EDITORIAL.

Trapezoidal Rule for Computing Seasonal Indices.

The following method for computing seasonals is suggested by the Detroit Edison article on *"A Mathematical Theory of Seasonals"* that appeared in Vol. I, No. 1 of the *Annals*.

We shall likewise define "the seasonal index for any month as the ratio of the total of the variates for the month in question to the total that would have been experienced if neither accidental nor seasonal influences were present", that is, the seasonal index for the i-th month is

$$(1) \qquad s_i = \frac{\Sigma_o y_i}{\Sigma \psi_i} \ .$$

The numerator presents no difficulties: the obstacle is met in determining the denominator, since $\psi(x)$ is the unknown function that is the consequence of only trend and cycle influences. According to accepted concepts the trend may be represented by some smooth analytic function, the cycle is a smooth though not a mathematically periodic function—but the seasonal and residual influences may inject all sorts of disturbances into a time series. We shall make but two further assumptions,—

(a) The smooth function $y = \psi(x)$, representing the combined effect of trend and cycle, may be approximated by the upper sides of a series of trapezoids as in figure (1). The area of each trapezoid is to equal the area under the function $\psi(x)$ limited by the common ordinates.

(b) Neither seasonal nor accidental influences affect annual totals. Thus we might assume that the seasonal activity in the production of coal does not affect the total coal mined within the year, but merely concentrates production within certain months and compensates this with a corresponding under-production in others. Although accidental disturbances within the year would merely attribute production to one month rather than the next, we must admit that if one of these months is the last of one year, and the other the first of the following year, such an accidental fluctuation will affect the annual totals. Usually, however, such perturbations represent but a small percent of the monthly production, and a negligible part of the annual total.

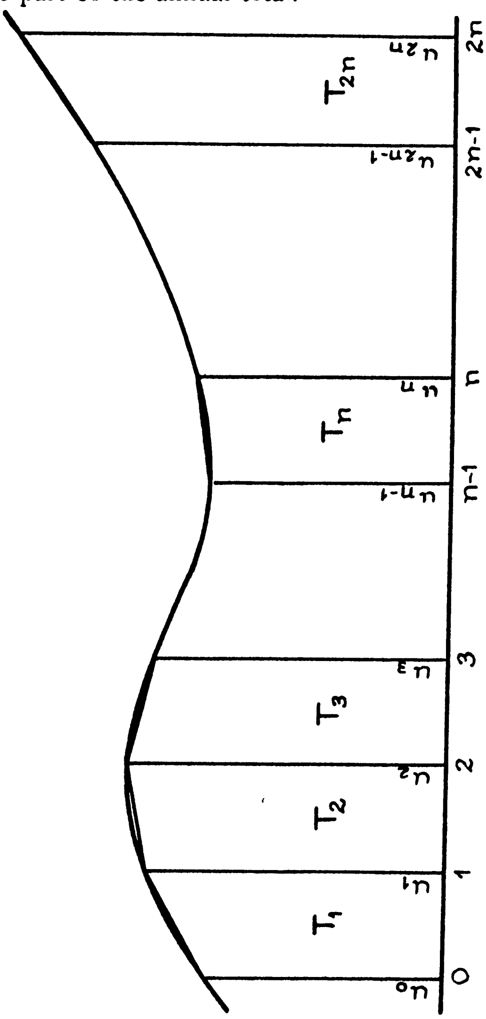

Fig. 1

Let us assume that we are dealing with $2n$ consecutive years: then by the above assumptions the area of the n-th trapezoid equals the total production for the n-th year, T_n .

Designating the ordinates corresponding to the n-th trapezoid by u_{n-1} and u_n , it follows since we are dealing with equal unit bases,

$$T_n = \frac{1}{2}(u_{n-1} + u_n), \qquad \text{that is}$$

$$u_n = 2T_n - u_{n-1}, \qquad \text{so that}$$

$$(2)\begin{cases} u_1 = 2T_1 - u_0 \\ u_2 = 2T_2 - u_1 = 2T_2 - 2T_1 + u_0 \\ u_3 = 2T_3 - u_2 = 2T_3 - 2T_2 + 2T_1 - u_0 \\ u_4 = 2T_4 - u_3 = 2T_4 - 2T_3 + 2T_2 - 2T_1 + u_0 \\ \cdot \\ u_{2n-1} = 2T_{2n-1} - u_{2n-2} = 2T_{2n-1} - 2T_{2n-2} + \cdots + 2T_1 - u_0 \\ u_{2n} = 2T_{2n} - u_{2n-1} = 2T_{2n} - 2T_{2n-1} + \cdots + 2T_2 - 2T_1 + u_0 \end{cases}$$

By elementary geometry, the area corresponding to the twelve months of the n-th year are

$$M_1 = \frac{1}{288}(23u_{n-1} + u_n)$$
$$M_2 = \frac{1}{288}(21u_{n-1} + 3u_n)$$
$$M_3 = \frac{1}{288}(19u_{n-1} + 5u_n)$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$M_{11} = \frac{1}{288}(3u_{n-1} + 21u_n)$$
$$M_{12} = \frac{1}{288}(u_{n-1} + 23u_n)$$

According to definition, $\Sigma \psi_i$ equals the sum of the values of M_i, one for each of the $2n$ years. Thus.

$$\Sigma\psi_1 = \frac{1}{288}(23u_0 + u_1$$
$$+23u_1 + u_2$$
$$+23u_2 + u_3$$
$$\cdots\cdots$$
$$+23u_{2n-2} + u_{2n-1}$$
$$+23u_{2n-1} + u_{2n}),$$ that is

(3) $$\Sigma\psi_1 = \frac{1}{288}\ 24(u_0 + u_1 + \cdots + u_{2n-1}) + (u_{2n} - u_0)$$

and in precisely the same manner

$$\Sigma\psi_2 = \frac{1}{288}\ 24(u_0 + u_1 + \cdots + u_{2n-1}) + 3(u_{2n} - u_0)$$
$$\Sigma\psi_3 = \frac{1}{288}\ 24(u_0 + u_1 + \cdots + u_{2n-1}) + 5(u_{2n} - u_0)$$

But by (2)

$$u_0 + u_1 + u_2 + \cdots + u_{2n-1} = 2T_1 + 2T_3 + 2T_5 + \cdots + 2T_{2n-3} + 2T_{2n-1}$$
$$= 2 \cdot O,$$

where O designates the sum of the totals for the odd years.

Again,

$$u_{2n} - u_0 = 2T_{2n} - 2T_{2n-1} + 2T_{2n-2} - \cdots + 2T_2 - 2T_1$$
$$= 2 \cdot E - 2 \cdot O,$$

E representing the corresponding sum for the even years.

We have finally that

$$(4) \quad \begin{cases} \Sigma \psi_1 = \frac{1}{144}(23 \cdot O + E) \\ \delta = \frac{1}{72}(E - O). \end{cases}$$

where δ represents the common difference $\Sigma \psi_{i+1} - \Sigma \psi_i$, which from equation (3) is seen to be

$$= \frac{1}{288}(2u_{2n} - 2u_0) = \frac{1}{72}(E - O).$$

It should be observed that we have not imposed the condition that the long time trend is a straight line—no matter what the law of growth may be, the assumption that the trend-cycle function, $\psi(x)$, may be approximated within each year by a secant line is alone responsible for the equal differences δ.

As an illustration let us compute the seasonals for the theoretical series presented in the Detroit Edison article, and reproduced below.

TABLE 1

THEORETICAL SERIES.

	1904	1905	1906	1907	1908	1909
Jan.	906	1662	1908	2030	1242	1714
Feb.	814	1582	1860	1855	1052	1831
Mar.	1138	1913	2052	2077	1283	1831
Apr.	1215	1976	2027	2088	1210	2077
May	1343	1892	2122	2043	1203	2143
June	1236	1700	1672	2093	1166	2058
July	1254	2092	2041	2060	1240	2320
Aug.	1702	1757	1846	2163	1334	2413
Sept.	1457	1906	2102	2262	1279	2502
Oct.	1564	1899	2304	1946	1364	2643
Nov.	1596	1611	2303	1475	1341	2378
Dec.	1836	2163	2170	1153	1564	2595
Total	16061	22153	24407	23245	15278	26505

TABLE 1 *Continued*

	1910	1911	1912	1913	1914	1915
Jan.	2392	1933	2052	2554	2041	1687
Feb.	2514	1746	2061	2330	1780	1532
Mar.	2417	1895	2267	2554	2194	1906
Apr.	2830	1865	2490	2800	2037	1796
May	2702	2167	2419	2845	2156	2539
June	2475	1732	2571	2696	1730	2674
July	2211	1742	2657	2314	1577	2566
Aug.	2249	2011	2469	2525	1649	2661
Sept.	2108	1976	2591	2377	1652	2952
Oct.	2203	2144	2516	2850	1753	3342
Nov.	1875	2168	2389	2494	1585	3093
Dec.	1899	2092	2435	2150	1779	3182
Total	27875	23471	28917	30489	21933	29930

Here

$$E = 155\ 793$$
$$O = 134\ 471$$
$$\Sigma\Psi_1 = 22\ 559.90$$
$$\delta = 296.14$$

Column 1 of table 2 is obtained by adding the items of table 1 horizontally. Column 2 is found by repeated adding the common difference, $\delta = 296.14$ to the value 22 559.90.

TABLE 2
SEASONALS BY TRAPEZOIDAL RULE.

Month	$\Sigma_o y_i$	$\Sigma \phi_i$	s
Jan.	22 121	22 560	.981
Feb.	20 957	22 856	.917
Mar.	23 527	23 152	1.016
Apr.	24 411	23 448	1.041
May	25 574	23 744	1.077
June	23 803	24 041	.990
July	24 074	24 337	1.015
Aug.	24 779	24 633	1.006
Sept.	25 164	24 929	1.009
Oct.	26 528	25 225	1.052
Nov.	24 308	25 521	.952
Dec.	25 018	25 817	.969
Total	290 264	290 263	12.025

The following table presents the mean and standard errors for the results obtained by Link-relative method, the Interpolation method suggested in the Detroit Edison article, and the Trapezoidal method. Obviously, the last requires by far the least time in application.

TABLE 3

Method	M.D.	σ
Link Relative	.0277	.0338
Interpolation	.0269	.0337
Trapezoidal	.0227	.0255

For weekly indices the formulae by the trapezoidal rule are

$$\Sigma \psi_1 = \frac{1}{52^2}(103 \cdot O + E)$$

$$\delta = \frac{1}{2 \cdot 52^2}(E - O)$$

If one has data for $2n+1$ years, he may either disregard the most distant year and then compute seasonals for the final $2n$ years, or he may combine the results for the first $2n$ years (neglecting the last year) and the results for the last $2n$ years (neglecting the first year). This yields an almost equally simple formula.

Applying the Trapezoidal rule to successive overlapping periods will reveal the presence of a shifting seasonal. The change in the seasonal for Automobile production, caused by the advent of good roads and closed models, affords a good example.

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