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NOTE ABOUT "FIRST ORDER CORRECTION FOR THE HYDRODYNAMIC LIMIT OF SYMMETRIC SIMPLE EXCLUSION PROCESSES WITH SPEED CHANGE IN DIMENSION $d \geq 3$"

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We refer to [1] for notation.

In [1], we proved that the first order correction for the hydrodynamic equation of symmetric simple exclusion processes with speed change in dimension $d \geq 3$ is

$$\partial_t \rho^N = \Delta_u\big(\rho^N(1+\alpha\rho^N)\big) - \frac{\alpha}{N}\sum_{i,j=1}^{d} \partial_{u_i}^2\big(R_{ij}(\rho^N)\partial_{u_j}\rho^N\big),$$

where R_{ij} is a continuous function on (0, 1).

We now prove that $R = 0$, which means that there is no correction of order N^{-1} to the hydrodynamic limit.

Indeed, from Lemma 9.1 of [1], there exists a sequence of functions F_k^i that satisfies

$$\lim_{k\to\infty} E_{\nu_m}\bigg[W_{0,e_l}\sum_x \tau_x F_k^i\bigg] = -R_{i,l}(m)m(1-m),$$

where ν_m is the product Bernoulli measure of parameter m and W_{0,e_l} is the current between 0 and e_l. Notice that ν_m is translation invariant. Therefore, since our process is gradient, we obtain

$$E_{\nu_m}\bigg[W_{0,e_l}\sum_x \tau_x F_k^i\bigg] = E_{\nu_m}\bigg[\bigg(\sum_x \tau_x W_{0,e_l}\bigg)F_k^i\bigg] = 0,$$

and thus $R_{i,l}(m) = 0$.

THEOREM 1. *For symmetric simple exclusion processes with speed change, under diffusive rescaling, the density of particles $q^N(t,u)$ at time t around $u \in \mathbb{R}^d$ satisfies*

$$N\big(q^N(t,\cdot) - m(t,\cdot)\big) \to 0 \qquad \textit{for dimension } d \geq 3,$$

where $m(t,u)$ is the solution of the hydrodynamic limit $\partial_t m = \sum_i \partial_{u_i}^2(m(1+\alpha m))$.

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Of course, knowing that $R = 0$, one can simplify the proof of Theorem 1:

There is no need to introduce the coefficient R. We have to prove (see Section 3 of [1]) that $\lim_{N\to\infty} E_N[N^{1-d}\sum_x J(T, x/N)[\eta_T(x) - m(T, x/N)]] = 0$, where $J_0 : T^d \to \mathbb{R}$ is a smooth function and $J : \mathbb{R}_+ \times T^d \to \mathbb{R}$ is the solution of the linear equation

$$\partial_s J(s,u) + \phi'\bigl(m(s,u)\bigr)\Delta_u J(s,u) = 0 \tag{1}$$

with final condition $J(T,u) = J_0(u)$.

There is no need to introduce corrections of order N^{-2} in the density ψ_t appearing in the specific relative entropy (see Section 4 of [1]). Indeed, one can prove directly that the specific relative entropy $N^{-d}H_N(t)$ is $o(N^{-1})$ by adding terms of the form

$$\int_0^{t_0} dt\, E_N\biggl[N^{1-d}\sum_{x,i} L_N\bigl\{\alpha\partial_{u_i}^2\lambda(t,x/N)F_i(\tau_x\eta_t) + \bigl(\partial_{u_i}\lambda(t,x/N)\bigr)^2 G_i(\tau_x\eta_t)\bigr\}\biggr]$$

with $F_i, G_i \in \mathcal{G}$, in the bound for $N^{1-d}H_N(t)$ obtained in Section 7 of [1].

This does not change anything since by the martingale property we have the following identity:

$$\begin{aligned}
&\int_0^{t_0} dt\, E_N\biggl[N^{-d-1}\sum_{x,i} N^2 L_N\bigl\{\partial_{u_i}^2\lambda(t,x/N)F_i(\tau_x\eta_t)\bigr\}\biggr]\\
&\quad= -\int_0^{t_0} dt\, E_N\biggl[N^{-d-1}\sum_{x,i}\partial_t\bigl\{\partial_{u_i}^2\lambda(t,x/N)F_i(\tau_x\eta_t)\bigr\}\biggr]\\
&\qquad + E_N\biggl[N^{-d-1}\sum_{x,i}\partial_{u_i}^2\lambda(t_0,x/N)F_i(\tau_x\eta_{t_0})\biggr]\\
&\qquad - E_N\biggl[N^{-d-1}\sum_{x,i}\partial_{u_i}^2\lambda(0,x/N)F_i(\tau_x\eta_0)\biggr]
\end{aligned}$$

and, as $N \uparrow \infty$, the right-hand side of the last expression converges to 0.

REFERENCE

[1] JANVRESSE, E. (1998). First order correction for the hydrodynamic limit of symmetric simple exclusion processes with speed change in dimension $d \geq 3$. *Ann. Probab.* **26** 1874–1912.

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