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HARALD CRAMÉR 1893–1985

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Harald Cramér died on October 5, 1985. He was a great scientist and a good man.

1. Harald Cramér—his life.

1.1. I had my last conversation with my teacher and friend, Harald Cramér in April 1985 when I visited him in his attic apartment in an old house in beautiful Djurgården, outside Stockholm. He had moved there after the death of his beloved wife. There were books in all corners. I admired the view across the water to the north and to the south. Ships passed on their way to and from the Stockholm harbour. However, Harald could not discern them, for his eyesight was failing. He was also nearly deaf. But his intellect was clear and his memory excellent. We spoke about things old and new. I showed him some research papers recently written by young Swedish research workers, and he was actively interested in everything. But there was melancholy in the air, and I think we both felt that it was the last time.

1.2. Harald Cramér was born on September 25, 1893, in Stockholm. In 1918 he married Marta Hanssow. She died in 1973. They had one daughter, Marie-Louise, who lives in Finland, and two sons, Tomas and Kim, who live in Stockholm.

1.3. Cramér began his studies at the University of Stockholm in 1912, and was particularly interested in chemistry and mathematics. During 1913–1914 he was a research assistant in biochemistry and published his first paper together with H. von Euler, later a Nobel laureate [1]. (The numbers in square brackets refer to the bibliography at the end of the paper.)

Cramér soon abandoned chemistry for mathematics and received a Ph.D. in 1917 for a thesis on Dirichlet series [6]. He was an assistant professor of mathematics from 1917–1929 at the University of Stockholm.

1.4. In the years before 1920, Cramér became interested in insurance, especially in problems connected with the risk of ruin of an insurance business. In 1919 he held a short appointment at Försäkringsinspektionen (Swedish Private Insurance Supervisory Service). I would guess it was there that he first become acquainted with the ruin problem. From 1920–1948 he was an actuary, first for the life insurance company Svenska livförsäkringsbolaget, and from 1929 on for the reinsurance company Sverige. From 1949–1961 he was a consulting actuary for Sverige.

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HARALD CRAMER

Insurance problems turned Cramér's attention to probability theory. His first paper on a probabilistic problem dates from 1923. In 1927 he was ready to publish an elementary textbook in Swedish, *Probability Theory and Some of Its Applications*.

1.5. In the middle of the 1920's, there were chairs in statistics at some Swedish universities, but none in mathematical statistics. Influential leaders of Swedish insurance companies recognized the need for such a chair. A professorship for Actuarial Mathematics and Mathematical Statistics was established at the University of Stockholm. In 1929, Harald Cramér became its first holder and head of the Institute of Actuarial Mathematics and Mathematical Statistics.

The institute attracted not only regular students, but also actuaries, biologists and other people interested in applications of probability theory. During 1934–1935 Cramér, possibly as the first scientist outside Russia, gave a series of lectures founded upon Kolmogorov's work. As H. Wold has remarked, this series became very important for the research of Cramér's students.

In 1934 W. Feller came to Stockholm and stayed for four years. It was of great value for Cramér personally and for his graduate students that Feller, with his mathematical roots in pre-1933 Germany and his skill as a researcher, was present at the institute during this period.

During this decade, Cramér was entrusted with the task of adjusting the bases of insurance premiums and reserves to the decreasing rates of interest and, also, with the broader task of constructing a new base ("the technical base of 1938"). In addition, he was appointed to a state commission for preparing a new insurance business act. These were time-consuming extra jobs for a professor at a newly founded institute.

1.6. Regarding the 1940's, I will be more detailed and more personal, which is natural, since in the autumn term of 1939 I had begun studying at the institute. One of my first memories is the move to new premises at Norrtullsgatan in northern Stockholm, built with funds obtained from insurance companies. The building was small and cosy; there was one room for the professor, a library, a small combined lecture and computing room and a few minor rooms—that was all. But we young students enjoyed the time we spent there and in the nearby café. We followed the vicissitudes of the war, performed long calculations using manual Odhner calculators and listened to Cramér's lectures on basic insurance mathematics. Touching the dull formulas with his magic wand, he turned them into poetry.

We progressed with our studies and were soon attending the seminars. Cramér was then busy with *Mathematical Methods* and provided us with subjects for our own seminars, taken from the third part of his book, in particular from the chapters on analysis of variance and regression analysis. It was a privilege to have this early contact with his book: The theory behind the standard statistical methods became clear and formed an essential part of our training.

During the war, Cramér showed great hospitality. Harald Bohr, the eminent mathematician, worked at the institute for some years, sheltered from the ill winds over Denmark, and Olav Reiersoel came from Norway.

After the war, Cramér spent a sabbatical year in the United States. For part of this time he was a visiting professor for the Bicentennial Year at Princeton University.

When Cramér returned to Sweden, guest speakers began to visit the institute, and we graduate students profited from their learning and experience. It was interesting to see people we had hitherto only read about. Indeed, they really existed! Still, most of all we liked Cramér's lectures given each Friday and Saturday morning, also attended by well-established actuaries and statisticians. After all these years, I can still feel the thrill in my mind when Cramér entered the lecture room and started speaking, distinctly, inspiringly, and always without a manuscript. It was sometimes said that Cramér was so clever that his listeners had a feeling of understanding his message even if, deep down, they did not grasp anything at all!

The seminars ended in May each year with a finale in Cramér's magnificent home in Djursholm, where Marta acted as a queen-like hostess. A lecture was given in Harald's library, we had a sumptuous meal and our hosts told us over coffee about their travels and encounters.

Cramér was a true gentleman. All his students liked him, but in a distant and respectful way. He gave the graduate students strong support and much encouragement. Niels Arley, the Danish scientist, writes in his *Memoirs* that Cramér was "one of the noblest and, at the same time, heartiest persons I have ever met."

Let me mention in passing that Cramér was fakultetsopponent (external examiner) when, in 1943, Arley defended his doctor's dissertation in public at the University of Copenhagen. In spite of the war, Cramér was allowed an entry permit to Denmark. His appearance on this occasion was, according to Arley, a display of Swedish academic elegance with doctor's tailcoat, doctor's hat and all that.

Let me return to the institute. To his students, Cramér was above all a theoretician, but he did not forget applications. Indeed, he was also an actuary! The written examinations revealed his dual interest in theory and real data. These exams were divided into one day for theoretical problems and one day for numerical and other applications. On the second day, one single large problem was given. For example: Given some raw mortality data, fit a Makeham curve by minimum chi-square. The second day was feared by some students including myself. If I cannot get started, what shall I do?

For some years in the 1940's, Cramér was occupied with a plan for adding to his institute a section for applied statistics. An organization was outlined by a committee chaired by Cramér, but the idea was never carried out. Instead, in 1948, we graduate students started an applied activity, known as the Statistical Research Group, which still exists. Cramér was the first Inspector of the Group and was helpful with both good advice and the acquisition of grants.

1.7. After this nostalgic retrospect of the 1940's, let me look, more briefly, at the 1950's.

Cramér was a good administrator and, as has already been said, an outstanding speaker. It was therefore not surprising that, in 1950, his colleagues

elected him President of the University of Stockholm. After eight years Cramér became Chancellor of the Swedish universities. He was then 65 and at the same time retired from his professorship. I have a lively recollection of Harald's appointment as Chancellor, for we discussed it at the dinner given when I defended my Ph.D. thesis in May 1958. (By the way, this publication was the last of the 10 theses written during Cramér's time as a professor.) It was certainly an honour for Cramér to become Chancellor. It gave him many opportunities to express his views concerning the administration of the universities, but I think it was not an ideal task for an active scientist like him. He left the Chancellor's office after three years and returned to research.

In the 1950's Cramér again visited the United States. In 1953 he lectured at Berkeley for some time.

Harald's friends celebrated his 65th birthday with a Festschrift: *Probability and Statistics. The Harald Cramér Volume* (1959). (References with years in parentheses refer to the list of additional references at the end of the paper.)

1.8. In the 1960's Cramér was again very active in research. From 1962–1965 he worked for several periods of time at the Research Triangle Institute of North Carolina, with M. R. Leadbetter as his assistant and later as his coauthor.

1.9. In the 1970's Cramér continued to travel and to do research. In 1970 he gave the first S. S. Wilks lecture at Princeton, in 1976 he lectured (in French) at the European Meeting of Statisticians in Grenoble and in 1977 he lectured in Calcutta. He published 10 articles during this decade, at the age of 76 and over.

1.10. Even in the 1980's, after the age of 86, Harald spoke on several occasions in Sweden and elsewhere. In 1980 he was invited to give the Pfizer Colloquium Lecture at the University of Connecticut [112]; in the summer of 1983 he gave the main address at the opening ceremony of the Berkeley Conference in Honor of Jerzy Neyman and Jack Kiefer; in November of the same year he talked at the Royal Swedish Academy of Sciences in Stockholm about "Probability theory and statistical methodology—memories from sixty years." In June 1984, when almost 91, he delivered a lecture, memorable to all present, at the Conference on Stochastic Processes and Their Applications in Gothenburg: The subject was "Some remarks on the early development of the theory of stochastic processes."

1.11. Harald Cramér received many distinctions. He was awarded honorary university degrees at Princeton (1947), Copenhagen (1950), Stockholm (1964), Helsinki (1971), Edinburgh (1972), Calcutta (1977) and Paris (1977). The Royal Statistical Society elected him an Honorary Fellow and in 1972 awarded him The Guy Medal in Gold. He received a prize for actuarial mathematics from Accademia dei Lincei in Rome. He was an Honorary Member of the International Statistical Institute and of the American Academy of Arts and Sciences. He was a member of the Swedish, Danish, Finnish, Norwegian and Spanish Academies of Sciences.

1.12. Cramér was Chief Editor of *Skandinavisk Aktuarietidskrift* (now *Scandinavian Actuarial Journal*) from 1940–1963. He was the President of the Swedish Actuarial Society from 1935–1964 and in 1964 became its Honorary President.

2. Harald Cramér's scientific contributions—an overview. Cramér's scientific output is rich and varied. Some results belong to the history of mathematical statistics, while some bear his name and are used over and over again even today. All second and third year students of statistics know about the Cramér–Rao inequality; some also know about the Cramér–von Mises statistics. Graduate students hear about Cramér's contributions to the theory of the central limit theorem, some use the Cramér–Wold device and a few know the Cramér–Lévy theorem, stating that if a sum of two independent random variables is normally distributed, then so are the components. However, these are only a few examples of Cramér's contributions to different fields of probability and statistics.

As I have mentioned, Cramér worked as a chemist, pure mathematician, actuary, probabilist and statistician, and this is reflected in his publications spanning a period of 70 years.

Together with B. Matérn, I have compiled a bibliography of Cramér's works. Harald assisted us. On the occasion of his 90th birthday, we gave him a mimeographed version, which was later printed [Blom and Matérn (1984)]. It is divided into two parts, the first comprising 114 publications in foreign languages (from a Swedish point of view!) and the second containing 38 publications in Swedish.

The first part of the bibliography is reprinted at the end of this article. It contains four books, *Random Variables and Probability Distributions* [53], *Mathematical Methods of Statistics* [61], *The Elements of Probability Theory* [71] and *Stationary and Related Stochastic Processes* [96]; and 110 scientific papers and other publications, 19 of which may be classified as pure mathematics, 23 as actuarial mathematics including risk theory, 56 as probability and stochastic processes and 12 as other subjects.

The contributions to pure mathematics concern analytic functions and analytic number theory and will not be considered here, nor will his papers on actuarial mathematics, apart from some remarks on risk theory. Cramér's scientific actuarial work is commented upon in the obituaries by Bohman (1985) and Lundberg (1986).

3. Harald Cramér—the probabilist.

3.1. It is not altogether easy for me to assess the importance of Cramér's extensive work as a probabilist. Being myself a statistician, not a probabilist, I would hardly have undertaken to write this article were it not for the fact that (in [108]) Cramér had assembled personal recollections from his life as a probabilist. This is an admirable paper: Apart from what it tells us about Cramér's writings, it has a lasting value as a document in the history of probability.

With Cramér's recollections before me, the best thing I can do is, once more, to be subjective and convey to the reader some personal views formed during my years as a student at Cramér's institute and later as a university teacher.

When Cramér became interested in probability in the 1920's, this subject was not an accepted branch of mathematics. Isolated efforts to base probabilistic results on good mathematics had been made, but no general mathematical theory of probability existed.

Cramér felt the need for a radical change, as appeared in one of his Swedish papers written in 1926: "The probability concept should be introduced by a purely mathematical definition, from which its fundamental properties and the classical theorems are deduced by purely mathematical operations."

As we know, something like a revolution took place in the 1930's, brought about in the first place by Kolmogorov, but also by other scientists such as Khintchine, Lévy and Cramér. In this decade, probability theory became a respectable part of mathematics with probability defined as a set function, a stochastic process a well-defined concept used as a model for random changes in space or time and so forth.

In 1937 Cramér published *Random Variables and Probability Distributions*, a booklet read by a whole generation of probabilists. In a way, Cramér's dream of 1926 materialized in this book. Probability was given a clear foundation in the spirit of Kolmogorov and was studied by rigorous mathematical methods. Cramér's whole subsequent work was governed by this attitude, which proved immensely fruitful for the development of probability theory.

3.2. I will now describe some of Cramér's special achievements, beginning with the central limit theorem. Cramér is a great name in the history of this theorem. It was his work as an actuary that inspired him to study the behaviour of a large number of random variables when added and suitably standardized. In particular, he was interested in estimating the error of replacing the distribution function of a standardized sum by that of the standard normal distribution function.

At the beginning of the century, Charlier had expanded the error in an asymptotic series. Cramér showed that Charlier's proof was incorrect and replaced the series by a more satisfactory one. Generalizations to multidimensional random variables were given by Cramér and Wold in [50], where the well-known Cramér–Wold device for proving multidimensional normality is found. Cramér's first paper on the central limit theorem is [23] and the most important one is, I think, [30]. A pioneering paper [55] deals with a limit theorem for large deviations of standardized sums of random variables. A condensed version of many of Cramér's results can be found in *Random Variables*. Another good source of knowledge (on a less sophisticated level) is *Mathematical Methods*, Chapter 17, where I, like so many students, first learned about these things. I loved Cramér's way of proving the central limit theorem in this book by means of the continuity theorem.

Let me add in passing that I have always regarded the continuity theorem as the finest in probability theory. Lévy was first with the idea in *Calcul des*

Probabilités, but the final formulation is due to Lévy and Cramér. The theorem and its proof given in the first edition of *Random Variables* were not entirely correct and were corrected in the second edition. In *Mathematical Methods* a very clear proof is given that ought to be read by all serious students of probability, not only for its content but also as an example of Cramér's art of writing.

3.3. The second area I will dwell upon is the theory of stochastic processes. Two aspects are important.

First, Cramér wrote many papers and one book on stochastic processes. Second, by his enthusiasm and expert knowledge he stimulated others to work in the field. This influence was felt in the 1930's by his graduate students working with time series [Wold (1938)], and processes of importance for insurance [Segerdahl (1939) and Lundberg (1940)] and by those of my own generation working with inference for stochastic processes [Grenander (1950)] and with stationary processes on the line and in the plane [Hanner (1949) and Matérn (1960)]. Of course, this influence was felt also by many other of Cramér's readers and listeners.

I have a vivid memory of Cramér's authoritative lectures on stochastic processes given some time during the second half of the 1940's. It is a pity that these lectures did not result in a companion volume to *Mathematical Methods*.

In the sequel I will mention three of Cramér's specific contributions to the theory and development of stochastic processes.

3.4. First, Cramér's research in the field of risk theory greatly clarified this theory and stimulated its progress. The work of the father of risk theory, the Swede F. Lundberg, was difficult to read. Cramér took the trouble to penetrate it, and in *Skandinavisk Aktuarietidskrift* (1926) wrote a 22-page review (in Swedish) of one of Lundberg's papers. This article, written in a pungent style seldom used by the amiable Cramér, is worth quoting (the translation is mine): Lundberg "has an outstanding talent for putting forward even rather simple mathematical arguments in such a way that the readers get the very lively feeling of not understanding anything at all... The reader waits in vain for these small valuable hints in the form of comments to the proofs, these glimpses backwards and forwards that a benevolent author inserts in order to mitigate the troubles of the reader. Little help is to be expected from Lundberg in this respect; if such a remark is actually made, it is at times formulated with oracle-like abstruseness." Nevertheless, in the review Cramér extracted from Lundberg's work the essentials of what is now called the collective risk theory and gave it a modern formulation; see also the account of Lundberg's work which Cramér gave much later in [100].

Some of the most important publications on risk theory are [32], [69] and [72]. In the third paper, collective risk theory is treated as a special part of the general theory of stochastic processes. Among other things, Cramér studied the ruin functions for an insurance business, a task already begun by F. Lundberg. Cramér's estimate for ruin is often quoted. Later on, many other people,

especially in Sweden, worked on these problems, and the technique has proved valuable even outside the insurance business; see, e.g., Feller (1971).

3.5. Second, Cramér was active in the field of stationary and related processes.

The first wave of interest came in the 1940's, when Cramér wrote two now famous papers: In [58] he generalized the spectral representation of a covariance function of a univariate stationary process to vector processes; in [60] he derived a spectral representation of the process itself in terms of an orthogonal process.

There was a second wave in the 1960's after Cramér's retirement. With M. R. Leadbetter he published *Stationary and Related Stochastic Processes* [96], an excellent book both for students and specialists, and another example of Cramér's interest in both teaching and research. The first part (Chapters 1–8) contains a very lucid account of the general theory of stochastic processes with special chapters on particular processes. In the second part (Chapters 9–15), the authors present a multitude of results concerning level crossings for stationary Gaussian processes. For example, they prove the fine result, due partly to Cramér, that under mild conditions the upcrossings by a stationary Gaussian process of a high level follow approximately a Poisson process. The second part of the book has given rise to much further research.

3.6. Third, Cramér and others developed the multiplicity theory of stochastic processes. For a comprehensive review of this field see *Random Processes, Multiplicity Theory and Canonical Decompositions* (1973). Cramér's work on this part of the theory of processes bears witness to his vitality after retirement. From 1960 onward he discussed multiplicity theory in about 10 papers, several of which are reprinted in the above-mentioned book; his last scientific paper [113], published when he was 89, treats this subject. The theory throws light on much work done in the past by many research workers, among them Wold, Hanner and Cramér himself, and completes, in a logical way, his activity as a probabilist.

4. Harald Cramér—the author of *Mathematical Methods*.

4.1. In 1945 Cramér published *Mathematical Methods of Statistics* [61]. This was a landmark in the history of mathematical statistics. Before this time, no unified statistical theory based on probability existed. Cramér saw the need and the possibility for building statistical science on the foundations of mathematical probability, and the written result of his efforts was *Mathematical Methods*. After the message of this book had been spread over the statistical world, our subject never was the same again. Let me comment upon some of the resulting changes.

4.2. First, education of statisticians has become different. Cramér convinced us that the ingredients corresponding to the three parts of the book, namely, mathematics, probability and statistics, are indispensable in the studies of a mathematical statistician. For several decades now, *Mathematical Methods* has been much used as a textbook by students in many countries. Some of these

people later wrote books and papers themselves, in the same spirit, and this process will be repeated in years to come. In this way, the book has had, and will have, a tremendous influence on the training of mathematical statisticians.

Personally, I owe more to this book than I can briefly explain. About 40 years have passed since I first read *Mathematical Methods* at Cramér's institute, and I still use it now and then for contemplation and for reference. A single example of a section to which I have returned many times is 20.6, with its useful convergence theorem, sometimes called Cramér's theorem or the Cramér–Slutsky theorem.

4.3. Second, research in statistical theory has been profoundly affected by *Mathematical Methods*. Earlier, statistical ideas were often launched very loosely, even in scientific journals. With few exceptions, they are nowadays studied thoroughly, using rigorous mathematical methods, before they are published. It is my conviction that this new attitude toward research in statistics is largely due to Cramér's influence through this book.

A good example is the exposition in Chapter 27 of standard errors of sample moments and functions of such moments. Earlier, there was in this field a mess of calculations of doubtful validity, an Augean stable of formulas, which Cramér cleaned by letting in a stream of good quality mathematics.

4.4. Third, several new results in the third part of the book have led to further important research activities. One example concerns the asymptotic properties of maximum likelihood estimates in Chapter 33, where Cramér performed important research that was later continued by many people.

Another striking example is the introduction and proof in Chapter 32 of the Cramér–Rao inequality. Such a fundamental result is seldom given for the first time in a textbook. When reading this chapter in the 1940's, I was astonished that such a result was possible and I admired the proof immensely. Until then, statistical inference had been mainly a lot of approximations of unknown validity; here was a marvellous precise answer.

4.5. Finally, let me put forward the view that *Mathematical Methods* is unique among statistical textbooks for its formal qualities. Quite apart from its content, it is a joy to read such a clearly written book. In a way, I deplore that the development of statistics since 1945 has made it partly obsolete. Nevertheless, the book is still read by many people. A Russian mathematician has aptly said that *Mathematical Methods* is nowadays a necessary but not sufficient part of a statistician's education. In any case, this book will in the future be a rich source of knowledge regarding the history of mathematical statistics and will remain a masterpiece of its time.

Harald Cramér is dead but his memory will live on in the history of probability and statistics and in the hearts of his admirers and friends. He was a great scientist and a good man.

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Publications by Harald Cramér in languages other than Swedish. This is part of the Bibliography compiled by G. Blom and B. Matérn published in (1984) *Scand. Actuar. J.* **67** 1–10. It is reproduced here with the permission of Almqvist and Wiksell Periodical Company.

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VIDEOTAPE

A videotape containing a recording of a lecture by Harald Cramér on the history of probability is available from Continuing Education, American Statistical Association, 1429 Duke Street, Alexandria, Virginia 22314.

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