follow Fisher, ANOVA should be mainly concerned with a statistical analysis. The definition of models for which variances are to be analyzed should at least include what is often referred to as Eisenhart's models I, II and III, both "balanced" and "unbalanced," since they fit into the framework of analyzing variances, and are useful in modelling real world situations.

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Dr. Speed is to be congratulated on the work that is surveyed here. He has sought to describe the basic framework for balanced designs and, moreover, has, with collaborators, elucidated the complex structure consequent on nesting and the proper analysis following from that structure.

What one can argue about, however, is the appropriateneess of the title ANOVA for only this class of structures. Of course, once one begins to abstract it is difficult to know where to stop. Fourier methods provide an example. But in the present situation the position is somewhat reversed and cases are left out in the cold which to the people who regard analysis of variance as rather down to earth (and what could be more down to earth than a field experiment?) are near to the center of the idea. When a subset S of a full replication T is considered in Speed (1985), then it is assumed that the adjacency matrices when restricted to S continue to fulfill the requirements (4.1) in the discussion paper and, in particular, define a commutative algebra. It is interesting to observe that this algebra $\mathbf{A}$ of Section 4, is the same as that introduced in Bose and Mesner (1959) who did not, however, concern themselves with Γ but were instead concerned with the construction of partially balanced designs. (The class of objects for which the A_a are adjacency matrices are not the plots but, for Bose and Mesner, the varieties.) If the subset S is a subset constituting a partially balanced incomplete block and T corresponds to the fully replicated experiment, then the adjacency matrices of Section 4 for S will not produce a commutative algebra so that, in spite of Fisher and Yates (1948, page 19) the resulting analysis is not to be called ANOVA. The algebra $\mathbf{A}$ is often the commuting algebra of the representation of a group G by permutation of the points of T. The analysis of variance will be unique if that algebra is commutative, which is closely connected with the existence of a further permutation, h say, such that, for all $s, t \in T$, there is a g for which $t^g = s^h$ and $t^h = s^g$.

Of course Dr. Speed might argue that what was discussed in the previous paragraph was the aliasing from a partial replication of his full ANOVA. However, the problem with this is that for such an analysis a fair amount of symmetry has to remain if the result is to be of interest. Thus, in the description of aliasing that makes use of the Selberg trace formula [Hannan (1965)] this formula has a useful interpretation if H, the largest subgroup of G that maps S onto itself, is normal in G because only in this case does Clifford's theorem guarantee that each irreducible representation of G, when restricted to H, breaks up into conjugate irreducible representations of H of equal degree. However, in many cases the partial replication will be so lacking in symmetry that nothing useful arises in this way, yet one would usually think of some analysis of variance being associated with the experiment.

When a vector of observations $y(t)$ is made at each point of T, other cases can arise than those discussed in Section 8 of the discussion paper. In the case where **A** is the commuting algebra of the representation of G by permutation of points of T, then this is the representation of G induced by the identity representation of the isotropy group, K. At least in cases where T is a continuum, examples arise [Yaglom (1961)] where the action of G is of the form $y(t) \to U(g)y(t^g)$, $U(g)$ being a unitary representation of G in the vector space wherein $y(t)$ stands, and $\Gamma(s, t) = U(g)\Gamma(s^g, t^g)U(g^{-1})$. Then the relevant representation is that induced by the representation $U(k)$ of K obtained by restricting $U(g)$ to K. A discussion of this may be found in Hermann (1966) who views $y(t)$ as a cross section of a vector bundle on the coset space $T = G/K$. In our case the cross section is stochastically chosen. There seems no very good reason why, if ANOVA is to be extended as in Section 7, the identity representation of K should occupy such a privileged place.

Dr. Speed's work is a substantial contribution to the mathematical theory of ANOVA and his paper is most interesting.

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