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# CORRECTION

# THE RELATIONSHIP BETWEEN SUFFICIENCY AND INVARIANCE WITH APPLICATIONS IN SEQUENTIAL ANALYSIS

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We are indebted to Professors A. G. Nogales and J. A. Oyola of the Universidad de Extremadura, Badajoz, Spain, for pointing out an error in Lemma 3.3 of the above paper. They have called our attention to Example 1 of Landers and Rogge (1973) from which a counterexample can be constructed to show that in our Lemma 3.3 the implication (ii) $\Rightarrow$ (i) is false in general. Indeed, in the attempt to prove (ii) $\Rightarrow$ (i), sight was lost of the fact that the equivalence $\sim$ depends on $P$. Therefore, the most obvious additional condition under which Lemma 3.3 will be valid is to assume that the family $\mathscr{P}$ is dominated by one of its members, say $P_0$. Then $P_0$ can be taken as the $P$ in the proof of (ii) $\Rightarrow$ (i). (Note that it is not enough that $\mathscr{P}$ be dominated, as is seen from the counterexample contained in Example 1 of Landers and Rogge; there must be a $P_0 \in \mathscr{P}$ that dominates all $P \in \mathscr{P}$). In particular, Lemma 3.3 will be valid if $\mathscr{P}$ is homogeneous, that is, its members are mutually absolutely continuous (for instance, a family of univariate normal distributions).

Theorem 3.2 remains valid as stated since it only uses (i) $\Rightarrow$ (ii) and (ii) $\Leftrightarrow$ (iii) of Lemma 3.3. However, in a few places in our paper the assertion has been made that the conditional independence of $\mathscr{A}_S$ and $\mathscr{A}_I$ given $\mathscr{A}_{SI}$ implies the sufficiency of $\mathscr{A}_{SI}$ for $\mathscr{A}_I$. This uses (ii) $\Rightarrow$ (i) and requires, therefore, an additional condition that makes Lemma 3.3 valid, such as the extra assumption on $\mathscr{P}$ proposed above.

Here are a few misprints that have been spotted: On page 602, line 2 from the bottom, the first $\mathscr{A}_{Sn}$ should be $\mathscr{A}_{In}$; on page 606, line 5 from the bottom, $g$ should be $\bar{g}$.

We take the opportunity to mention that Lemma 3.1 and Theorem 3.1 can be strengthened somewhat, as pointed out by several of our colleagues. In Lemma 3.1, one needs only assume $f$ to be almost invariant, and in Theorem 3.1, $\mathscr{A}_I$ may be replaced by the subfield of almost invariant sets.

## REFERENCE

LANDERS, D. and ROGGE, L. (1973). On sufficiency and invariance. *Ann. Statist.* **1** 543–544.

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