The Annals of Statistics
1987, Vol. 15, No. 1, 385–397

ON SIMPLE ADJUSTMENTS TO CHI-SQUARE TESTS WITH SAMPLE SURVEY DATA[1]

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For testing the goodness-of-fit of a log-linear model to a multi-way contingency table with cell proportions estimated from survey data, Rao and Scott (1984) derived a first-order correction, $\delta.$, to Pearson chi-square statistic, X^2 (or the likelihood ratio statistic, G^2) that takes account of the survey design. It was also shown that $\delta.$ requires the knowledge of only the cell design effects (deffs) and the marginal deffs provided the model admits direct solution to likelihood equations under multinomial sampling. Simple upper bounds on $\delta.$ are obtained here for models not admitting direct solutions, also requiring only cell deffs and marginal deffs or some generalized deffs not depending on any hypothesis. Applicability of an F-statistic used in GLIM to test a nested hypothesis is also investigated. In the case of a logit model involving a binary response variable, simple upper bounds on $\delta.$ are obtained in terms of deffs of response proportions for each factor combination or some generalized deffs not depending on any hypothesis. Applicability of the GLIM F-statistic for nested hypotheses is also studied.

1. Introduction. Rao and Scott (1984) have studied the impact of sample survey design on standard multinomial-based methods for multi-way contingency tables, under general log-linear models. This work and previous investigations have shown that clustering in the survey design can have a substantial impact on the significance levels of the standard chi-square test, X^2, or the likelihood ratio test, G^2. Hence, some adjustment to X^2 or G^2 is necessary, without which one can get misleading results in practice. Rao and Scott (1984) obtained a simple adjusted statistic, $X^2/\delta.$, requiring the knowledge only of cell design effects (deffs) and the deffs of marginals to determine $\delta.$ provided the model admits a direct (explicit) solution to likelihood equations under multinomial sampling (see also Bedrick, 1983). The correction $\delta.$ is particularly useful if the researcher does not have access to micro-data files and hence has to perform secondary analyses from published multi-way tables reporting the deffs. Improved corrections to X^2 or G^2, based on the Satterthwaite approximation, can also be obtained when the full estimate covariance matrix, $\hat{\mathbf{V}}$, of cell estimates is available. Asymptotically valid methods, based on $\hat{\mathbf{V}}$ and using the Wald statistic, have also been proposed by Koch et al. (1975) and others. However, the Wald statistic tends to become unstable and leads to high significance levels compared to the nominal level α as the number of sample clusters

Received March 1985; revised June 1986.

[1]This work was supported by a research grant from the Natural Sciences and Engineering Research Council of Canada.

AMS 1980 *subject classifications*. Primary 62D05; secondary 62H15.

Key words and phrases. Multi-way contingency tables, log-linear models, logit models, design effect, corrections to chi-square tests.

decreases and the number of cells increases (Fay, 1983 and Thomas and Rao, 1984).

The first purpose of this article is to provide some simple adjustments to X^2 or G^2 in those cases not covered by Rao and Scott (1984), viz. the models not admitting direct solutions to multinomial likelihood equations. These corrections require only the cell deffs and the marginal deffs, as in the case of models admitting direct estimates, but lead to conservative tests or nearly conservative tests (as defined in Section 2.1) when compared to the corresponding $\delta.$ requiring the knowledge of $\hat{\mathbf{V}}$. The applicability of an F-statistic used in GLIM (McCullagh and Nelder, 1983) is also investigated.

The second purpose is to obtain similar simple adjustments in the case of logit models involving a binary response variable. Although such a model can be viewed as a special case of a suitable log-linear model, it is important to find corrections to X^2 or G^2 in terms of deffs and marginal deffs of estimated response proportions within factor combinations since the published tables for logit analysis usually report only the deffs of estimated response proportions. Moreover, as shown in Section 3, these adjustments lead to less conservative tests as compared to those based on the cell deffs in the extended table appropriate for log-linear model analysis. It may also be pointed out that logit models corresponding to log-linear models with direct estimates are often not realistic in practice.

2. Log-linear models. A log-linear model M on the population cell proportions π_t is a multi-way table may be written as

$$\boldsymbol{\mu} = \tilde{u}(\boldsymbol{\theta})\mathbf{1} + \mathbf{X}\boldsymbol{\theta}. \tag{2.1}$$

Here $\boldsymbol{\mu}$ is the T-vector of log probabilities $\mu_t = \ln \pi_t$, $t = 1, \dots, T$, $\Sigma\pi_t = 1$, $\mathbf{X}$ is a known $T \times r$ matrix of full rank r ($< T - 1$) and $\mathbf{X}'\mathbf{1} = 0$, $\boldsymbol{\theta}$ is the r-vector of parameters, $\mathbf{1}$ is the T-vector of 1's and $\tilde{u}(\boldsymbol{\theta})$ is the normalized factor to ensure that $\Sigma\pi_t = 1$. Let $\hat{\mathbf{p}}$ denote a consistent estimator (typically, a ratio estimator) of $\boldsymbol{\pi} = (\pi_1, \dots, \pi_T)'$ under the given survey design, and let $\sqrt{n}(\hat{\mathbf{p}} - \boldsymbol{\pi})$ converge in distribution to $N_T(\mathbf{0}, \mathbf{V})$ as the sample size $n \to \infty$, where $\mathbf{V}$ is p.s.d. $T \times T$ matrix. If $\hat{\mathbf{p}}$ is a post-stratified estimator adjusted for known population counts of a supplementary variable (or variables) and if these variables are included in the multi-way table, then the rank of $\mathbf{V}$ is less than $T - 1$ (but assumed to be greater than the rank of $\mathbf{X}$); otherwise the rank of $\mathbf{V}$ is $T - 1$, noting that $\Sigma\hat{p}_t = 1$. In the example of Section 2.2, the estimate $\hat{\mathbf{p}}$ was a post-stratified ratio estimator adjusted for projected census age–sex counts at the provincial level.

We obtain "pseudo m.l.e." $\hat{\boldsymbol{\pi}}$ of $\boldsymbol{\pi}$ by solving the likelihood equations appropriate under multinomial sampling, viz. $\mathbf{X}'\hat{\boldsymbol{\pi}} = \mathbf{X}'\hat{\mathbf{p}}$.

2.1. *Goodness-of-fit.* The Pearson chi-square statistic for testing the goodness-of-fit of model M, say $\pi_t = \pi_t(\boldsymbol{\theta})$, is obtained as

$$X^2 = n\sum(\hat{p}_t - \hat{\pi}_t)^2/\hat{\pi}_t. \tag{2.2}$$

Alternatively, the likelihood ratio statistic is given by

$$G^2 = 2n\sum \hat{p}_t \ln(\hat{p}_t/\hat{\pi}_t). \tag{2.3}$$

Rao and Scott (1984) have shown that X^2 or G^2 is asymptotically distributed as ($\approx$) a weighted sum of $T - r - 1$ independent χ_1^2 variables W_i under the model, where the weights $\delta_1 \geq \cdots \geq \delta_{T-r-1} > 0$ are the eigenvalues of the generalized design effects matrix $\mathbf{V}_{0\phi}^{-1}\mathbf{V}_{\phi}$. Here $\mathbf{V}_{0\phi} = \mathbf{C}'\mathbf{D}_{\pi}^{-1}\mathbf{C}/n$ and $\mathbf{V}_{\phi} = \mathbf{C}'\mathbf{D}_{\pi}^{-1}\mathbf{V}\mathbf{D}_{\pi}^{-1}\mathbf{C}/n$ are the asymptotic covariance matrices of $\hat{\boldsymbol{\phi}} = \mathbf{C}'\hat{\boldsymbol{\mu}}$ under multinomial sampling and under the survey design, respectively, where $\hat{\boldsymbol{\mu}} = (\hat{\mu}_1, \ldots, \hat{\mu}_T)'$, $\hat{\mu}_i = \ln \hat{p}_i$, $\mathbf{D}_{\pi} = \operatorname{diag}(\pi_1, \ldots, \pi_T)$ and $\mathbf{C}$ is any $T \times (T - r - 1)$ matrix of rank $T - r - 1$ with $\mathbf{C}'\mathbf{X} = 0$ and $\mathbf{C}'\mathbf{1} = \mathbf{0}$. In particular, $\mathbf{C}$ may be chosen as the matrix which complements $\mathbf{X}$ to form the model matrix of the saturated model provided it is orthogonal to $\mathbf{X}$ as in the standard parametrization of a log-linear model.

A first-order correction to X^2 or G^2 is given by $X^2/\hat{\delta}.$ (or $G^2/\hat{\delta}.$), where $\hat{\delta}.$ is a consistent estimator of

$$\delta. = \sum \delta_i/(T - r - 1)$$

and

$$(T - r - 1)\delta. = \operatorname{tr}\left(\mathbf{C}'\mathbf{D}_{\pi}^{-1}\mathbf{C}\right)^{-1}\left(\mathbf{C}'\mathbf{D}_{\pi}^{-1}\mathbf{V}\mathbf{D}_{\pi}^{-1}\mathbf{C}\right)$$

(see Rao and Scott, 1984, Section 2.4). This correction, in general, requires the knowledge of estimated covariance matrix, $\hat{\mathbf{V}}/n$, of $\hat{\mathbf{p}}$. However, in the case of a model M leading to direct estimates $\hat{\boldsymbol{\pi}}$ of the form

$$\hat{\pi}_\theta = \prod_i \hat{p}_{\theta_i} \Big/ \prod_j \hat{p}_{\phi_j},$$

Rao and Scott (1984) have proved that

$$(T - r - 1)\delta. = \sum_\theta (1 - \pi_\theta)\, d_\theta - \sum_i \left\{\sum_{\theta_i} (1 - \pi_{\theta_i})\, d_{\theta_i}\right\} + \sum_j \left\{\sum_{\phi_j} (1 - \pi_{\phi_j})\, d_{\phi_j}\right\}. \tag{2.4}$$

Here θ denotes the set of subscripts for an arbitrary cell, θ_i is a subset of subscripts in θ and ϕ_j is the set of subscripts common to θ_i and θ_t for some i and t. Also $d_\theta = \operatorname{var}(\hat{p}_\theta)/[\pi_\theta(1 - \pi_\theta)/n]$, $d_{\theta_i} = \operatorname{var}(\hat{p}_{\theta_i})/[\pi_{\theta_i}(1 - \pi_{\theta_i})/n]$, $d_{\phi_j} = \operatorname{var}(\hat{p}_{\phi_j})/[\pi_{\phi_j}(1 - \pi_{\phi_j})/n]$ are the deffs of cell estimates $\hat{p}_\theta$ and the marginals $\hat{p}_{\theta_i}$ and $\hat{p}_{\phi_j}$, respectively, and $\pi_\theta = \prod \pi_{\theta_i} / \prod \pi_{\phi_j}$ under M, where π_{θ_i} and π_{ϕ_j} are the marginals of π_θ. Thus only the cell deffs and the deffs of marginals are needed to compute $\delta.$.

In the case of a model M not admitting direct estimates, Rao and Scott (1984) made a somewhat ad hoc proposal to use the $\delta.$ corresponding to a nested model M^* "closest" to M and permitting direct estimate of $\boldsymbol{\pi}$, i.e., a model $\pi_t = \pi_t(\boldsymbol{\theta}^*)$ given by $\boldsymbol{\mu} = \tilde{u}\binom{\boldsymbol{\theta}^*}{\mathbf{0}}\mathbf{1} + \mathbf{X}^*\boldsymbol{\theta}^*$, where $\mathbf{X}\boldsymbol{\theta} = \mathbf{X}^*\boldsymbol{\theta}^* + \mathbf{X}^{**}\boldsymbol{\theta}^{**}$, $\mathbf{X}^*$ is $T \times r^*$ of rank r^*, $\mathbf{X}^{**}$ is $T \times (r - r^*)$ of rank $r - r^*$ and $r - r^*$ is as small as possible.

We now obtain an improvement to this suggestion which leads to an approximate upper bound to $\delta.$ under M. Let $G^{*2} = 2n\Sigma\hat{p}_t \ln(\hat{p}_t/\hat{\pi}_t^*)$ denote the likelihood ratio test of the goodness-of-fit of model M^*, where $\hat{\pi}_t^*$ is the pseudo m.l.e. of π_t under M^*. We have $\bar{E}G^2 = (T - r - 1)\delta.$ and $\bar{E}^* G^{*2} = (T - r^* - 1)\delta.^*$, where $\bar{E}$ and $\bar{E}^*$, respectively, denote the asymptotic expectation (i.e., the limit of sequence of expectations as $n \to \infty$) under M and the asymptotic expectation under M^*, and $\delta.^*$ is of the form (2.4) since M^* admits direct estimates.

Since the models M and M^* are close, let $\pi_t(\theta) - \pi_t(\theta^*) = n^{-1/2}a_t$. Now following Rao and Scott (1984, pages 54–55) it is easily seen that

$$\bar{E}G^{*2} = (T - r^* - 1)\delta.^* + \sum_{t=1}^{T} a_t^2/\pi_t(\theta).$$

Hence, noting that $G^{*2} > G^2$ we get

$$\delta. < (T - r - 1)^{-1}(T - r^* - 1)\delta.^* + (T - r - 1)^{-1} \sum_{t=1}^{T} a_t^2/\pi_t(\theta). \tag{2.5}$$

One could estimate $(T - r - 1)^{-1}\Sigma a_t^2/\pi_t(\theta)$ from the sample data but it is likely to be small relative to the first term on the right-hand side of (2.5) when M and M^* are close. Hence, we suggest the "nearly conservative" correction

$$X^2\Big/\left[(T - r - 1)^{-1}(T - r^* - 1)\hat{\delta}.^*\right] \quad \text{or} \qquad \qquad (2.6)$$
$$G^2\Big/\left[(T - r - 1)^{-1}(T - r^* - 1)\hat{\delta}.^*\right],$$

where $\hat{\delta}.^*$ is a consistent estimator of $\delta.^*$. This correction is conservative compared to using $X^2/\hat{\delta}.^*$ or $G^2/\hat{\delta}.^*$, as suggested by Rao and Scott (1984), since $(T - r - 1)^{-1}(T - r^* - 1)\delta.^* > \delta.^*$. Empirical results given in Section 2.1 seem to support the hypothesis that (2.6) leads to a "nearly conservative" test.

An exact upper bound to $\delta.$ under M can also be obtained from some separation inequalities for eigenvalues (Scott and Styan, 1985). We have

$$\delta. \le \sum_{1}^{T-r-1} \lambda_i/(T - r - 1), \tag{2.7}$$

where $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_{T-1} > 0$ are the nonzero eigenvalues of $\mathbf{D}_\pi^{-1}\mathbf{V}$. A practical implication of this result is that one could calculate an exact upper bound on $\hat{\delta}.$, and hence obtain a conservative test compared to $X^2/\hat{\delta}.$ or $G^2/\hat{\delta}.$, if the published tables also report the estimates $\hat{\lambda}_i$ of all nonzero eigenvalues, λ_i, of $\mathbf{D}_\pi^{-1}\mathbf{V}$. This should be feasible since the λ_i do not depend on any hypothesis, unlike the δ_i. If the λ_i are not available, a simple upper bound on $\delta.$ depending only on the cell deffs d_t can be obtained from (2.7) as follows:

$$\begin{aligned}(T - r - 1)\delta. &\le \sum_{1}^{T-r-1} \lambda_i \le \sum_{1}^{T-1} \lambda_i = (T - 1)\lambda. = \operatorname{tr} D_\pi^{-1}V \\ &= \sum_{1}^{T}(1 - \pi_t)\, d_t,\end{aligned} \tag{2.8}$$

where $d_t = \text{Var}(\hat{p}_t)/[\pi_t(1-\pi_t)/n]$ are the cell deffs. However, the upper bound $(T-1)\lambda_. /(T-r-1)$ on $\delta_.$ is not likely to be good unless r is small relative to T.

Nathan (1984) expressed $\hat{\delta}_.$ as a linear function of the elements of estimated covariance matrix, $n^{-1}\hat{\mathbf{V}}$, of cell estimates $\hat{\mathbf{p}}$ and then proposed to minimize $\hat{\delta}_.$ subject to constraints on the elements of $\hat{\mathbf{V}}$. He applied standard linear programming methods (after linearizing a nonlinear inequality constraint) to obtain lower and upper bounds on $\hat{\delta}_.$, say $\hat{\delta}_m$ and $\hat{\delta}_M$, respectively. He proposed to use $\hat{\delta}_M$ to correct X^2, or the average of $\hat{\delta}_m$ and $\hat{\delta}_M$ when $\hat{\delta}_M$ is expected to lead to a very conservative test, e.g., when little information about cell and marginal deffs is available.

2.2. *Example.* Hidiroglou and Rao (1983) have considered a $2 \times 5 \times 4$ table from the Canada Health Survey, 1978 with the following variables: (1) sex (male, female); (2) drug use (0, 1, 2, 3, 4 + drugs in a 2-day period); (3) age group (0–14, 15–44, 45–64, 65 +). They obtained $\hat{\lambda}_. = 1.614$ and the $\hat{\delta}_.$ values given in Table 1 for the following seven hypotheses: (a) complete independence ($\bar{1} \otimes \bar{2} \otimes \bar{3}$); (b) independence of one variable from the remaining variables jointly ($\bar{1} \otimes \overline{23}$; $\bar{2} \otimes \overline{13}$; $\bar{3} \otimes \overline{12}$); (c) conditional independence ($\bar{1} \otimes \bar{2}|\bar{3}$; $\bar{2} \otimes \bar{3}|\bar{1}$; $\bar{1} \otimes \bar{3}|\bar{2}$). For the hypothesis of no three-factor interaction, we do not have direct estimates. The nested model M^*: $\bar{1} \otimes \bar{3}|\bar{2}$ is closest to this hypothesis with $u = 3$. Noting that $T - r - 1 = (2-1)(5-1)(4-1) = 12$ and $T - r^* - 1 = (2-1)(4-1)5 = 15$, we get $\tilde{\delta}_. = [(T - r^* - 1)/(T - r - 1)]\hat{\delta}_.^* = 1.74$ as compared to true $\hat{\delta}_. = 1.66$ which was computed from an actual estimate, $\hat{V}/n$ of the full covariance matrix of $\hat{p}$. Hence, the correction (2.6) to X^2 is excellent in this example. On the other hand, the exact upper bound $(T-1)\hat{\lambda}_. /(T-r-1) = 5.4$, thus leading to a very conservative test of no three-factor interaction.

Suppose the published tables reported only the cell deffs and the deffs of the one-way marginals in the three-way table. Then, one could compute $\delta_.$ from (2.4) only for the hypothesis of complete independence ($\bar{1} \otimes \bar{2} \otimes \bar{3}$) since the remaining hypotheses admitting direct estimates also require the deffs of two-way marginals. We can, however, compute the bound $\tilde{\delta}_.$ for the latter hypotheses by treating $\bar{1} \otimes \bar{2} \otimes \bar{3}$ as M^*. The resulting $\tilde{\delta}_.$-values are also given in Table 1. The bound $\tilde{\delta}_.$ is satisfactory for the hypotheses $\bar{2} \otimes \overline{13}$, $\bar{3} \otimes \overline{12}$ and $\bar{2} \otimes \bar{3}|\bar{1}$ with small

TABLE 1

$\hat{\delta}_.$-values for seven hypotheses admitting direct estimates: Canada Health Survey data

	Hypothesis						
	$\bar{1} \otimes \bar{2} \otimes \bar{3}$	$\bar{1} \otimes \overline{23}$	$\bar{2} \otimes \overline{13}$	$\bar{3} \otimes \overline{12}$	$\bar{1} \otimes \bar{2}\|\bar{3}$	$\bar{1} \otimes \bar{3}\|\bar{2}$	$\bar{2} \otimes \bar{3}\|\bar{1}$
$\hat{\delta}_.$:	2.09	1.40	2.25	2.09	1.63	1.39	2.31
$\tilde{\delta}_.$:	—	3.41	2.31	2.40	4.05	4.32	2.70
d.f.:	31	19	28	27	16	15	24

u, but it leads to overly conservative test for the remaining three hypotheses (and also for the no three-factor interaction hypothesis).

2.3. *Nested hypotheses.* To test a nested hypothesis $H_{2.1}$: $\boldsymbol{\theta}_2 = \mathbf{0}$ given the model M: $\boldsymbol{\mu} = \tilde{u}(\boldsymbol{\theta})\mathbf{1} + \mathbf{X}_1\boldsymbol{\theta}_1 + \mathbf{X}_2\boldsymbol{\theta}_2$, the Pearson chi-square statistic is given by

$$X^2(2|1) = n\sum\left(\hat{\pi}_t - \hat{\hat{\pi}}_t\right)^2/\hat{\hat{\pi}}_t, \tag{2.9}$$

where $\hat{\hat{\pi}}_t$ are the pseudo m.l.e. under $H_{2.1}$ obtained from $\mathbf{X}_1'\hat{\hat{\boldsymbol{\pi}}} = \mathbf{X}_1'\hat{\mathbf{p}}$. Alternatively, the likelihood ratio statistic

$$\begin{aligned} G^2(2|1) &= 2n\sum\hat{p}_t\ln\left(\hat{\pi}_t/\hat{\hat{\pi}}_t\right) = G^2(2) - G^2(1) \\ &= 2n\sum\hat{\pi}_t\ln\left(\hat{\pi}_t/\hat{\hat{\pi}}_t\right) \end{aligned} \tag{2.10}$$

can be used to test $H_{2.1}$, where $G^2(1) = 2n\Sigma\hat{p}_t\ln(\hat{p}_t/\hat{\pi}_t)$ and $G^2(2) = 2n\Sigma\hat{p}_t\ln(\hat{p}_t/\hat{\hat{\pi}}_t)$. Rao and Scott (1984) have shown that

$$G^2(1) \sim n(\hat{\mathbf{p}} - \boldsymbol{\pi})'\mathbf{A}'\mathbf{D}_\pi^{-1}\mathbf{A}(\hat{\mathbf{p}} - \boldsymbol{\pi}) \tag{2.11}$$

and

$$G^2(2|1) \sim n(\hat{\mathbf{p}} - \boldsymbol{\pi})'\mathbf{X}\mathbf{E}_2'\left(\tilde{\mathbf{X}}_2'\mathbf{P}\tilde{\mathbf{X}}_2\right)\mathbf{E}_2\mathbf{X}'(\hat{\mathbf{p}} - \boldsymbol{\pi}) \tag{2.12}$$

under $H_{2.1}$: $\boldsymbol{\theta}_2 = \mathbf{0}$. Here $\sim$ denotes asymptotic equivalence,

$$\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2), \qquad \mathbf{A} = \mathbf{I} - \mathbf{P}\mathbf{X}(\mathbf{X}'\mathbf{P}\mathbf{X})^{-1}\mathbf{X}',$$

$$(\mathbf{X}'\mathbf{P}\mathbf{X})^{-1}\mathbf{X}'(\hat{\mathbf{p}} - \boldsymbol{\pi}) = \begin{pmatrix}\mathbf{E}_1 \\ \mathbf{E}_2\end{pmatrix}\mathbf{X}'(\hat{\mathbf{p}} - \boldsymbol{\pi}), \quad \text{say},$$

where $\mathbf{E}_2$ is $u \times T$, and

$$\tilde{\mathbf{X}}_2 = \left(\mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{P}\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{P}\right)\mathbf{X}_2.$$

Since $\sqrt{n}(\hat{\mathbf{p}} - \boldsymbol{\pi})$ is asymptotically $N_T(\mathbf{0}, \mathbf{V})$, it follows from (2.11) and (2.12) that both $G^2(1)$ and $G^2(2|1)$ are asymptotically distributed as weighted sums of independent χ_1^2 variables W_i and W_i^* under $H_{2.1}$, say $\Sigma_1^{T-r-1}\eta_i W_i$ and $\Sigma_1^u\gamma_i W_i^*$, respectively, $\eta_i > 0$ and $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_u > 0$. A first-order correction to $G^2(2|1)$ is given by $G^2(2|1)/\hat{\gamma}.$, where $\hat{\gamma}.$ is the estimate of $\gamma. = \Sigma\gamma_i/u$, i.e., we treat $G^2(2|1)/\hat{\gamma}.$ or $X^2(2|1)/\hat{\gamma}.$ as χ_u^2 under $H_{2.1}$.

In the case *both* M and M_1: $\boldsymbol{\mu} = \tilde{u}\binom{\boldsymbol{\theta}_1}{\mathbf{0}} + \mathbf{X}_1\boldsymbol{\theta}_1$ admit direct estimates, the adjustment factor $\gamma.$ can be computed knowing only the cell deffs and the deffs of marginals. This follows from the result

$$u\gamma. = \overline{E}G^2(2|1) = \overline{E}G^2(2) - \overline{E}G^2(1) \tag{2.13}$$

and noting that both $\overline{E}G^2(2)$ and $\overline{E}G^2(1)$ have the form (2.4). For instance, in a three-way table, we can get a simple adjustment $\gamma.$ for testing the hypothesis $\overline{1} \otimes \overline{23}$ given $\overline{1} \otimes \overline{2}|\overline{3}$. On the other hand, this method does not work if M does not admit direct estimates, as in the case of testing for $\overline{1} \otimes \overline{2}|\overline{3}$ given the hypothesis of no three-factor interaction in a three-way table.

If M corresponds to a closed-form model and M_1 does not admit direct estimates, then a test similar to (2.6) can be obtained by bounding M_1 above by a similar closed-form model with fewer parameters and using the exact expression of the form (2.4) for $\bar{E}G^2(1)$. If M does not admit direct estimates and M_1 corresponds to a closed-form model, then a similar test can be obtained if M could be bounded below by a similar closed-form model with more parameters.

If both M and M_1 do not admit direct estimates, then a similar test can also be obtained if M could be bounded below by a similar closed-form model with more parameters and M_1 bounded above by a similar closed-form model with fewer parameters.

An exact upper bound on $\gamma.$, similar to (2.8), can also be obtained. Rao and Scott (1984) have shown that $\gamma_1 \geq \cdots \geq \gamma_u > 0$ are the eigenvalues of $(\tilde{\mathbf{X}}_2'\mathbf{P}\tilde{\mathbf{X}}_2)^{-1}(\tilde{\mathbf{X}}_2'\mathbf{V}\tilde{\mathbf{X}}_2)$. Using the matrix result of Scott and Styan (1985) again, an upper bound on $\gamma.$ is given by

$$\gamma. \leq \sum_1^u \lambda_i / u. \tag{2.14}$$

The practical implication of the result (2.14) is that one could calculate an upper bound on $\hat{\gamma}.$ if the published tables also report the estimates of all λ_i, the eigenvalues of $\mathbf{D}_\pi^{-1}\mathbf{V}$, or at least of the few largest ones since u is likely to be small. If the λ_i are not available, a simple upper bound depending only on the cell deffs, d_t, can be obtained from (2.14) as follows:

$$u\gamma. \leq \sum_1^u \lambda_i \leq (T-1)\lambda. = \sum (1-\pi_t)\, d_t. \tag{2.15}$$

However, the bound $(T-1)\lambda./u$ on $\gamma.$ is not satisfactory since u is usually small relative to T.

2.4. *GLIM method.* We now study the effect of survey design on the method used in GLIM (see McCullagh and Nelder, 1983). The statistic

$$F = \frac{G^2(2|1)/u}{G^2(1)/(T-r-1)} \tag{2.16}$$

is used to test $H_{2.1}$, by treating it as a F-variable with d.f. u and $T-r-1$, respectively. Using the first-order adjustment to $G^2(1)$ and $G^2(2|1)$, we get

$$F \doteq \frac{\gamma.\chi_u^2/u}{\eta.\chi_{T-r-1}^2/(T-r-1)}, \tag{2.17}$$

where $\eta. = \Sigma\eta_i/(T-r-1)$ and $\gamma. = \Sigma\gamma_i/u$. Hence, F reduces to a F-variable provided $\gamma. \doteq \eta.$, and χ_u^2 and χ_{T-r-1}^2 are stochastically independent. Noting that $\sqrt{n}(\hat{\mathbf{p}} - \boldsymbol{\pi})$ is asymptotically $N_T(\mathbf{0}, \mathbf{V})$ and applying the well-known Craig theorem for independence of quadratic forms in normal variables to (2.11) and (2.12), we find that $G^2(1)$ and $G^2(2|1)$ are asymptotically independent if and only if

$$(\mathbf{A}'\mathbf{D}_\pi^{-1}\mathbf{A})\mathbf{V}\big(\mathbf{X}\mathbf{E}_2'(\tilde{\mathbf{X}}_2'\mathbf{P}\tilde{\mathbf{X}}_2)\mathbf{E}_2\mathbf{X}'\big) = \mathbf{0}. \tag{2.18}$$

The condition (2.18) holds if $V = \lambda P$ for some constant λ (> 0) since

$$\mathbf{AVX} = \lambda\left[\mathbf{I} - \mathbf{PX}(\mathbf{X}'\mathbf{PX})^{-1}\mathbf{X}'\right]\mathbf{PX} = \mathbf{0}.$$

In this case, $\gamma. = \eta. = \lambda.$ and F is asymptotically distributed as F-variable with u and $T - r - 1$ d.f., under $H_{2.1}$. It is important to note that F does not require any adjustment factor, unlike $G^2(2|1)$. Altham (1976), Brier (1980), Cohen (1976) and Rao and Scott (1981) proposed random effects models for two-stage cluster sampling leading to $\mathbf{V} = \lambda\mathbf{P}$. The condition $\mathbf{V} = \lambda\mathbf{P}$, however, is somewhat restrictive in practice since it implies a constant deff for all cell estimates $\hat{p}_i$ and all linear combinations of $\hat{p}_i$. Even if this condition is not satisfied, the statistic F might work well if $\eta.$ is close to $\gamma.$ and $T - r - 1$ is large, since the denominator in (2.16) then converges to $\eta.$ and the effect of stochastic dependence would be minimal. GLIM, however, does not provide a statistic for testing the goodness-of-fit of model M, considered in Section 2.1.

3. Logit models. Suppose that the first variable among $k + 1$ variables can be considered as a binary response variable and the remaining as factors affecting the response. If the log-linear model contains the saturated submodel for all the k factors, then it is well known that it is equivalent to a logit model. That is, the log-linear model (2.1) can be written as

$$\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_0 \end{pmatrix} = u(\boldsymbol{\theta})\begin{pmatrix} \mathbf{1} \\ \mathbf{1} \end{pmatrix} + \begin{pmatrix} \mathbf{Z}_0 \\ \mathbf{Z}_0 \end{pmatrix}\boldsymbol{\theta}_0 + \begin{pmatrix} \mathbf{Z} \\ -\mathbf{Z} \end{pmatrix}\bar{\boldsymbol{\theta}}, \tag{3.1}$$

where $\boldsymbol{\mu}_i = (\mu_{i1}, \dots, \mu_{iR})'$ is the vector of $\ln \pi_t$ having level i of the response variable, $i = 0, 1$, $R = T/2$, $\mathbf{Z}_0$ is a known matrix (of rank $R - 1$) corresponding to the saturated submodel with associated parameters $\boldsymbol{\theta}_0$, and $\boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_0 \\ \bar{\boldsymbol{\theta}} \end{pmatrix}$. The equivalent logit model is given by

$$\boldsymbol{\ell} = \mathbf{Z}\boldsymbol{\beta}. \tag{3.2}$$

Here $\boldsymbol{\beta} = 2\bar{\boldsymbol{\theta}}$ is the m-vector of parameters, $\boldsymbol{\ell}$ is the R-vector of logits $l_j = \mu_{1j} - \mu_{2j}$, $\mu_{ij} = \ln \pi_{ij}$, $j = 1, \dots, R$, and $\mathbf{Z}$ is the $R \times m$ model matrix of rank m.

3.1. *Goodness-of-fit.* It is a common practice to report the survey estimates $\hat{P}_j = \hat{p}_{1j}/\hat{p}_{+j}$ of response proportions $P_j = \pi_{1j}/\pi_{+j}$ for each factor combination, j, along with their standard errors or estimates of deffs

$$D_j = \operatorname{Var}(\hat{P}_j)\left[P_j(1 - P_j)/(n\pi_{+j})\right]^{-1},$$

where $\hat{p}_{+j} = \hat{p}_{1j} + \hat{p}_{2j}$ and $\pi_{+j} = \pi_{1j} + \pi_{2j}$. It is important, therefore, to express the first-order correction $\delta.$ or the simple upper bounds to $\delta.$ in terms of the deffs D_j or the eigenvalues of the design effects matrix $\mathbf{V}_{0P}^{-1}\mathbf{V}_P$, where $\mathbf{V}_{0P} = \operatorname{diag}(P_1(1 - P_1)/\pi_{+1}, \dots, P_R(1 - P_R)/\pi_{+R})$ and $\mathbf{V}_P/n$ is the asymptotic covariance matrix of $\hat{\mathbf{P}} = (\hat{P}_1, \dots, \hat{P}_R)'$ under the survey design.

In the context of logit models, the only log-linear models admitting direct estimates correspond to those logit models saturated with respect to $k - 1$ or less factors with the remaining factors excluded (Haberman, 1974 and Roberts, 1985). Such models are usually not very realistic in practice since they do not even cover the often-used models with all factors (two or more) acting additively on the response. Hence, the expression (2.4) for $\delta_.$, corresponding to log-linear models yielding direct estimates, is not very useful. Moreover, the only nested logit model yielding direct estimates and "closest" to a logit model with two or more factors acting additively is the model with only one factor. Hence, the correction (2.6), based on $\delta_.^*$, will be overly conservative in the context of logit models.

The exact upper bound (2.8) in terms of the cell deffs d_t (assuming that d_t are available) also leads to overly conservative tests since the corresponding log-linear model must contain the saturated submodel containing all the k factors, thus leading to a large value of r. However, as shown below, we can find good upper bounds on $\delta_.$ in terms of the deffs D_t of the response proportions, $\hat{P}_t$.

As stated in Section 2.1, the δ_i are eigenvalues of $\mathbf{V}_{0\phi}^{-1}\mathbf{V}_{\phi}$, where $\mathbf{V}_{0\phi}/n$ and $\mathbf{V}_{\phi}/n$ are the asymptotic covariance matrices of $\hat{\boldsymbol{\phi}} = \mathbf{C}'\hat{\boldsymbol{\mu}}$ under multinomial sampling and under the survey design, respectively, and $\mathbf{C}'\mathbf{X} = 0$, $\mathbf{C}'\mathbf{1} = 0$. Noting that

$$\mathbf{X} = \begin{pmatrix} \mathbf{Z}_0 & \mathbf{Z} \\ \mathbf{Z}_0 & -\mathbf{Z} \end{pmatrix}$$

is the log-linear model (2.1), we can choose $\mathbf{C}' = \tilde{\mathbf{W}}'(\mathbf{V}_{0l}^{-1}, -\mathbf{V}_{0l}^{-1})$, where $\tilde{\mathbf{W}} = \mathbf{W} - \mathbf{Z}(\mathbf{Z}'\mathbf{V}_{0l}^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{V}_{0l}^{-1}\mathbf{W}$, $\begin{pmatrix} \mathbf{W} \\ -\mathbf{W} \end{pmatrix}$ is the matrix which complements $\mathbf{X}$ to form the model matrix of the saturated log-linear model, and $\mathbf{V}_{0l}/n$ is the asymptotic covariance matrix of $\hat{\ell} = (\hat{l}_1, \ldots, \hat{l}_R)'$ under multinomial sampling. Here $\hat{l}_j = \ln[\hat{P}_j/(1 - \hat{P}_j)] = \ln(\hat{p}_{1j}/\hat{p}_{2j})$ and

$$\mathbf{V}_{0l} = (\mathbf{I} \quad -\mathbf{I})\mathbf{D}_{\pi}^{-1}\begin{pmatrix} \mathbf{I} \\ -\mathbf{I} \end{pmatrix} = (\mathbf{D}_w\mathbf{V}_{0P}\mathbf{D}_w)^{-1},$$

where $\mathbf{D}_w = \text{diag}(w_1, \ldots, w_R)$ and $w_j = \pi_{+j}$. Since $\hat{\boldsymbol{\phi}}$ reduces to $\hat{\boldsymbol{\phi}} = \tilde{\mathbf{W}}'\mathbf{V}_{0l}^{-1}\hat{\ell}$ under the above choice for $\mathbf{C}$, we get

$$\mathbf{V}_{0\phi} = \tilde{\mathbf{W}}'\mathbf{V}_{0l}^{-1}\tilde{\mathbf{W}} = \tilde{\mathbf{W}}'\mathbf{D}_w\mathbf{V}_{0P}\mathbf{D}_w\tilde{\mathbf{W}}.$$

Also, $\mathbf{V}_l$ can be expressed in terms of $\mathbf{V}_P$ as

$$\mathbf{V}_l = \mathbf{D}_w^{-1}\mathbf{V}_{0P}^{-1}\mathbf{V}_P\mathbf{V}_{0P}^{-1}\mathbf{D}_w^{-1} = \mathbf{V}_{0l}\mathbf{D}_w\mathbf{V}_P\mathbf{D}_w\mathbf{V}_{0l}$$

so that

$$\mathbf{V}_{\phi} = \tilde{\mathbf{W}}'\mathbf{V}_{0l}^{-1}\mathbf{V}_l\mathbf{V}_{0l}^{-1}\tilde{\mathbf{W}} = \tilde{\mathbf{W}}'\mathbf{D}_w\mathbf{V}_P\mathbf{D}_w\tilde{\mathbf{W}}.$$

Hence, the δ_i, $i = 1, \ldots, T - r - 1$, are the eigenvalues of $(\tilde{\mathbf{W}}'\mathbf{D}_w\mathbf{V}_{0P}\mathbf{D}_w\tilde{\mathbf{W}})^{-1} \cdot (\tilde{\mathbf{W}}'\mathbf{D}_w\mathbf{V}_P\mathbf{D}_w\tilde{\mathbf{W}}) = (\tilde{\mathbf{W}}'\mathbf{A}_0\tilde{\mathbf{W}})^{-1}(\tilde{\mathbf{W}}'\mathbf{A}\tilde{\mathbf{W}})$, say. We also note that $T - r - 1 = R - m$.

An upper bound on the first-order correction, $\delta_.$, to X^2 or G^2 in terms of the largest $R - m$ eigenvalues $\alpha_1 \geq \cdots \geq \alpha_{R-m}$ of $\mathbf{V}_{0P}^{-1}\mathbf{V}_P = \mathbf{A}_0^{-1}\mathbf{A}$ can be obtained from the matrix result of Scott and Styan (1985) or more simply as follows. We can write

$$(R-m)\delta_{\cdot} = \operatorname{tr}(\tilde{\mathbf{W}}'\mathbf{A}_0\tilde{\mathbf{W}})^{-1}(\tilde{\mathbf{W}}'\mathbf{A}\tilde{\mathbf{W}}) = \operatorname{tr}(\mathbf{Y}\mathbf{A}_0\mathbf{Y}')^{-1}(\mathbf{Y}\mathbf{A}\mathbf{Y}')$$
$$= \sum_{i=1}^{R-m} \frac{\mathbf{y}_i'\mathbf{A}\mathbf{y}_i}{\mathbf{y}_i'\mathbf{A}_0\mathbf{y}_i},$$

where

$$\mathbf{Y} = \mathbf{B}\tilde{\mathbf{W}}' = (\mathbf{y}_1,\dots,\mathbf{y}_{R-m})', \qquad \mathbf{y}_i'\mathbf{A}_0\mathbf{y}_j = 0, \qquad i \neq j,$$

and $\mathbf{B}$ is a $R-m \times R-m$ nonsingular matrix such that

$$\mathbf{Y}\mathbf{A}_0\mathbf{Y}' = \mathbf{B}\tilde{\mathbf{W}}'\mathbf{A}_0\tilde{\mathbf{W}}\mathbf{B}' = \mathbf{D},$$

where $\mathbf{D}$ is a diagonal matrix. Such a matrix $\mathbf{B}$ exists since $\tilde{\mathbf{W}}'\mathbf{A}_0\tilde{\mathbf{W}}$ is symmetric. Now, using a well-known matrix result (see Rao, 1973, page 59), we get

$$(R-m)\delta_{\cdot} \le \sup_{\substack{\mathbf{y}_1,\dots,\mathbf{y}_{R-m}\\ \mathbf{y}'_i\mathbf{A}_0\mathbf{y}_j=0\\ i\neq j}} \sum_{i=1}^{R-m} \frac{\mathbf{y}_i'\mathbf{A}\mathbf{y}_i}{\mathbf{y}_i'\mathbf{A}_0\mathbf{y}_i} = \sum_{i=1}^{R-m}\alpha_i. \tag{3.3}$$

It should be feasible to report all the eigenvalues α_i since they do not depend on any hypothesis, unlike the δ_i. However, if the α_i are not available, we can obtain an upper bound to $\delta_{\cdot}$ in terms of the deffs, D_j, of the response proportions $\hat{P}_j$ as follows:

$$(R-m)\delta_{\cdot} \le \sum_{i=1}^{R-m}\alpha_i \le \sum_{i=1}^{R}\alpha_i = R\alpha_{\cdot} = \operatorname{tr}\mathbf{V}_{0P}^{-1}\mathbf{V}_P = \sum_{1}^{R} D_j = RD_{\cdot} \tag{3.4}$$

The bound $[R/(R-m)]D_{\cdot}$ on $\delta_{\cdot}$ is likely to be much better than the bound $[(T-1)/(T-r-1)]\lambda_{\cdot}$, based on the cell deffs d_t, since r is large and m is small in the context of logit models.

The statistics X^2 and G^2 may be expressed in terms of the estimated response proportions, $\hat{P}_j$ and $P_j(\hat{\boldsymbol{\beta}}) = \hat{\pi}_{1j}/\hat{\pi}_{+j}$, as

$$X^2 = n\sum_{j=1}^{R} \frac{\hat{p}_{+j}\big(\hat{P}_j - P_j(\hat{\boldsymbol{\beta}})\big)^2}{P_j(\hat{\boldsymbol{\beta}})\big(1-P_j(\hat{\boldsymbol{\beta}})\big)} \tag{3.5}$$

and

$$G^2 = 2n\sum_{j=1}^{R}\hat{p}_{+j}\left\{\hat{P}_j\ln\frac{\hat{P}_j}{P_j(\hat{\boldsymbol{\beta}})} + \big(1-\hat{P}_j\big)\ln\frac{\big(1-\hat{P}_j\big)}{\big(1-P_j(\hat{\boldsymbol{\beta}})\big)}\right\}, \tag{3.6}$$

respectively. Note that $\hat{\pi}_{+j} = \hat{p}_{+j}$.

3.2. *Nested hypotheses.* We wish to test the nested hypothesis $\bar{\boldsymbol{\theta}}_2 = \mathbf{0}$ given the log-linear model

$$\boldsymbol{\mu} = \begin{pmatrix}\boldsymbol{\mu}_1\\ \boldsymbol{\mu}_0\end{pmatrix} = \mu(\boldsymbol{\theta})\begin{pmatrix}\mathbf{1}\\ \mathbf{1}\end{pmatrix} + \begin{pmatrix}\mathbf{Z}_0\\ \mathbf{Z}_0\end{pmatrix}\boldsymbol{\theta}_0 + \begin{pmatrix}\mathbf{Z}_1 & \mathbf{Z}_2\\ -\mathbf{Z}_1 & -\mathbf{Z}_2\end{pmatrix}\begin{pmatrix}\bar{\boldsymbol{\theta}}_1\\ \bar{\boldsymbol{\theta}}_2\end{pmatrix}, \tag{3.7}$$

where

$$\mathbf{Z} = (\mathbf{Z}_1, \mathbf{Z}_2) \quad \text{and} \quad \bar{\theta} = \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \end{pmatrix}$$

in (3.1), or equivalently $H_{2.1}$: $\beta_2 = 0$ given the model

$$\ell = \mathbf{Z}_1\beta_1 + \mathbf{Z}_2\beta_2,$$

where $\beta_1 = 2\bar{\theta}_1$ and $\beta_2 = 2\bar{\theta}_2$. Note that in the notation of Section 2.3 we have the following correspondence:

$$(3.8)\quad \mathbf{X}_1 = \begin{pmatrix} \mathbf{Z}_0 & \mathbf{Z}_1 \\ \mathbf{Z}_0 & -\mathbf{Z}_1 \end{pmatrix}, \quad \mathbf{X}_2 = \begin{pmatrix} \mathbf{Z}_2 \\ -\mathbf{Z}_2 \end{pmatrix}, \quad \theta_1 = \begin{pmatrix} \theta_0 \\ \bar{\theta}_1 \end{pmatrix}, \quad \theta_2 = \bar{\theta}_2,$$

where $\mathbf{Z}_2$ is $R \times u$ matrix.

The statistics $X^2(2|1)$ and $G^2(2|1)$ may be expressed in terms of $P_j(\hat{\beta})$ and $P_j(\hat{\hat{\beta}}) = \hat{\hat{\pi}}_{1j}/\hat{\hat{\pi}}_{+j}$ as

$$(3.9)\qquad X^2(2|1) = n \sum_{j=1}^{R} \frac{\hat{p}_{+j}\left(P_j(\hat{\beta}) - P_j(\hat{\hat{\beta}})\right)^2}{P_j(\hat{\hat{\beta}})\left(1 - P_j(\hat{\hat{\beta}})\right)}$$

and

$$(3.10)\quad G^2(2|1) = 2n \sum_{j=1}^{R} \hat{p}_{+j}\left\{P_j(\hat{\beta})\ln\frac{P_j(\hat{\beta})}{P_j(\hat{\hat{\beta}})} + \left(1 - P_j(\hat{\beta})\right)\ln\frac{\left(1 - P_j(\hat{\beta})\right)}{\left(1 - P_j(\hat{\hat{\beta}})\right)}\right\},$$

respectively. Roberts (1985) has shown that an upper bound on $\gamma.$, the first-order correction factor, is given by $\sum_{i=1}^{u}\alpha_i/u$. If the α_i are not available, an upper bound on $\gamma.$ is given by $[R/u]D.$. This bound, however, is not good since u is likely to be small relative to R. As in the case of log-linear models, the GLIM F-statistic (2.16) may be used when the α_i are not available. In the context of logit models, F may be written as

$$(3.11)\qquad F = \frac{G^2(2|1)/u}{G^2(1)/(R-m)}.$$

Under $H_{2.1}$, the statistic F is treated as a F-variable with d.f. u and $R - m$, respectively.

3.3. *Example.* Roberts, Rao and Kumar (1985) fitted a logit model with two factors, age and education, to explain the variation in unemployment rates among males estimated from the October 1980 Canadian Labour Force Survey. Age-group levels were formed by dividing the interval $[15, 64]$ into ten groups and then using the midpoint of each interval, A_j, as the value of age for all persons in that age group. Similarly, six levels of education, E_k, were formed by assigning to each person a value based on median years of schooling. Thus, the age by education cross-classification provided a two-way table of $R = 60$ cell proportions, P_{jk}, of employed males. Roberts, Rao and Kumar found that the model

$$l_{jk} = \ln \frac{P_{jk}}{1 - P_{jk}} = \beta_0 + \beta_1 A_j + \beta_2 A_j^2 + \beta_3 E_k + \beta_4 E_k^2, \tag{3.12}$$

$$j = 1, \dots, 10;\ k = 1, \dots, 6,$$

provides an adequate fit to the estimated cell proportions, $\hat{P}_{jk}$. The estimate of $\delta.$ is equal to 1.88, while the upper bound on $\delta.$ is estimated as

$$\frac{R}{R-m}\hat{D}. = \frac{60}{55}(1.905) = 2.07$$

noting that $m = 5$. Hence, the upper bound, depending only on the estimated cell deffs $\hat{D}_t$, provides an excellent approximation to $\hat{\delta}.$ The value $G^2/\hat{\delta}. = 101.2/1.88 = 53.7$ is not significant at 5% level when compared to $\chi^2_{55}(0.05) = 73.3$, the upper 5% point of χ^2 with $R - m = 55$ d.f.

Given the model (3.12), the nested hypothesis $H_{2.1}$: $\beta_4 = 0$ was also tested. The first-order correction factor, $\gamma.$, is estimated as $\hat{\gamma}. = 1.67$ and

$$G^2(2|1)/\hat{\gamma}. = 0.77/1.67 = 0.46,$$

which is not significant at the 5% level when compared to $\chi^2_1(0.05) = 3.84$. The upper bound on $\gamma.$, $(R/u)\hat{D}. = 60(1.905) = 114.3$, is very conservative here since $R = 60$ is much larger than $u = 1$. On the other hand, the F-statistic used in GLIM gives

$$F = \frac{0.77/1}{101.2/55} = 0.41,$$

which is compared to $F_{1,55}(0.05) \doteq 4.03$, the upper 5% point of F random variable with 1 and 55 d.f., respectively. The GLIM procedure has performed extremely well for testing a nested hypothesis in the present example since $R - m = 55$ is large and $\hat{\gamma}. = 1.67$ is close to $\hat{\eta}. = \hat{\delta}. = 1.88$ (0.46 compared to 3.84 versus 0.41 compared to 4.03).

Acknowledgments. The authors wish to thank Gad Nathan for helpful comments. Thanks are also due to a referee for constructive suggetions.

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