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CORRECTION

PATH PROPERTIES OF INDEX-β STABLE FIELDS

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There is an error in Theorem 3.1(i) on page 1600. The correct statement is that for a symmetric p-stable process $\{X(t), t \in [0,1]\}$ with $p > 1$ and $\beta > 1/p$, $\|X(t+h) - X(t)\|_p = o(h^\alpha)$ for all $\alpha < \beta$ implies X has a version whose paths satisfy $|X(t+h) - X(t)| \le c(\omega)|h|^\gamma$ for all $\gamma < \beta - 1/p$. We originally stated that this Hölder condition held for all $\gamma < \beta$, and referred to Theorem 1.1 of Pisier (1983). Pisier's result does establish continuity of the process, but does not yield the claimed Hölder condition. Furthermore, a counterexample when $\beta - 1/p < \gamma < \beta$ can be found in the corollary after Theorem 3.4 in Takashima (1989).

The corrected statement has several proofs. One is to apply Theorem 3 of Kono and Maejima (1990). A second is to apply Lemma 1 of Garsia (1972). Follow the proof of Garsia's Theorem 2 using qth moments, $0 < q < p$, instead of second moments and the fact that

$$E|X(t+h) - X(t)|^q = \text{const.}\|X(t+h) - X(t)\|_p^q = o(h^{\alpha q}).$$

A third proof can be constructed using the stochastic integral representation for symmetric stable processes: The process has a version given by $X(t) = \int_0^1 f(t,u)W(du)$, where $\{f(t,\cdot), t \in [0,1]\} \subset L^p([0,1], du)$ and W is the symmetric p-stable Lévy process. Now

$$\|f(t+h,\cdot) - f(t,\cdot)\|_{L^p} = \|X(t+h) - X(t)\|_p = o(h^\alpha),$$

so we can apply Garsia's Lemma 1 to the kernel function $f(\cdot,\cdot)$ to conclude that there is a version $f_0(\cdot,\cdot)$ with $|f_0(t+h,u) - f_0(t,u)| \le B(u)|h|^\gamma$ for all $\gamma < \beta - 1/p$ and $B(u) \in L^p(du)$. The unanticipated result here is that the moment condition on the increments of the process lead to Hölder conditions on the kernel in the stochastic integral representation. Furthermore, any stochastic integral representation for X will have a kernel with the same property. Once we have such regularity conditions on the kernel, we can use Proposition 4 of Nolan (1989) to prove the process is Hölder continuous a.s. of all orders less than $\beta - 1/p$.

We note that all three approaches fail when $p \le 1$ or $\beta \le 1/p$. In general, increment conditions on a p-stable process do not guarantee regularity of the sample paths, so the first two proofs cannot be extended. However, regularity of the kernel $f(\cdot,\cdot)$ does generally lead to regularity of the sample paths, for example, Theorems 1 and 3 of Nolan (1989). The third proof above shows that

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the regularity of the kernel follows from the original increment condition and thus relates the different approaches.

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