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A Conversation with Harald Bergström

Lennart Råde

Abstract. Harald Bergström was born on April 1, 1908, in Mölltorp, situated in the middle of Sweden. A Master of Philosophy degree was awarded in 1931 at the University of Uppsala (in mathematics, physics and chemistry, extended to theoretical physics in 1932). He taught in secondary schools (gymnasiums) from 1932 to 1934, and then returned to Uppsala to pursue his research in mathematics toward a doctorate degree, which he received in 1938. He had a permanent lectureship in mathematics at the University of Uppsala from 1938 to 1945, and at a military college in 1945. In 1946, he was asked to hold a new established professorship in applied mathematics at the Chalmers University of Technology in Gothenburg, where he became full Professor in 1949. In 1960, the professorship in applied mathematics was expanded to two professorships—one in numerical analysis and one in mathematical statistics; he occupied the latter until his retirement in 1974.

The following conversation took place in Gothenburg in April, 1992.

THE EARLY YEARS

Råde: Harald, I appreciate very much that you agreed to do this interview. I started to work for you 40 years ago in 1952 as your assistant, so we have had a very long time of cooperation. I am looking forward to this conversation with you. You were born in Mölltorp in 1908 so let us start from the beginning. Is there something you would like to tell us about your first school years in Mölltorp?

Bergström: Yes, I attended a rather primitive school in Mölltorp. We went to school only every second day and there were two classes on different levels at the same time.

Råde: I should think that at such a school the students had to do some of the teaching. Did you have a special interest in mathematics at this stage?

Bergström: Yes, I did quite well and I was asked to help the other students. This gave me my first teaching experience. I was very interested in mathematics even if it was simple mathematics that we encountered. I was especially interested in our religion studies. We read the Bible a lot so I still remember very well the contents of the Old Testament. Then after this school, I went to a somewhat higher school in Karlsborg—the well-known fortress place—which I attended for four years. Because of my poor economic situation, it was difficult for me to continue my studies. It was suggested that I apply to private schools which I did, and after two years of study, I graduated with my gymnasium degree.

Råde: Did you then go to Uppsala to start your university studies?

Bergström: No, it was really a problem for me to decide what to do next due to my economic situation. First I tried to get a job but that was very difficult and finally I decided to become an elementary school teacher for grades 3 through 6. So for one year, to prepare myself, I studied drawing and music (organ playing) because these subjects—and also gymnastics, arts and crafts—were not in my secondary school degree and these were required for the teacher training college in Växjö. In the entrance examinations for the college, I did quite well in drawing and music, but failed in gymnastics and was not accepted.

UPPSALA AND GÖTTINGEN

Råde: So instead of being an elementary school teacher, you had to do something else.

Bergström: Well, yes; as it happened, I met a former teacher on the train to the teacher training college, and when he heard that I planned to be an

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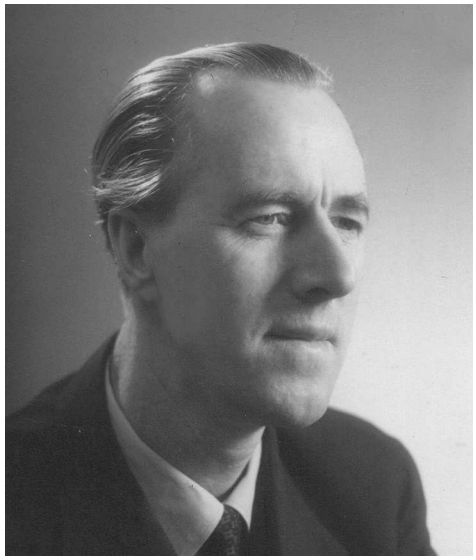

FIG. 1. *Young Harald Bergström.*

elementary school teacher, he urged me not to do so, but to go to Uppsala University and study mathematics, physics and chemistry. So I went to Uppsala, where I studied chemistry. As a matter of fact, I was very interested in chemistry mainly because I had a very good teacher in secondary school in that subject, and I did some exciting work on chrome synthesis. At the same time, we had a very poor teacher in mathematics.

Råde: Did you anticipate a career in chemistry at that time?

Bergström: I studied mathematics at the same time as chemistry, and then I started studying physics. Within two years, in 1930, I took my first degree in these three subjects.

Råde: How was the teaching in mathematics in Uppsala at that time? Were there many lectures or did you study mainly from books, and if so, what books were you using?

Bergström: First we studied De La Vallée-Poussin's classical book *Cours d'Analyse Infinitesimale.* There were only a few lectures, three hours a week, and every second week we had a seminar with exercises. It was strange that there was no information about requirements. We had to try to find out from previous students, the actual content of the courses and what the requirements were.

Råde: Later on you became interested in algebra. Had you studied algebra at this time?

Bergström: No, at that time there was very little interest in algebra in Uppsala; analysis was the important mathematical area. I continued my mathematics studies after my first degree and received the highest grade, but I had no algebra. I followed lectures by Professor Erik Holmgren on partial differential equations and I read a lot from the excellent book on function theory by Ludwig Bieberbach.

Råde: When did you do your one-year compulsory military service?

Bergström: I did not do it until I had obtained my Master of Philosophy degree. I was fortunate to be asked to do civil research work at the Nobel Prize Winner, The Swedberg's Institute for Physical Chemistry in Uppsala. For one year I did research on the indication of chemical agents, but I also had time to study theoretical physics.

Råde: So at this time of your life you still did not know that you were going to devote your life to mathematics.

Bergström: No. It was necessary for me to earn some money so I started teaching in high schools; and for two years I taught at Södertälje, close to Stockholm, and in Östersund, in the Northern part of Sweden. The first year I taught mathematics, physics and some chemistry, and the second year I taught only physics. I enjoyed this kind of teaching very much.

Råde: But then you returned to Uppsala.

Bergström: Yes, then came my second period in Uppsala, starting in 1934. During my military service year I had started to follow lectures in elementary number theory by Trygve Nagel, and during my school years in Södertälje and Östersund I studied the wonder book *Theorie der Algebraischen Zahlen* by Erich Hecke. So I was prepared to continue my studies in algebra and algebraic number theory when I returned to Uppsala. Again, I followed Trygve Nagel's lectures, and he suggested a problem about the arithmetic of biquadratic number fields as the subject for my thesis. This problem brought me into the algebraic class field theory

FIG. 2. *H. Bergström and his assistant, Lennart Råde, on a sailing tour along the Swedish west coast in 1952.*

and the related group theory. I read Helmut Hasse's "Berichte" and papers by the Japanese mathematician, Teiji Tacago. Then I got a grant to study in Göttingen in the summer terms 1937–39. The summer of 1937 was especially important for me since I was introduced by Hasse to the work of Emil Artin and I could apply his characters for my description of the related arithmetics of the biquadratic field and its universal field. In 1938, Rolf Nevanlinna from Helsinki was a guest professor in Göttingen and I followed his lectures on analytic complex functions. During my Master's degree, I had a course on this subject according to Ludwig Bieberbach's book *Funktionentheorie.* Nevanlinna brought with him several students and I spent a lot of time with him and his students. I kept up with these relationships later on.

Råde: Is there more to tell about your summers in Göttingen?

Bergström: It was very important for me because I could read the Rockefeller Institute's lecture notes written by students after lectures given by, for example, Felix Klein and Henri Poincaré. I studied lecture notes by Helmut Hasse on algebraic function theory. I also met Karl Ludvig Siegel in Göttingen. At that time, it was common among German mathematicians to do a lot of walking. For instance, I walked with Siegel and his students all the way from Göttingen to Hannovers-Münden, which is about a 30-km walk. During these walks, we talked a lot about mathematics but also about the difficult political situation in the country. Siegel and his students were very much opposed to the Nazi development in the country.

Råde: I understand that you have been greatly influenced by mathematical books even though you had personal contact with several profound mathematicians.

Bergström: Yes, books have meant very much to me. There are books to which I have returned many times. I should especially like to mention the classical French books *Cours d'Analyse* by De La Valle-Poussin and Goursat, *Appelle et d'Auteville, Precis de Mècanique Rationelle*, Courant-Hilbert's *Methoden der Mathematichen Physik* and algebra books by Oskar Perron, Erich Hecke and B. L. van der Waerden, etcetera.

Råde: But you did not only work in algebra. I think you worked also on some hydrodynamical problems.

Bergström: Yes, I did some work on water flow in tunnels and through generators, and some other applied problems mainly dealing with boundary value problems for partial differential equations. Later, when I was Professor of Applied Mathematics at Chalmers University of Technology, I did more of this kind of work.

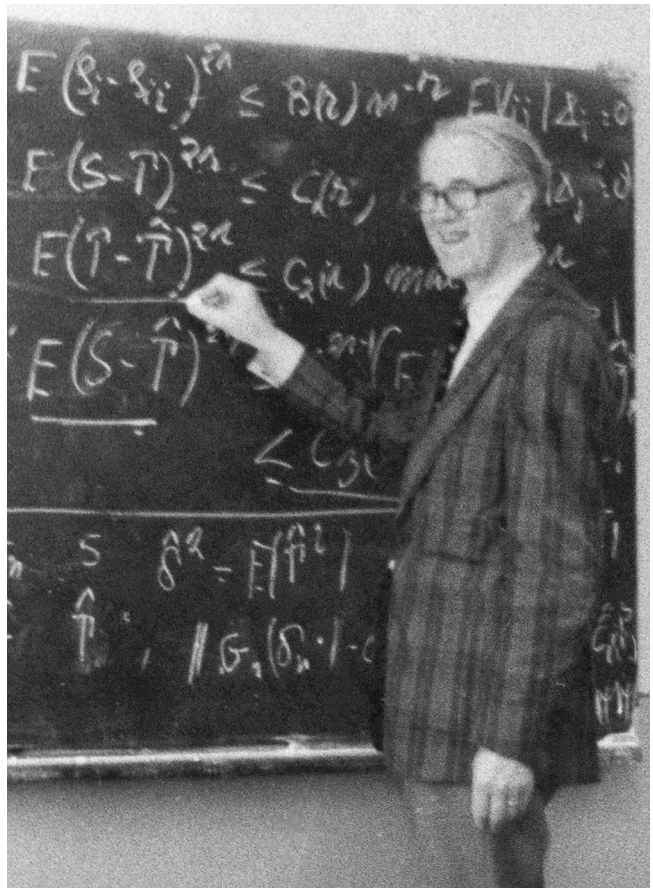

FIG. 3. *Harold Bergström at the blackboard in Tashkent in Uzbekistan, 1974.*

Råde: As a student in one of your classes, I know that for several years you were responsible for the basic mathematics courses in Uppsala. Tell us about this.

Bergström: During the years 1938–1945, I gave the basic course in calculus. The textbook was De La Valleé-Poussin's *Cours d'Analyse Infinitesimale.* However, during the war years, it was impossible for the students to obtain this book so I published my lectures in a primitive form and this book was used for several years. This was also a time when there was a great interest in mathematics and every year I had more than 100 students in my course.

Råde: So far you have not said anything about probability and your well-known work in this field—especially on the central limit theorem.

Bergström: Well, algebraic number theory and group theory seem to be a field far from probability theory, but probability theory has relations to many branches of mathematics, and many mathematicians have entered the field of probability theory

FIG. 4. *Ann-Marie and Harald Bergström with Yu. V. Linnik in Nice, France, 1974.*

from different directions. For instance, Bernoulli, Laplace, Poincaré, Kolmogorov, Van der Waerden and, in Sweden, Cramér, Esseen and myself. It was Carl-Gustaf Esseen's work on the central limit theorem that sparked my interest in this problem, and I developed the method which I should like to call the *direct convolution method* for dealing with limit theorems in probability theory. Together with the Weierstrass singular integral, this is a very useful method. The algebraic component is an essential part of this method. In 1942, 1945 and 1949, I published my results in *Skandinavisk Aktuarietidskrift.* My second paper dealt with the multidimensional central limit theorem. It was reviewed by Kai-Lai Chung, who pointed out possible extensions of my results, which I followed up in my 1949 paper.

Råde: Your first contributions to probability theory had, as far as I know, to do with the remainder term in the central limit theorem. Can you tell us a little more about your work here and your direct convolution method.

Bergström: The distribution function, and more generally the probability measure of a sum of independent random variables, is the convolution product of the distributions of the terms. Hence, we have to do with products. We can deal with general products as with products of numbers when we have a suitable norm or seminorm. I used the Gaussian transform in order to smooth the product and the factors, and I gave a product-sum identity for the difference between two products. In the first two papers, I considered convolution products with identical factors and very elementary expansions. The transformed terms in the sum had derivatives of all orders and could be approximated, and then the transform could be removed in order to give the remainder term of the difference. This was the case when one of the products was the convolution of Gaussian factors. This removing depends on the fact that the Gaussian transform is the Weierstrass singular integral where the variance σ^2 is permitted to tend to zero. The method worked as well for multidimensional distributions as for one-dimensional distributions and it was possible to give the remainder terms dependence not only on the number of terms but also on the dimension. Later, I give a general and more complex product-sum identity which has also been used in analysis, and which I have used for the theory of convolution products convergence to infinitely divisible distributions. This product-sum identity in different forms has been applied to get asymptotic expansions by Nagaev. It seems to me that the "direct convolution method" is an appropriate name for this method.

Råde: What are your views about the distinction between pure and applied mathematics?

Bergström: It is hard to tell the difference between pure and applied mathematics. I once read a long article by Felix Klein about the relation be-

FIG. 5. *Bergström sitting opposite Helmut Hasse at a conference in Salzberg, Austria, 1952.*

tween pure and applied mathematics. I think the article was in Crelle's journal. Felix Klein also discussed this relation in many other articles and talks (see his collected works). He has given an interesting picture of the situation when he compares mathematics with a fortification where formal pure mathematics is the citadel (a fortress in a city usually not considered so important in Roman cities), pure mathematics, in general, is the real fortress and applied mathematics the outwork. But there are bridges leading in and out of these parts of the fortification. Klein also pointed out that applied mathematics is always connected with other sciences. I think that these bridges have often been passed in both directions.

My work in probability theory is surely pure mathematics, but it has direct applications. As I have mentioned, I have done some applied work—most of which was because I was approached by engineers for help. I worked on a hydrodynamic problem concerning the control of water flow with the aid of a basin (of importance for power generating stations) and on a related problem (but mathematically very different) on subduing a high-voltage direct current when transmitted through water. I have also studied a stress problem in connection with tunnels in rocks. The main part of these problems have been applications of well-known boundary solutions of partial differential equations and numerical approximations of solutions to ordinary differential equations. I once worked on an unusual application of probability theory on the distribution of simultaneous discharges of missiles from a destroyer at sea. That work also gave me the opportunity to travel on a destroyer on the open sea. I liked the sea but not the shooting.

CHALMERS UNIVERSITY OF TECHNOLOGY

Råde: Let us now turn to your time as Professor in Applied Mathematics at Chalmers University of Technology in Gothenburg.

Bergström: At the beginning of the academic year 1946–1947, I was approached by Karl Gustaf Hössjer, Rector and Professor of Mathematics, to hold a new chair of applied mathematics. The program for this chair was rather disconnected. It contained courses in linear algebra, analytic geometry, vector analysis, nomography, partial differential equations, with applications and some probability theory, and mathematical statistics "for the estimation of errors." I had experience in all these areas with the exception of nomography, of which I knew nothing. This old method of representation of functions by diagrams (nomograms) was important for engineers before the time of computers. I found my resource in Maurice d'Ocagne's *Traité de Nomographie* (1899, 1921) and I learned a lot from that book but also from my students. But I soon stopped lecturing on this subject and left all teaching to practical exercises by an able assistant. I introduced a substantial compulsory course in mathematical statistics for all engineering students

FIG. 6. *Harald Bergström with Marta and Harald Cramér in Moscow, 1974.*

(without any formal permission to do so) as a subject in its own. This was quite new at a university of technology at that time. I also introduced vectors in the study of analytic geometry, which was usually not the case at universities in Sweden at that time.

On a more advanced level, I gave a seminar on Hilbert spaces, which was attended by several mathematicians, engineers and physicists. At that time the great importance of Hilbert spaces was not well known.

The application of mathematics to the natural sciences has always interested me very much—from a mathematical and from a philosophical point of view. In this field I have been guided by two excellent books: Hermann Weil's *Philosophy of Mathematics and Natural Science* and Henry Margenau's *The Nature of Physical Reality.* I once had a particularly exciting coincidence relating to Margenau. I was traveling by train in Germany. Opposite me in the compartment was a gentleman. We started to talk and pretty soon our discussion went into natural science. It seemed that we did not agree on a certain point. Then I said: "But Margenau said so and so." His reply was: "I am Margenau." And so he was. He obviously knew his book much better than I did. Unfortunately for me, our travel ended shortly thereafter.

I should mention that during my first years at Chalmers I was quite alone for all this teaching. My only help was a "half assistant" for exercises in nomography. But later I had several very able assistants and lecturers, including my interviewer, who played an important role for the improvement of all these courses.

Råde: After the war, there was a great interest to introduce statistical quality control and other statistical methods in the Swedish manufacturing industry. Did you not participate in a committee to promote the use of statistical methods in Swedish industry?

Bergström: At Chalmers, I started a seminar on statistical quality control, together with people from industry—especially from SKF, the Swedish ball bearing company in Gothenburg, where statistical process control is used to a great extent today. I was also a member of a committee set up by an organization for the Swedish mechanical industry. Mainly, we discussed standardization of statistical measures and methods, but we also discussed some other statistical methods.

Råde: There are still universities of technology where the students have very short courses in probability and statistics or no such courses at all. In your opinion, why should engineering students learn about probability and statistics?

Bergström: Well, probability theory and statistics play an important role in the natural sciences and in many engineering applications. Surely, engineering students should learn about these things. In my opinion, the theoretical background is very important for the user of probability and statistics to solve practical industrial problems.

Råde: For some time you were Chairman of the Swedish Statistical Association and, as such, you organized its first summer school. Can you tell us something about this?

Bergström: The Swedish Statistical Association started in 1962 and I was Chairman in 1966–67. In 1966, there were 76 members. Today there are about 800 members. The Association was quite young when I was Chairman and it was a time of planning. We discussed the Nordic Conferences on Mathematical Statistics, the establishment of a Scandinavian Journal of Statistics, and the concept of summer schools. I organized the first such school and the theme was inference on stochastic processes. The school was located high atop one of our most beautiful mountains in the North, the Areskutan, from where we had a most splendid view.

Råde: You have, of course, attended a large number of conferences.

Bergström: Yes, my first conference was the 1936 World Mathematical Conference in Oslo. I gave a paper there on algebraic number theory. I remember a reception attended by all 900 participants where we were welcomed by King Hakon,

FIG. 7. *Bergström with Boris Gnedenko in Moscow, 1974.*

who shook hands with all of us. Since then, I have attended all World Mathematical Conferences until 1970, but of course there were no such conferences during the war. I attended the World Conference in Cambridge, USA, in 1950, which was the first such after the war. I did that in connection with a study tour to the United States with the object of studying the use of computers. I visited several universities and also IBM's large establishment in New York, and I also attended a conference in Washington, D.C., on computers. The conference was in a hotel which was hooked up to a computer at the Bureau of Standards and we could give problems to that computer. I was very impressed with Harold Hotelling, who knew the (then) largest known prime number, which he gave to the computer for checking of primality. In quite a short time, the computer reported that it was indeed a prime number.

At the Cambridge conference, I attended the famous lecture by Laurent Schwartz on distributions, and on the ship back to Europe I spent nine days with Harald Bohr, who had introduced Laurent Schwartz when he received the Fields medal at the conference. With Harald Bohr—who was very enthusiastic about the lecture by Schwartz—I had many interesting mathematical conversations but we also played a lot of table tennis.

Råde: Please tell us about your experiences as Visiting Professor.

Bergström: Yes, in 1959 I was invited by Erik Sparre Andersen to Aarhus University, where I had the opportunity to collaborate with him and with Monroe Donsker and Klaus Krickeberg. I had met Andersen several times and we still meet. We all gave seminar lectures. I spent the academic year 1960–61 at the Catholic University in Washington, D.C., at the invitation of Eugene Lukacs, who was very interested in my work on stable distributions, especially my derivation of their characteristic functions. In Washington, I was with Daniel Dugue from Paris, and Batchelet from Basel. On that occasion, I also started to work on my book, *Limit Theorems for Convolutions*. I also visited Chapel Hill at the invitation of Harold Hotelling and later I was there at the invitation of Ross Leadbetter.

I was invited by Ralph Bradley and Richard Savage to the University of Florida in Tallahassee in 1966. Here I had the opportunity to learn about many interesting applications in statistics. In 1974, I was invited by Madan Puri in Bloomington, and we wrote a paper together on convergence and remainder terms in linear rank statistics. It was in Bloomington that I got to know Paul Halmos, whose book on measure theory I had used. In 1975 I was invited to Bowling Green by Eugene Lukacs and there I gave lectures on the central limit theorem.

Råde: I think that probabilists in the former Soviet Union have been very interested in your limit theorem method and that you have visited the Soviet Union several times.

Bergström: Yes, in 1974 I went directly to the Soviet Union from Bloomington, and stayed there three months with visits to Leningrad, Moscow, Vilnius and Novosibirsk. Altogether, I have been several times to the Soviet Union. I think that Vjacheslav Vasiijevich Sazonov was my first contact there. I met him at the World Conference in Amsterdam in 1954, where he gave an invited lecture and described my method. Other people I have collaborated with are Zolatarev, Ibragimov, Borovkov, Nagaev, Paulaskas and Bikelis. From Borovkov and Nagaev in Novosibirsk, I learned about Alexandrov's method—which has been very important for my book on weak convergence of measures. I also met Gnedenko and Kolmogorov, and once I had dinner in Gnedenko's home with Kolmogorov and Harald Cramér. Unfortunately, I never picked up any knowledge of the Russian language. I remember that in order to make myself understood in Tashkent, I had to give my lecture in French. In 1968, Yu. V. Linnik visited our department at Chalmers for two months and we had long and valuable discussions.

Perhaps it should be mentioned that all these visits to other countries were not concerned only with mathematics, but also close personal ties, wonderful sceneries, cultural activities and architecture. My wife, Ann-Marie, has often guided me on these explorations.

FIG. 8. *The Bergströms outside their summer house on the Swedish west coast.*

AFTER RETIREMENT

Råde: You have been very active since your retirement. Recently you did research on an economics problem.

Bergström: Yes, I was elected to be an accountant for a scientific society. Such an accountant is not supposed to have either large responsibilities nor ability for his commission since there is also a professional accountant. However, I got interested in different possible investments such as stockholding and bearing investments, and I made—together with my colleague Christer Borell—a comparison of different investments and we found that in this field we have to deal with interesting stochastic processes. Recently I learned about options on shares and I found that we were facing an interesting gambling situation which, under certain assumptions, is fair. Thus the value of the option at any time can be given in explicit form as a rather simple function of the value of the share. I have written a paper where I generalize earlier results. This work has given me great fun since I have been able to deal with the problem as a partial parabolic difference boundary value problem, comparable to the classic boundary value problem for the well-known partial differential equation for the conduction of heat.

Råde: Finally, Harald, I would like to ask about your hobbies.

Bergström: Yes, I like very much sailing, skating, skiing and tennis, and I still play tennis every week and I do some sailing.

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