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CORRECTION

MODERATE DEVIATIONS OF DEPENDENT RANDOM VARIABLES RELATED TO THE CLT

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1. The simplicity of the eigenvalue 1 should be interpreted as the algebraic simplicity, that is, the eigenprojection associated with 1 is of rank one.

2. About the proof of Theorem 2.1. To derive the key (2.11) in page 430, I claimed in (2.10) that there is some $\gamma > 0$ and a small neighborhood U of 0 in $\mathbb{C}$ such that for all $z \in U$ and for all $n \geq 1$,

$$\left\| P_{zf}^n - \left[G^1(z)\right]^n J_0(z) \right\| = \left\| \left(P_{zf}[I - J_0(z)]\right)^n \right\| \leq e^{-\gamma n},$$
$$\left|G^1(z)\right| > e^{-\gamma}.$$

The second inequality above should be replaced by $\inf_{z \in U} |G^1(z)| e^{-\gamma}$ [for deriving (2.11)]. The first inequality above lacks a constant factor M at its right-hand side, given by

$$(1) \quad M := \sup_{\xi \in S(e^\gamma),\, z \in U} \|R(\xi, z)\| \quad \text{with} \quad R(\xi, z) := \left(I - \xi P_{zf}[I - J_0(z)]\right)^{-1},$$

where $S(r) = \{\xi \in \mathbb{C}; |\xi| = r\}$. This follows from the Cauchy integral formula below:

$$\left(P_{zf}[I - J_0(z)]\right)^n = \frac{1}{n!} \frac{\partial^n}{\partial \eta^n} R(\eta, z)|_{\eta=0} = \frac{1}{2\pi i} \int_{S(e^\gamma)} \frac{R(\xi, z)}{\xi^{n+1}}\, d\xi,$$

and the boundedness of M follows from the fact that the spectrum of $P_{zf}(I - J_0(z))$ is contained in $\{c \in \mathbb{C}; |c| < e^{-\gamma}\}$ for all $z \in U$ and the fact that the resolvent $(I - \xi P_{zf}(I - J_0(z)))^{-1}$ is bounded in $(\xi, z) \in S(e^\gamma) \times U$ for U sufficiently small by Kato [(1984), Chapter VII, Theorem 1.3, page 367].

3. About the identification (2.13) and (2.17) of the rate function. In Theorem 2.3 (discrete time) and Theorem 2.3′ (continuous time), it is proved that the moderate deviation principle holds with the rate function given by

$$(2) \qquad I(\nu) = \sup\left\{\nu(f) - \tfrac{1}{2}\langle f, Af\rangle_m;\, f \in L_0^2(m)\right\},$$

where $Af = (Q + Q^* - I)f$ or $Af = (Q + Q^*)f$ according to the discrete or continuous time case.

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I claimed in the Note after Theorem 2.3 and in the remarks after Theorem 2.3′ that the self-adjoint definite nonnegative and bounded operator A is invertible or injective on $L_0^2(m)$, which leads to the identification (2.13) and (2.17). But in general A *is not injective* (I found a counterexample recently). Thus we must interpret $\langle f, A^{-1}f\rangle_m$, appearing in (2.13) and (2.17), as

$$\langle f, A^{-1}f\rangle_m = \int_{[0,+\infty)} \frac{1}{\lambda} d\langle E_\lambda f, f\rangle_m \in [0,+\infty], \tag{3}$$

where $A = \int_{[0,+\infty)} \lambda \, dE_\lambda$ is the spectral decomposition of A on $L_0^2(m)$.

See de Acosta (1997) for a careful identification of the rate function in the discrete time case, in terms of P, P^* only.

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REFERENCES

DE ACOSTA, A. (1997). Moderate deviations for empirical measures of Markov chains: Lower bounds. *Ann. Probab.* **25** 259–284.

KATO, T. (1984). *Perturbation Theory of Linear Operators*, 2nd ed. (2nd corrected printing). Springer, Berlin.

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