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CORRECTION

A LARGE SAMPLE STUDY OF GENERALIZED MAXIMUM LIKELIHOOD ESTIMATORS FROM INCOMPLETE DATA VIA SELF-CONSISTENCY

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Richard Gill and others have pointed out an error in Theorem 4.1 of our paper. The problem is that the condition in Lemma 4.1 as stated is not sufficient to guarantee the almost sure consistency of the self-consistent estimator. A new Theorem 4.1 follows.

THEOREM 4.1. *Suppose the condition of Proposition* 3.1 *is satisfied. If* F_x *is the unique solution of* $H(F_y, G_x) = 0$, *then* $H(F_y^n, G_x) = 0$ *has a solution* $\hat{F}_x^n$ *with probability* 1 *for* n *large which converges uniformly to* F_x *almost surely.*

PROOF. For n large, $\|F_y^n - F_y\|_\infty < \varepsilon$. Therefore, from Proposition 3.1(b), there exists a continuously differentiable mapping T such that $F_x = T(F_y)$ and $H(F_y^n, T(F_y^n)) = 0$. That is, $T(F_y^n)$ is a solution of $H(F_y^n, G_x) = 0$. Convergence of $T(F_y^n) = \hat{F}_x^n$ follows easily from the argument in Definition 3.1.

A more serious problem has also been pointed to us by Richard Gill. For most estimating equations of interest, including those in our examples of the product limit estimator and doubly censored data, the mapping H is not continuously differentiable, and thus the conditions of Proposition 3.1 (the implicit function theorem) are not satisfied. There are several possible solutions to this problem:

1. In special cases an expansion (in terms of T') of the self-consistent estimator can be found explicitly and the remainder term shown to be asymptotically negligible, as for the product limit estimator.
2. A weaker definition of differentiability can be used for which more mappings are continuously differentiable. This approach also fails, however, for our two examples.

We believe that the general approach outlined in our paper is still useful as a framework in which to study the large sample properties of nonparametric maximum likelihood estimators, but in order for the approach to be truly practical new developments in the theory of statistical differentials are required.

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