

Correction to “Bayesian Model Averaging: A Tutorial”

A printing malfunction caused all minus signs and some left parentheses to be omitted from the paper “Bayesian Model Averaging: A Tutorial” by Jennifer A. Hoeting, David Madigan, Adrian E. Raftery and Chris T. Volinsky in the November 1999 issue of *Statistical Science* (volume 14, pages 382–417). These errors occurred after the proof stage and were not the fault of the authors. Corrections to the paper are listed below. A corrected version of the paper is also available at <http://www.stat.washington.edu/www/research/online/hoeting1999.pdf>. Please cite this article as follows: Hoeting, J. A., Madigan, D., Raftery, A. E. and Volinsky, C. T. (1999) “Bayesian Model Averaging: A Tutorial (with discussion)” *Statistical Science* 14:4, 382–417. Corrected version available at <http://www.stat.washington.edu/www/research/online/hoeting1999.pdf>.

p. 383, last display of second column: replace the formula for $\text{Var}[\Delta|D]$ by

$$\text{Var}[\Delta|D] = \sum_{k=0}^K (\text{Var}[\Delta|D, M_k] + \hat{\Delta}_k^2) \text{pr}(M_k|D) - E[\Delta|D]^2,$$

p. 385, 24 lines from top of first column: replace

$$“O_L = O_R^{-1}” \quad \text{by} \quad “O_L = O_R^{-1}”$$

p. 386, last display of second column: replace by

$$Y^{(\rho)} = \begin{cases} \frac{y^\rho - 1}{\rho}, & \rho \neq 0, \\ \log(y), & \rho = 0. \end{cases}$$

p. 386, 11 lines from bottom of second column: replace

$$“(1, 0, 0.5)” \quad \text{by} \quad “(-1, 0, 0.5)”$$

p. 387, first column: equation (8) should read

$$(8) \quad \varepsilon \sim \begin{cases} N(0, \sigma^2), & \text{w.p. } (1 - \pi), \\ N(0, K^2 \sigma^2), & \text{w.p. } \pi. \end{cases}$$

p. 387, last line of second column: “ $O(n^{-1})$ ” should be “ $O(n^{-1})$.”

p. 388, first column: equation (11) should read

$$(11) \quad 2 \log B_{10} \approx \chi^2 + (E_1 - E_0).$$

p. 388, sentence that follows equation (11) should read:

$$“\text{In (11), } \chi^2 = 2\{\ell_1(\hat{\beta}_1) - \ell_0(\hat{\beta}_0)\}”$$

p. 388, second display in first column should read:

$$\begin{aligned} E_k &= 2\lambda_k(\hat{\beta}_k) + \lambda'_k(\hat{\beta}_k)^T (F_k + G_k)^{-1} \\ &\cdot \{2 - F_k(F_k + G_k)^{-1}\} \lambda'_k(\hat{\beta}_k) \\ &- \log |F_k + G_k| + p_k \log(2\pi) \end{aligned}$$

p. 388, 2 lines under second display in first column: replace

$$“G_k = W_k^{-1}” \quad \text{by} \quad “G_k = W_k^{-1}”$$

p. 388, 6 lines under second display in first column: replace

$$“O(n^{-1/2})” \quad \text{by} \quad “O(n^{-1/2})”$$

p. 388, 9 lines under second display in first column: replace

$$“O(n^{-1})” \quad \text{by} \quad “O(n^{-1})”$$

p. 388, equation (13): this equation should read

$$(13) \quad \log \text{pr}(D|M_k) \approx \log \text{pr}(D|\hat{\beta}_k, M_k) - d_k \log n,$$

p. 390, equation (16): this equation should read

$$(16) \quad \text{pr}(M_i) = \prod_{j=1}^p \pi_j^{\delta_{ij}} (1 - \pi_j)^{1 - \delta_{ij}},$$

p. 390, equation (17): this equation should read

$$(17) \quad - \sum_{d \in D^T} \log \text{pr}(d | M, D^B),$$

p. 390, equation (18): this equation should read

$$(18) \quad - \sum_{d \in D^T} \log \left\{ \sum_{M \in \mathcal{M}} \text{pr}(d | M, D^B) \text{pr}(M | D^B) \right\}.$$

p. 392, table 1: this table should read

TABLE 1
PBC example: summary statistics and BMA estimates

Variable	Range	Mean	Mean βD	SD βD	$P(\beta \neq 0 D)$
Bilirubin (log)	-1.20-3.33	0.60	0.784	0.129	100
Albumen (log)	0.67-1.54	1.25	-2.799	0.796	100
Age (years)	26-78	49.80	0.032	0.010	100
Edema	0 = no edema 0.5 = edema but no diuretics 1 = edema despite diuretics	$n = 263$ $n = 29$ $n = 20$	0.736	0.432	84
Prothrombin time	2.20-2.84	2.37	2.456	1.644	78
Urine copper (log)	1.39-6.38	4.27	0.249	0.195	72
Histologic stage	1-4	3.05	0.096	0.158	34
SGOT	3.27-6.13	4.71	0.103	0.231	22
Platelets	62-563	262.30	-0.000	0.000	5
Sex	0 = male	0.88	-0.014	0.088	4
Hepatomegaly	1 = present	0.51	0.006	0.051	3
Alkaline phosphates	5.67-9.54	7.27	-0.003	0.028	3
Ascites	1 = present	0.08	0.003	0.047	2
Treatment (DPCA)	1 = DPCA	0.49	0.002	0.028	2
Spiders	1 = present	0.29	0.000	0.027	2
Time observed (days)	41-4556	2001			
Status	0 = censored 1 = died	0.40			

p. 392, table 2: this table should read

TABLE 2
PBC example: results for the full data set¹

Model no.	Age	Edema	Bili	Albu	UCopp	SGOT	Prothromb	Hist	PMP	Log lik
1	•	•	•	•	•		•		0.17	-174.4
2	•	•	•	•	•		•	•	0.07	-172.6
3	•	•	•	•	•			•	0.07	-172.5
4	•		•	•	•		•		0.06	-172.2
5 ²	•	•	•	•			•		0.05	-172.0
6	•	•	•	•	•				0.05	-172.0
7	•	•	•	•	•	•	•		0.04	-171.7
8	•	•	•	•		•	•		0.04	-171.4
9	•	•	•	•		•	•	•	0.04	-171.3
10	•	•	•	•	•	•	•	•	0.03	-170.9
$Pr_{MA}[\beta_i \neq 0]$	1.00	0.84	1.00	1.00	0.72	0.22	0.78	0.34		

¹ PMP denotes the posterior model probability. Only the 10 models with the highest PMP values are shown.

² Model selected by FH.

p. 395, table 7: this table should read

TABLE 7
Body fat example: least squares regression results from the full model¹

Predictor		Coef	Std error	t-statistic	p-value
Intercept		-17.80	20.60	-0.86	0.39
X ₁	age	0.06	0.03	1.89	0.06
X ₂	weight	-0.09	0.06	-1.50	0.14
X ₃	height	-0.04	0.17	-0.23	0.82
X ₄	neck	-0.43	0.22	-1.96	0.05
X ₅	chest	-0.02	0.10	-0.19	0.85
X ₆	abdomen	0.89	0.08	10.62	<0.01
X ₇	hip	-0.20	0.14	-1.44	0.15
X ₈	thigh	0.24	0.14	1.74	0.08
X ₉	knee	-0.02	0.23	-0.09	0.93
X ₁₀	ankle	0.17	0.21	0.81	0.42
X ₁₁	biceps	0.16	0.16	0.98	0.33
X ₁₂	forearm	0.43	0.18	2.32	0.02
X ₁₃	wrist	-1.47	0.50	-2.97	<0.01

¹Residual standard error = 4, R² = 0.75, N = 251, F-statistic = 53.62 on 13 and 237 df, p-value <0.0001.

p. 396, table 8: this table should read

TABLE 8
Body fat example: comparison of BMA results to model selected using standard model selection methods¹

Predictor		Bayesian model averaging			Stepwise model
		Mean β D	SD β D	P(β ≠ 0 D)	p-value
X ₆	abdomen	1.2687	0.08	100	<0.01
X ₂	weight	-0.4642	0.15	97	0.03
X ₁₃	wrist	-0.0924	0.08	62	<0.01
X ₁₂	forearm	0.0390	0.06	35	0.01
X ₄	neck	-0.0231	0.06	19	0.05
X ₁₁	biceps	0.0179	0.05	17	
X ₈	thigh	0.0176	0.05	15	0.02
X ₇	hip	-0.0196	0.07	13	0.12
X ₅	chest	0.0004	0.02	6	
X ₁	age	0.0029	0.02	5	0.05
X ₉	knee	0.0020	0.02	5	
X ₃	height	-0.0015	0.01	4	
X ₁₀	ankle	0.0011	0.01	4	

¹Stepwise, minimum Mallows's C_p, and maximum adjusted R² all selected the same model. The predictors are sorted by P(β_i ≠ 0|D) which is expressed as a percentage. The results given here are based on standardized data (columns have means equal to 0 and variances equal to 1).

p. 412, last line of second column: replace

“σ^d” by “σ^{-d}”

p. 414, display near bottom of second column: replace by

$$\beta_1 = \frac{1}{2} (\bar{Y}_{21}^{\text{new}} - \bar{Y}_{11}^{\text{new}}) + \frac{1}{2} (\bar{Y}_{22}^{\text{new}} - \bar{Y}_{12}^{\text{new}}) + O_p \left(n_{\text{new}}^{-\frac{1}{2}} \right),$$