The Annals of Statistics
1994, Vol. 22, No. 3, 1633–1634

CORRECTION

DISTRIBUTION FUNCTIONS OF MEANS OF A DIRICHLET PROCESS

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The Annals of Statistics (1990) **18** 429–442

There is an error in Theorem 1 of our paper. The problem is that the condition $A(\tau) \in [0, 1)$ does not imply the expression of $\mathcal{M}$ stated in part (ii) of Theorem 1, page 436. In fact, such an expression holds under the further hypothesis that A has no jump with size greater than or equal to 1. On the other hand the expression stated in part (i) of the same theorem is true even if $A(\tau) \in [0, 1)$, provided that $\alpha^* > 1$. The one and only case previously not considered [i.e., $\alpha^* > 1$ and $S(\alpha) = -\infty$] is covered by part (iii) of the following proposition, which represents a correct complete reformulation of the aforementioned Theorem 1.

THEOREM 1. *Let χ be a random probability measure chosen by a Dirichlet process on $(\mathbb{R}, \mathfrak{B})$ with parameter α, and satisfying*

$$P\left(\int_{\mathbb{R}} |x| \chi(dx) < +\infty\right) = 1.$$

Write $\mathcal{M}$ for the probability distribution function of $Y = \int_{\mathbb{R}} x\chi(dx)$, $S(\alpha)$ for the support of α, $A(\cdot)$ for the corresponding distribution function and α^* for $\alpha(\mathbb{R})$. Then if α is degenerate at ξ, $\mathcal{M}$ is also degenerate at the same point. On the other hand, if α is not degenerate, we obtain the following:

(i) *For* $\inf S(\alpha) = \tau > -\infty$ *and* $\alpha^* > 1$,

$$\mathcal{M}(x) = \begin{cases} 0, & \text{if } x < r, \\ \displaystyle\int_\tau^x \frac{2^{\alpha^*-3}(\alpha^*-1)}{\pi(u-\tau)}\,du \\ \quad \times \displaystyle\int_{-\pi}^{\pi} \left\{\cos\left(\frac{y}{2}\right)\right\}^{\alpha^*-2} \cos\left\{\int_\tau^\infty q(v;u,y)(u-\tau)\sin y\,dv - \frac{\alpha^* y}{2}\right\} \\ \quad \times \exp\left\{-\displaystyle\int_\tau^\infty q(v;u,y)\big[(u-\tau)\cos y + v - \tau\big]dv\right\} dy, & \text{if } x \ge \tau, \end{cases}$$

where

$$q(v;u,y) = \frac{\alpha^* - A(v)}{(u-\tau)^2 + (v-\tau)^2 + 2(v-\tau)(u-\tau)\cos y}.$$

Received July 1993; revised January 1994.

(ii) *For* $\inf S(\alpha) = \tau \geq -\infty$ *and* A *without jumps with size greater than or equal to* 1,

$$\mathcal{M}(x) = \begin{cases} 0, & \text{if } x \leq \tau, \\ \dfrac{1}{\pi}\displaystyle\int_{\tau}^{x} (x-u)^{\alpha^*-1} \sin\{\pi A(u)\} h(u; -\infty)\, du, & \text{if } x > \tau, \end{cases}$$

where h *is any function from* $\mathbb{R}$ *to* $[0, \infty]$ *such that*

$$h(y; -\infty) = \exp\left\{ -\int_{[\tau,\infty)} \ln|v-y|\, dA(v) \right\} \quad a.e.\text{-}\lambda.$$

(iii) *For* $\alpha^* > 1$ *and* $\inf S(\alpha) = -\infty$,

$$\mathcal{M}(x) = 1 - \int_0^1 \left\{ \int_{\mathbb{R}} \underline{M}(\tau, t; u - x + \tau - 0)\, d_u \overline{M}(\tau, t; u) \right\} d_t \mathcal{D}\big(t; A(\tau), \alpha^* - A(\tau)\big),$$

where τ *is a continuity point of* A *such that* $0 < A(\tau) < \min\{1, \alpha^* - 1\}$; $\underline{M}(\tau, t; x)$ *coincides with the value at* $(x - \tau t)$ *of the distribution function assessed in part* (ii) *with* $(-\tau t)$ *in the place of* τ, $A(\tau)$ *in the place of* α^* *and* $\{A(\tau) - A(-\nu/t)\}I_{[-\tau t, \infty)}(\nu)$ *in the place of* $A(\nu)$; $\overline{M}(\tau, t; x)$ *coincides with the value at* $(x + \tau(1-t))$ *of the distribution function stated in part* (i) *with* $\tau(1-t)$ *in the place of* τ, $(\alpha^* - A(\tau))$ *in the place of* α^* *and* $\{A(\nu/(1-t)) - A(\tau)\}I_{[\tau(1-t), \infty)}(\nu)$ *in the place of* $A(\nu)$; $\mathcal{D}$ *is defined on page* 430 *in our paper.*

A detailed proof of this theorem is given in Cifarelli and Regazzini (1993).

In Corollary 1 on page 439, $\mathcal{M}_\psi$ coincides with the revised expression for $\mathcal{M}$ upon replacement of A by the distribution function corresponding to α_ψ.

REFERENCES

CIFARELLI, D. M. and REGAZZINI, E. (1993). Some remarks on the distribution function of means of a Dirichlet process. *Quaderno Istituto per le Applicazioni della Matematica e dell'Informatica del Consiglio Nazionale delle Ricerche* **93** 4.

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