points than is the trimmed least-squares estimator and it is inherently insensitive to a preliminary estimator, which is a potentially serious problem with Welsh's estimator. Even when p, the number of parameters being estimated, is large relative to n, TRQ adheres fairly closely to the behavior predicted by its asymptotic theory. Like Welsh's estimator and trimmed least squares, it is scale- and reparameterization-of-design equivariant and therefore offers most of the attractions of the Huber M estimator without the difficulties created by the necessity of joint estimation of a scale parameter. This is also an advantage with respect to the estimators proposed by Bickel (1973).

As Welsh notes, L estimation plays an extremely useful role in the analysis of the one-sample problem; I believe that it could play a similarly constructive role in analyzing linear models. I hope others, like Welsh, will help to build a theory that would justify this belief.

REFERENCES

ANDREWS, D. F., BICKEL, P. J., HAMPEL, F. R., HUBER, P. J., ROGERS, W. H. and TUKEY, J. W. (1972). *Robust Estimates of Location*. Princeton Univ. Press, Princeton, N.J.

BICKEL, P. J. (1973). On some analogues to linear combinations of order statistics in the linear model. *Ann. Statist.* **1** 597–616.

BICKEL, P. J. (1975). One-step Huber estimates in the linear model. *J. Amer. Statist. Assoc.* **70** 428–434.

FOX, P. (1976). The PORT mathematical subroutine library. Bell Laboratories, Murray Hill, N.J.

JUREČKOVÁ, J. and SEN, P. K. (1984). Adaptive scale-equivariant m-estimators in linear models. *Statist. Decisions, Suppl.* **1** 31–46.

KOENKER, R. W. and BASSETT, G. W. (1978). Regression quantiles. *Econometrica* **46** 33–50.

KOENKER, R. W. and D'OREY, V. (1985). Computing regression quantiles. To appear in *Appl. Statist.*

MOSTELLER, F. (1946). On some useful "inefficient" statistics. *Ann. Math. Statist.* **17** 377–408.

PORTNOY, S. (1986). Personal communication.

RUPPERT, D. and CARROLL, R. J. (1980). Trimmed least squares estimation in the linear model. *J. Amer. Statist. Assoc.* **75** 828–838.

WU, C. F. J. (1986). Jackknife, bootstrap and other resampling methods in regression analysis. *Ann. Statist.* **14** 1261–1295.

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REJOINDER

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The discussants have provided valuable insights into the nature of the one-step trimmed mean in the regression problem and made original proposals of their own. Their empirical results are both helpful and encouraging.

The choice of initial estimator for one-step estimators is important as both discussants note. In addition to the technical requirement that the initial estimator be $n^{1/2}$ consistent, it is desirable for it to have the right invariance properties, to be robust and to be relatively simple to calculate, preferably without requiring the simultaneous estimation of scale to achieve scale invariance. The L_1 estimator or "regression median" is philosophically appealing, meets the above criteria and, from the empirical results provided by the discussants, seems to perform well. The increase in the complexity of the calculation of the L_1 estimator over that of the least squares estimator is clearly more than compensated for by its superior quality in a wide range of circumstances. Koenker and D'Orey (1985) give references to and a discussion of some recent algorithms.

Koenker's investigation of the relationship between the one-step trimmed mean and Huber's M-estimator is interesting—his comment that the trimmed mean essentially trims the intercept and "Winsorises" the slope is particularly nice. While the adoption of a symmetric model here yields useful insights, the results of Ruppert and Carroll (1980) for the naively trimmed mean indicate the extent of the simplication this affords; the symmetric model should not be pursued too far. A major difference between L- and M-estimation is that L-estimators do not require concomitant scale estimates for their calculation. For the trimmed mean, this reflects the fact that specifying a proportion of observations to trim is different from specifying a scale on which to trim. This difference is likely to be important both in inference problems and in the application of analogues of more general L-estimators such as those with redescending influence curves.

The approach of the present paper generalizes to the construction of general L-estimators for regression. *Koenker's* interesting new proposal TRQ opens the possibility of parallel extensions based on the regression quantiles. The algorithm in Koenker and D'Orey (1985) will make the regression quantiles more easily attainable and thus facilitate their use in constructing estimators and as an exploratory/diagnostic tool. Even in very large samples, the computation of selected regression quantiles could prove useful.

de Jongh and de Wet have proposed a bounded influence version of the trimmed mean and suggested the use of a modified L_1 estimator as the initial estimator. Their construction exploits the close link between the trimmed mean and its influence curve. The application they provide is most encouraging and suggests that further study could be valuable.

I would like to thank the discussants for their detailed and stimulating contributions. I am also grateful to an Associate Editor and the Editor for organizing the discussion.

REFERENCE

KOENKER, R. W. and D'OREY, V. (1985). Computing regression quantiles. To appear in *Appl. Statist.*

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