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NOTE

CORRECTION TO

"CONDITIONS FOR CONTINUITY OF RANDOM PROCESSES WITHOUT DISCONTINUITIES OF THE SECOND KIND"

BY OLAV KALLENBERG

University of Göteborg

The proof of statement (i) in Lemma 1 of my paper [*Ann. Probability* **1** (1973) 519–526] is incorrect. However, we may argue as follows:

Suppose that (i) is false, i.e. that

$$\lim_{h\to 0+} h^{-1}P(t \leqq \eta \leqq t+h) = 0\,, \qquad \text{for all} \quad t \in T\,.$$

Writing $F(t) \equiv P(\eta \leqq t)$, it follows that F is continuous with vanishing right-hand derivative F^+. Choose $t', t'' \in T$ with $F(t') < F(t'')$, and put

$$G(t) = F(t) - t\,\frac{F(t'') - F(t')}{t'' - t'}\,, \qquad t \in T\,.$$

Then G is continuous with $G(t') = G(t'')$ and $G^+ < 0$, so it must have a local minimum at some point $t_0 \in (t', t'')$, and we reach the contradiction $G^+(t_0) \geqq 0$. □

We take this opportunity to state the following extension (with the same proof) of my Theorem 1:

THEOREM. *Let ξ be a random process in $D(R)$, and let $0 < a < b \leqq \infty$. Then ξ has* a.s. *no positive jump with size in (a, b) if any one of the following conditions holds for all $t \in R$:*

(i) $\lim_{h\to 0+} h^{-1}P(a < \xi_{t+h} - \xi_{t-} < b) = 0$,
(ii) $\lim_{h\to 0+} h^{-1}P(a < \xi_{t+} - \xi_{t-h} < b) = 0$,
(iii) $\liminf_{h\to 0+} h^{-1}P(a < \xi_{t+h} - \xi_{t-h} < b) = 0$.

This yields in particular new conditions for the simplicity of point processes on R. Conversely, the classical results by Korolyuk and Dobrushin on the atom structure of point processes have obvious counterparts for the jump structure of general random processes. We omit the details, but refer to Section 2 in [1] for a first step.

REFERENCE

[1] KALLENBERG, O. (1975). *Random Measures*. Schriftenreihe des Zentral-instituts für Mathematik und Mechanik der ADW der DDR. Akademie-Verlag, Berlin (to appear).