AN EXPERIMENT REGARDING THE χ^2 TEST

By

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1. *Introduction*

R. A. Fisher has proposed that in case the hypothesis being tested has been partially obtained from the data, the Elderton table[1] for χ^2 should be entered with n' equal to, not the number of frequency classes, but the number of frequency classes minus the number of statistics computed from the data. It has been proved under certain restrictions that this theory holds in the limit as the size of the sample approaches infinity.[2]

For samples of moderate size our only guide is experimental evidence, which indicates that Fisher's method is satisfactory in practice.[3] This is true in particular of the evidence presented in the present paper which describes a coin tossing experiment suggested by Professor H. L. Rietz.

2. *The Experiment*

The experimental work here considered was done by seventy students each of whom tossed seven coins 128 times. In any one of the seventy experiments, this results in a frequency distribution of 128 items divided into eight frequency classes. But we lumped together the classes of zero heads and one head and likewise for six heads and seven heads, so that we had six frequency classes. If for every coin on every throw the probability of heads

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[1] Karl Pearson, Tables for statisticians and biometricians, (1914), Table XII.

[2] J. Neyman and E. S. Pearson, Biometrika. V. 20A (1928), pp. 263-294.

[3] Yule, Journal of the Royal Statistical Society, V. 85, pp. 95-104; Brownlee, *ibid.*, V. 87. p. 76; Neynam and Pearson, Loc. Cit.; Sheppard, Phil. Trans., A. V., 228, p. 115.

is one-half, the expected numbers $\tilde{m}_i$ in the six frequency classes are 8, 21, 35, 35, 21, 8. The divergence of the actual distribution n_1, n_2, ..., n_6, from the theoretical one is measured by

$$\chi^2 = \sum_{i=1}^{i=6} \frac{(n_i - \tilde{m}_i)}{\tilde{m}_i} .$$

If we had possessed only one sample of 128 throws, we would have looked in Elderton's table with the value of χ^2 and with $n' = 6$ (the number of frequency classes) to find the probability P_χ that a sample would by chance deviate from the expected distribution so much as the actual sample had deviated. But having seventy samples, we compared the distribution of our seventy values of χ^2 with that expected from $n = 6$. The arithmetic mean of our seventy values of χ^2 was 4.62 whereas the expected value was five; a deviation which could very well occur by chance. So our results are consistent with the hypothesis that the probability of a head is always one-half.

We considered next the following composite hypothesis: the probability p of heads is the same for all coins on all throws. For any sample of 128 throws, we took as the estimate of p the actual proportion of heads among the 7 x 128 possibilities. From this value we calculated the expected frequencies, $m_1, \ldots, m_6$. For each of the seventy samples, we computed

$$\chi_1^2 = \sum_{i=1}^{i=6} \frac{(n_i - m_i)^2}{m_i} .$$

When we used the χ^2 test to compare this distribution of χ_1^2s with that expected for $n' = 6$, we found that $P_\chi = .01$. But when we compared our distribution with that expected for $n' = 6 - 1 = 5$ we found that $P_\chi = .82$. The mean value of our values of χ_1^2 was 3.97 compared with $6-1-1=4$ demanded by Fisher's theory, and with five by the theory that χ_1^2 is distributed as

χ^2. If the latter theory were correct the probability of the mean of seventy values of χ_1^2 being so far away from five, is .007. That our distribution of χ_1^2 corresponds to that expected for $n' = 5$ whereas our distribution of the values of χ^2 corresponds to $n' = 6$, can be seen from the following table.[4]

Values of χ^2 (or χ_1^2).	Observed frequency of χ^2	Expected frequency $n' = 6$	Observed frequency of χ_1^2	Expected frequency $n' = 5$
0—1	3	2.6	7	6.3
1—2	8	8.0	12	12.2
2—3	11	10.4	14	12.5
3—4	14	10.5	12	10.6
4—5	12	9.3	10	8.3
5—7	11	13.7	7	10.6
7—9	5	7.8	3	5.2
greater than 9	6	7.6	5	4.3

[4] In computing P_χ we combined the first two classes of this table and also the last two, thus making $n' = 6$.

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