ISOMORPHIC $L_2(7)$ DESIGNS

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Raghavarao (*Ann. Statist.* 1973) has given an L_2 design with $b = v = 49$, $r = k = 9$, $\lambda_1 = 3$, $\lambda_2 = 1$. It is shown that this design is isomorphic with a design given by Archbold and Johnson (*Ann. Math. Statist.* 1956).

Raghavarao (1973) has produced a partially balanced design with the L_2 scheme having parameters $b = v = 49$, $r = k = 9$, $\lambda_1 = 3$, $\lambda_2 = 1$.

Archbold and Johnson (1956) also obtained a design with these parameters. Raghavarao points out that it is not known whether or not these two designs are isomorphic. In this note we show that they are indeed isomorphic.

The incidence matrix of the Archbold–Johnson design, which we shall henceforth call the AJ design, is presented in diagram 3 of their paper with blocks for rows and varieties for columns. For convenience we shall number the blocks 1 through 49; the authors number them differently by associating each block with a 'prime' in a geometry. The first seven rows of the incidence matrix consist of seven square submatrices $\mathbf{A}, \mathbf{0}, \mathbf{B}, \mathbf{C}, \mathbf{0}, \mathbf{0}, \mathbf{0}$, where $\mathbf{0}$ has every element zero and

$$
\mathbf{A} = \begin{matrix}
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1
\end{matrix}
\qquad
\mathbf{B} = \begin{matrix}
0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 1 & 0
\end{matrix}
\qquad
\mathbf{C} = \begin{matrix}
0 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 & 0
\end{matrix}.
$$

The first block consists of varieties 01 02 04 18 19 21 24 25 27. To obtain the next set of seven blocks (8 through 14) these submatrices are moved cyclically one place to the right, giving $\mathbf{0}, \mathbf{A}, \mathbf{0}, \mathbf{B}, \mathbf{C}, \mathbf{0}, \mathbf{0}$, and so on. We denote these sets of seven blocks each by I, II, $\cdots$, VII.

In Raghavarao's design the varieties are represented, following Bose (1939) by i_j where $0 \leq i \leq 6$ and $1 \leq j \leq 7$. He obtains seven sets of seven blocks by developing cyclically seven initial blocks. We shall show in the next section that these sets are isomorphic with I, II, $\cdots$, VII.

The isomorphism. In Table 1 we show the mapping of the varieties from Raghavarao's scheme into the AJ design. Raghavarao's 0_2 maps into 18 in the AJ design, and so on.

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TABLE 1

Raghavarao							Archbold–Johnson						
0_1	1_1	2_1	3_1	4_1	5_1	6_1	01	02	03	04	05	06	07
0_2	1_2	2_2	3_2	4_2	5_2	6_2	18	21	20	19	16	15	17
0_3	1_3	2_3	3_3	4_3	5_3	6_3	41	40	42	36	39	38	37
0_4	1_4	2_4	3_4	4_4	5_4	6_4	24	25	23	27	22	28	26
0_5	1_5	2_5	3_5	4_5	5_5	6_5	33	31	29	30	35	32	34
0_6	1_6	2_6	3_6	4_6	5_6	6_6	44	48	46	49	45	47	43
0_7	1_7	2_7	3_7	4_7	5_7	6_7	14	08	12	10	13	09	11

The jth of Raghavarao's seven initial blocks is

$$(0_j, 1_j, 3_j, 0_x, 1_x, 3_x, 0_y, 1_y, 3_y)$$

with $1 \leq j \leq 7$, $x = j + 1$ and $y = j + 3$ (x, y are reduced mod seven).

The set of blocks generated by the second initial block is mapped into the blocks of set III of the AJ design, taken in the order 18, 20, 19, 16, 15, 21, 17. The complete correspondence of the blocks is shown in Table 2.

TABLE 2

Initial block	Set in AJ	Order of blocks in AJ
1	I	1, 2, 3, 4, 5, 6, 7
2	III	18, 20, 19, 16, 15, 21, 17
3	IV	24, 22, 27, 26, 28, 25, 23
4	II	14, 12, 9, 10, 13, 8, 11
5	V	30, 35, 29, 34, 32, 31, 33
6	VII	48, 45, 46, 49, 44, 47, 43
7	VI	40, 39, 42, 36, 38, 37, 41

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