because we have no such custom as to pay an honorarium for a seminar. You would be glad to be invited. It's an honor.

**Bickel:** But your travel is paid by. . .

**Ibragimov:** It depends. Usually you do it in such a way where your work pays for it so it's not very expensive. Moscow and Leningrad are not so expensive.

**Le Cam:** I wish we could do that here. Travel expenses can be quite big.

**Bickel:** Actually, travel in this country is deregulated and is maybe even less expensive. Not less expensive than travel within the Soviet Union, but say between the Soviet Union and Europe.

Speaking of Europe, there have been some links developing between the Soviet Union and Western

Europe through the Bernoulli Society. The [First] World Congress [of the Bernoulli Society] was held in Tashkent. How do you see those connections developing?

**Ibragimov:** They develop, but what can I tell you? But you know Shirayev is President of the Bernoulli Society. And now we have the Bernoulli Society in the Soviet Union. It has just been created and I don't know how it will work. Anyway, I hope it will be connected with Europe. In time it will be better, but even now it is not so bad. You know Europe is much closer than the United States. It's much easier to come to Europe. Not so much time, not so much money.

**Bickel:** But there are difficulties because of currency problems in the same way there are in this country.

**Ibragimov:** Exactly.

**Bickel:** But you can see more meetings of the Bernoulli Society being held in the Soviet Union.

**Ibragimov:** Yes. For example, the money issue [suggests that more meetings will be held]. Every year we pay dues. But they're in rubles and you just have to organize a meeting in the Soviet Union of course [to use the rubles].

**Bickel:** Is there a Soviet Statistical Society, or is it just part of the Mathematical Society?

**Ibragimov:** No, no, there is [even] no mathematical society. For example, we have the Leningrad Mathematical Society. There is a Moscow Mathematical Society, Kharkov Mathematical Society, but there is no common mathematical society in a formal way. Of course, such a community exists. But it's not organized. We have no chairman for example. No common mathematical society.

**Bickel:** But you do have regular meetings?

**Ibragimov:** Not now. We had a congress for all mathematicians of the country, but the last was many years ago. Because there are so many mathematicians, it's almost impossible to organize such a congress. So now they are mostly probabilists, number theorists, like this. In probability, even that is huge.

**Bickel:** The most extreme example of that in this country is the meeting that you gave your Wald lectures at, the ASA/IMS/Biometric Society national meetings.

**Ibragimov:** I think they are too big.

**Bickel:** Much too big.

**Ibragimov:** In fact, smaller conferences are much more useful.

**Bickel:** And much more pleasant.

**Ibragimov:** You had very good conferences here—the Berkeley Symposia. But they also are finished.

**Le Cam:** Yes, that's a pity. But they were getting too big and too hard to organize. On the last one in 1970, I really spent a lot of time at it, but when you



go through a list of 5–600 people who should speak, and you have to cut it down to maybe 100, it's very hard.

**Ibragimov:** You have the same problem everywhere. Too many people are working on statistics and probability and science as a whole.

## 10. CONCLUDING THOUGHTS ON KOLMOGOROV

**Bickel:** You said that Kolmogorov was an influence on your own work and your professional development, just as Linnik was. Are there any particular things you can point to, or maybe stories in connection with Kolmogorov's influence on your development?

**Ibragimov:** Difficult to say, because you know it was very great. I had many opportunities to speak with him, and in fact it was not so easy always to understand him. When I tried to ask him questions he just became silent; he listened to me very politely and then he began to speak at just the same point where he stopped before. Just ignoring my question mostly. But he did it so nicely. Sometimes he answered the question, maybe, when the question was more or less reasonable. But mostly he ignored it, so it was not so easy to understand him. At last I just had to memorize everything. And to think about it. But in fact Kolmogorov was really very interesting because he was the only person who understood my own work's meaning better than I. I talked about my work with some other mathematicians, but my own works I knew better than they. But Kolmogorov was different, he knew them better.

But for a more precise story. It was connected with my work about the sum of independent random variables. You have a stationary process, a stationary random sequence, and you are interested in under what condition the central limit theorem will hold. I had such conditions with some number of moments and some rates of mixing. And Kolmogorov was interested in how to find the precise relation between the rates of mixing and the number of moments. He told me about it, and once—I remember it was in his summer home—he told me how it would be possible to do it. And I misunderstood him. My opinion then was that he wasn't really interested in such problems. [I thought] he just told me how he would try to do it, and so I didn't try to do it in his direction. I remember what he explained. He told me it would be nice to consider just a series of specialized sequences, sequences of stationary processes with very long waves. It might be in this area we could find counterexamples [when the conditions were violated]. It was that I was very young; I had not yet my Ph.D. and so I forgot about it. Then, later, I had students. Some time passed, five years maybe. My student was Davidov, and he prepared for his Ph.D. I asked him more or less the same question, but I said that [forgetting

Kolmogorov's comments], maybe you could find such examples considering Markov chains with a long time of returning, i.e., without moments.

And Davidov really succeeded. He constructed very good examples. I was happy also because my old theorem was very precise. The rate of convergence, the number of moments, and the rate of mixing were just what was really needed. So it was all right. And when everything has been done I learned that it was the same as Kolmogorov, and that Kolmogorov was really absolutely correct. His idea was very vague. But just the same, he knew he needed some very long return times. I forgot about it, but somewhere inside me I remembered it, and it took such form. So it's one kind of story.

**Bickel:** A very interesting one.

**Le Cam:** So maybe we should close.

**Ibragimov:** Thank you for the opportunity to speak so much.

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## REFERENCES

- HÁJEK, J. (1972). Local asymptotic minimaxity and admissibility in estimation. *Proc. Sixth Berkeley Symp. Math. Statist. Probab.* **1** 175–194. Univ. California Press.
- KAGAN, A. M., LINNIK, YU. V. and RAO, C. R. (1973). *Characterization Problems in Mathematical Statistics*. Wiley, New York.

## SELECTED ARTICLES BY I. A. IBRAGIMOV

- (1956). On the composition of unimodal distributions. *Theory Probab. Appl.* **1** 255–260.
- (1962). Central limit theorems for stationary processes. *Theory Probab. Appl.* **7** 349–382.
- (1963). On the estimation of the spectral function of a stationary Gaussian process. *Theory Probab. Appl.* **8** 366–400.
- (1975). Properties of maximum likelihood and Bayes estimators for nonidentically distributed observations. *Theory Probab. Appl.* **20** 689–697.
- (1984). Nonparametric estimation of the value of a linear functional in a Gaussian white noise. *Theory Probab. Appl.* **29** 1–32 (with R. Z. Has'minskii).

## BOOKS BY I. A. IBRAGIMOV

- (1965). *Independent and Stationary Sequences of Random Variables*. Nauka, Moscow (with Yu. V. Linnik). (English translation, Walters-Noordhoff, Groningen, 1971.)
- (1970). *Gaussian Random Processes*. Nauka, Moscow (with Yu. A. Rozanov). (English translation, Springer, New York, 1978.)
- (1979). *Statistical Estimation: Asymptotic Theory*. Nauka, Moscow (with R. Z. Has'minskii). (English translation, Springer, New York, 1981.)