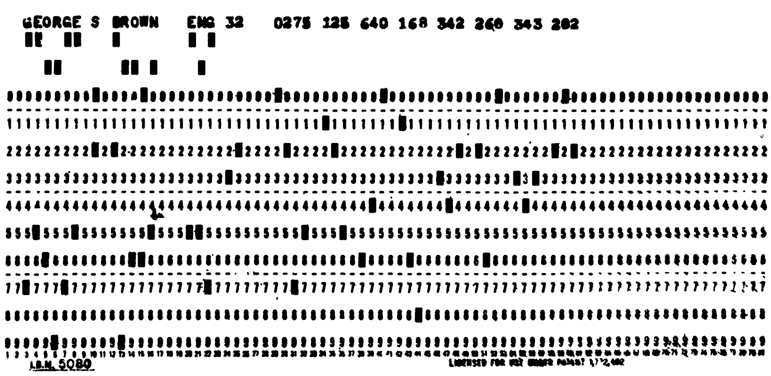
# EDITORIAL: H. C. CARVER

## PUNCHED CARD SYSTEMS AND STATISTICS

Because of the increasingly important part being played by mechanical devices in statistical methodology, it seems desirable to call attention in the *Annals* to some of the possibilities of punched-card systems.

The standard punched-card, illustrated below, is seven and one-half inches by three and one-quarter inches in size. To a certain extent the operation of a punched-card system is analogous to that of the Teletype machines used in wiring messages. In the latter case telegrams are written on a special typewriter which translates the message into a series of electrical impulses that in turn operate a distant typewriter which prints the message on a strip of paper; in the former, cards are automatically fed through a special typewriter that both prints the words or numbers on each card and also translates the information into properly punched holes in the card. The data on these cards may be totaled if desired by running the cards through a tabulating

FIG. 1



machine at a rate exceeding one per second; the total of the squares of the numbers appearing on consecutive cards may be obtained automatically; if the variates $x$ and $y$ be punched in respective columns on each card, the total of the $xy$ products for all the cards is likewise made available; the cards may be arranged in order of magnitude according to card number, date, or variates at a rate exceeding six per second, and finally the data on the cards may be printed on a scroll—the cards passing through the listing or printing machine at about 80 per minute.

In order to provide an actual problem to serve as an illustration, I secured the anthropometric records of one thousand first year male students who entered the University of Michigan in the fall of 1928. The 1,000 cards, of which Figure 1 is a sample, were punched by an operator of average ability in slightly less than three hours. The data for the students were selected at random and the cards, as punched, were numbered consecutively from 1 to 1,000,—the card selected for Figure 1 being the 275th. The weight to the nearest pound of this individual was 125 pounds, and the height, width of shoulders, and the circumferences of chest, waist, hips and right thigh were, respectively, 64.0, 16.8, 34.2, 26.0, 34.3 and 20.2,—linear measurements being made to the nearest one-tenth inch.

These 1,000 cards may now be placed in a *tabulating and listing* machine which can total all seven of the data fields simultaneously. If desired, the machine will also print all of the information of each card on a scroll together with the totals. The first and last parts of this scroll are reproduced below photographically, —the names of the individuals being omitted purposely. Because of both the number of columns involved and the magnitudes of the totals, the listed totals unfortunately run together. The vertical lines, inserted with a pen, facilitate the reading of the totals. The cards are totaled and listed simultaneously at the rate of 80 per minute.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 154 | 681 | 165 | 335 | 285 | 365 | 195 |
| 2 | 121 | 674 | 164 | 342 | 265 | 327 | 190 |
| 3 | 158 | 677 | 177 | 365 | 306 | 382 | 222 |
| 4 | 134 | 698 | 166 | 350 | 266 | 345 | 196 |
| 5 | 138 | 698 | 165 | 355 | 287 | 346 | 195 |
| 6 | 143 | 718 | 162 | 356 | 272 | 360 | 200 |
| 7 | 166 | 683 | 171 | 397 | 325 | 383 | 222 |
| 8 | 135 | 682 | 167 | 365 | 280 | 344 | 196 |
| 9 | 145 | 691 | 176 | 372 | 300 | 361 | 202 |
| 10 | 115 | 645 | 157 | 328 | 248 | 330 | 180 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 991 | 125 | 658 | 158 | 360 | 253 | 342 | 192 |
| 992 | 149 | 684 | 160 | 340 | 284 | 364 | 211 |
| 993 | 138 | 664 | 162 | 352 | 281 | 354 | 190 |
| 994 | 149 | 698 | 168 | 355 | 285 | 370 | 212 |
| 995 | 128 | 630 | 176 | 365 | 280 | 345 | 205 |
| 996 | 133 | 695 | 167 | 327 | 267 | 353 | 191 |
| 997 | 130 | 667 | 168 | 330 | 278 | 345 | 200 |
| 998 | 142 | 718 | 166 | 343 | 275 | 370 | 201 |
| 999 | 141 | 680 | 174 | 370 | 300 | 360 | 212 |
| 1000 | 135 | 686 | 174 | 345 | 280 | 365 | 206 |

139288 | 678896 | 165337 | 353241 | 281510 | 355165 | 201096

Fig. 2

An investigation of the correlation that may exist between height and weight will involve the numerical value of

$$\sum_{i=1}^{1000} x_i y_i$$

where $x_i$ and $y_i$ designate the height and weight, respectively, of the $i^{th}$ individual. The plugboard of an *Automatic Multiplying Punch* may be wired in a few seconds so that

(a) the data of columns 34, 35 and 36 of Figure 1 will feed into the multiplier of the punch,

(b) the data of columns 38, 39 and 40 will feed into the multiplicand, and then

(c) the product, $xy$, for any card run through the machine will appear on the *product summary counter*. As the cards pass through the machine, the total of the products is accumulated on this counter.

If desired, each product may be punched automatically in the card, provided of course the card contains a sufficiently large number of otherwise vacant columns. The maximum number of digits in current models that may occur in either multiplier or multiplicand is eight. The number of digits in the multiplicand does not affect the speed of the multiplication; for three or less digits in the multiplier the cards feed through the machine at the rate of three seconds per card,—for eight digits in the multiplier the speed is twelve cards per minute. One may therefore place our cards in the machine, press a button, resume other duties, and some fifty minutes later the 1,000 cards will have yielded the total

$$\Sigma xy = 9\ 477\ 433.6$$

To obtain the sum of the squares of the variates in question it is necessary only to double-wire the machine,—one wire going to the multiplier and the other to the multiplicand. We obtain then

$$\Sigma x^2 = 4\ 615\ 312.12 \qquad \Sigma y^2 = 19\ 692\ 452$$

By permitting the machine to punch each value of $x$ in the card, we may treat $x^2$ as the multiplicand and $x$ as the multiplier and then obtain the sum of the cubes of the variates; or by double-wiring $x^2$ obtain the sum of the fourth powers of the variates. If, while accumulating the cubes of the variates, we let the machine also punch each cube in the card, we may then obtain the sum of the powers of the variates up to and including that of the sixth order, etc. We are limited, of course, by the fact that the card contains eighty columns.

By running the punched cards through a sorting machine, we may obtain very readily the frequency distribution of the weights, and also the corresponding median, quartiles, etc. To accomplish this the cards must be run through a *sorting machine* three times, first sorting to column 35 of Figure 1, then to column 34 and finally to column 33. The cards pass through the sorting machine at the rate of 400 per minute, so that in approximately eight minutes—including time spent in handling the cards between sorts—these 1,000 cards will be perfectly arranged according to magnitude in weight. If the numbers with respect to which the sort is to be made contain $N$ digits, the cards must be run through the sorter $N$ times. We reproduce on the following page a photograph of the first part of a printed scroll obtained by running the cards through the listing machine after they had been sorted according to weight.

Figure 3.

| Card # | Weight | Height | Shoulder | Chest | Waist | Hip | Rt. Thigh |
|---|---|---|---|---|---|---|---|
| 232 | 89 | 597 | 140 | 295 | 253 | 305 | 171 |
| 146 | 100 | 607 | 157 | 300 | 242 | 312 | 170 |
| 691 | 101 | 690 | 150 | 301 | 245 | 308 | 165 |
| 358 | 102 | 676 | 154 | 327 | 262 | 322 | 174 |
| 555 | 102 | 662 | 153 | 303 | 242 | 307 | 165 |
| 941 | 102 | 640 | 154 | 313 | 262 | 313 | 168 |
| 209 | 103 | 663 | 154 | 310 | 285 | 313 | 172 |
| 801 | 103 | 649 | 150 | 302 | 247 | 317 | 168 |
| 14 | 104 | 638 | 147 | 313 | 253 | 300 | 178 |
| 513 | 105 | 635 | 148 | 310 | 250 | 328 | 187 |
| 720 | 105 | 637 | 155 | 330 | 258 | 319 | 178 |
| 563 | 106 | 648 | 152 | 302 | 245 | 320 | 169 |
| 672 | 106 | 638 | 162 | 323 | 254 | 380 | 181 |
| 153 | 107 | 623 | 143 | 317 | 250 | 322 | 172 |
| 75 | 108 | 630 | 160 | 335 | 245 | 332 | 183 |
| 235 | 108 | 669 | 157 | 320 | 247 | 324 | 172 |
| 322 | 108 | 624 | 165 | 305 | 260 | 325 | 188 |
| 505 | 108 | 637 | 152 | 330 | 246 | 330 | 183 |
| 31 | 109 | 620 | 161 | 334 | 265 | 340 | 192 |
| 393 | 109 | 625 | 152 | 314 | 233 | 336 | 187 |
| 30 | 110 | 631 | 168 | 345 | 265 | 387 | 180 |
| 160 | 110 | 667 | 153 | 320 | 247 | 382 | 175 |
| 185 | 110 | 627 | 152 | 323 | 274 | 320 | 172 |
| 631 | 110 | 630 | 153 | 335 | 255 | 330 | 180 |
| 802 | 110 | 623 | 149 | 317 | 257 | 328 | 181 |
| 15 | 111 | 650 | 158 | 312 | 265 | 344 | 194 |
| 151 | 111 | 637 | 157 | 322 | 258 | 325 | 172 |
| 273 | 111 | 651 | 156 | 332 | 270 | 326 | 180 |
| 447 | 111 | 655 | 160 | 314 | 252 | 324 | 176 |
| 20 | 112 | 696 | 155 | 300 | 230 | 335 | 174 |
| 426 | 112 | 647 | 154 | 333 | 262 | 328 | 170 |
| 507 | 112 | 691 | 161 | 323 | 262 | 322 | 180 |
| 716 | 112 | 667 | 148 | 330 | 254 | 322 | 185 |
| 831 | 112 | 651 | 147 | 373 | 262 | 333 | 182 |
| 308 | 113 | 661 | 155 | 318 | 250 | 330 | 180 |
| 383 | 113 | 665 | 154 | 338 | 240 | 333 | 178 |
| 449 | 113 | 651 | 151 | 328 | 258 | 311 | 163 |
| 541 | 113 | 675 | 152 | 316 | 257 | 340 | 186 |
| 591 | 113 | 650 | 166 | 341 | 265 | 333 | 180 |
| 826 | 113 | 654 | 147 | 331 | 260 | 333 | 183 |
| 898 | 113 | 677 | 156 | 320 | 253 | 335 | 187 |
| 933 | 113 | 656 | 153 | 315 | 240 | 326 | 179 |
| 947 | 113 | 637 | 146 | 315 | 262 | 337 | 188 |
| 967 | 113 | 696 | 151 | 320 | 258 | 330 | 169 |
| 148 | 114 | 654 | 153 | 320 | 247 | 322 | 183 |
| 177 | 114 | 633 | 160 | 342 | 255 | 321 | 172 |
| 257 | 114 | 653 | 162 | 318 | 270 | 320 | 185 |
| 368 | 114 | 679 | 145 | 318 | 243 | 382 | 173 |
| 462 | 114 | 653 | 153 | 322 | 261 | 381 | 171 |
| 464 | 114 | 627 | 150 | 310 | 262 | 332 | 180 |
| 545 | 114 | 612 | 157 | 334 | 260 | 318 | 181 |
| 741 | 114 | 662 | 161 | 338 | 262 | 338 | 186 |
| 951 | 114 | 681 | 153 | 311 | 242 | 333 | 174 |
| 987 | 114 | 667 | 152 | 342 | 250 | 330 | 188 |
| 10 | 115 | 645 | 157 | 328 | 248 | 330 | 180 |
| 139 | 115 | 665 | 146 | 332 | 265 | 339 | 188 |
| 159 | 115 | 636 | 160 | 328 | 297 | 318 | 174 |
| 336 | 115 | 663 | 153 | 321 | 252 | 323 | 167 |
| 743 | 115 | 639 | 158 | 322 | 278 | 338 | 182 |
| 949 | 115 | 686 | 154 | 330 | 242 | 330 | 182 |
| 970 | 115 | 653 | 152 | 313 | 250 | 337 | 172 |
| 988 | 115 | 692 | 155 | 330 | 252 | 352 | 172 |
| 52 | 116 | 640 | 157 | 316 | 260 | 337 | 193 |
| 186 | 116 | 606 | 159 | 330 | 258 | 337 | 186 |
| 226 | 116 | 665 | 166 | 341 | 262 | 335 | 185 |
| 268 | 116 | 636 | 165 | 350 | 265 | 350 | 192 |
| 347 | 116 | 630 | 163 | 321 | 265 | 325 | 195 |
| 510 | 116 | 662 | 162 | 337 | 275 | 330 | 190 |
| 523 | 116 | 695 | 158 | 325 | 243 | 340 | 182 |
| 926 | 116 | 641 | 154 | 338 | 257 | 340 | 182 |

A rough notion of the functional dependence that exists between weight and the other six variables recorded on the cards may be obtained by permitting the machine to total these ordered-with-respect-to-weight cards in consecutive groups of 100. That is, we obtain the averages for numerically equal groups selected according to the weight-deciles. The six regression lines may therefore be plotted, approximately, from the following results:

TABLE 1.
ANTHROPOMETRIC AVERAGES BASED ON WEIGHT DECILES.

| *Inter-decile Range* | *Weight* | *Height* | *Sh'der* | *Chest* | *Waist* | *Hips* | *Rt.Th.* |
|---|---|---|---|---|---|---|---|
| First | 112.98 | 65.133 | 15.576 | 32.607 | 25.748 | 32.968 | 18.133 |
| Second | 122.41 | 66.659 | 15.980 | 33.663 | 26.777 | 33.927 | 18.926 |
| Third | 127.85 | 67.087 | 16.161 | 34.421 | 26.978 | 34.387 | 19.206 |
| Fourth | 131.98 | 67.381 | 16.334 | 34.816 | 27.622 | 34.858 | 19.554 |
| Fifth | 135.62 | 67.937 | 16.406 | 34.860 | 27.954 | 35.081 | 19.893 |
| Sixth | 139.54 | 68.189 | 16.651 | 35.608 | 28.065 | 35.511 | 20.112 |
| Seventh | 143.87 | 68.576 | 16.789 | 35.766 | 28.513 | 36.006 | 20.438 |
| Eighth | 149.43 | 68.895 | 16.807 | 36.116 | 28.780 | 36.420 | 20.712 |
| Ninth | 156.01 | 69.185 | 17.022 | 36.788 | 29.537 | 37.181 | 21.444 |
| Tenth | 173.19 | 69.854 | 17.611 | 38.596 | 31.536 | 38.826 | 22.678 |

If we had arranged the cards numerically with respect to height, instead of weight, we would have obtained the following results:

TABLE 2.
ANTHROPOMETRIC AVERAGES BASED ON HEIGHT DECILES

| *Inter-decile Range* | *Weight* | *Height* | *Sh'der* | *Chest.* | *Waist* | *Hips* | *Rt.Th.* |
|---|---|---|---|---|---|---|---|
| First | 123.95 | 63.339 | 16.036 | 34.201 | 27.371 | 34.217 | 19.592 |
| Second | 130.97 | 65.295 | 16.261 | 34.856 | 27.984 | 34.944 | 19.929 |
| Third | 133.75 | 66.367 | 16.282 | 34.787 | 27.731 | 35.152 | 19.926 |
| Fourth | 136.28 | 67.021 | 16.570 | 35.429 | 28.329 | 35.302 | 20.084 |
| Fifth | 139.81 | 67.623 | 16.587 | 35.379 | 28.206 | 35.499 | 20.223 |
| Sixth | 140.60 | 68.189 | 16.532 | 35.510 | 28.184 | 35.560 | 20.008 |
| Seventh | 142.65 | 68.806 | 16.659 | 35.550 | 28.380 | 35.821 | 20.315 |
| Eighth | 143.44 | 69.498 | 16.638 | 35.328 | 28.065 | 35.858 | 20.205 |
| Ninth | 145.71 | 70.412 | 16.776 | 35.778 | 28.236 | 35.960 | 20.068 |
| Tenth | 155.72 | 72.346 | 16.996 | 36.423 | 29.024 | 36.852 | 20.746 |

A comparison of tables 1 and 2 reveals clearly that the right thigh measurements are more highly correlated with weight than with height and this phenomena appears also to be true—though possibly less pronounced—for the shoulder, chest, waist and hip measurements.

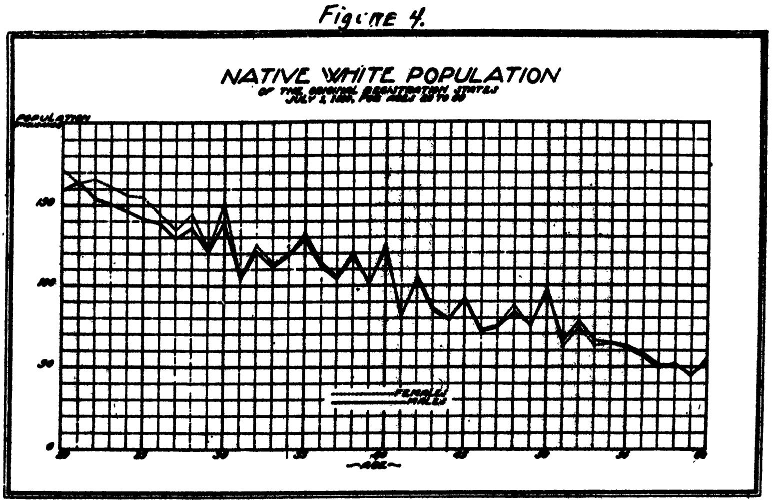
By sorting the cards according to the last recorded digit of each weight, the tendency of observers to state results as multiples of two and five is apparent. The results presented in table 3 indicate that final digits of 1, 3, 7 and 9 are decidedly unpopular and this casts a reflection upon the accuracy of the recorded measurements. This type of bias is well known—in fact, census and mortality staatistics usually present the same phenomena. Figure 4, which follows, clearly illustrates this fact.

TABLE 3.

RELATIVE FREQUENCY OF FINAL DIGITS IN OBSERVED VARIATES.

| *Final Digit* | *Frequency* |
|---|---|
| 0 | 142 |
| 1 | 69 |
| 2 | 117 |
| 3 | 84 |
| 4 | 108 |
| 5 | 112 |
| 6 | 112 |
| 7 | 69 |
| 8 | 117 |
| 9 | 70 |
| Total | 1000 |

Figure 4.



## SUMMARY

The punching, sorting and listing machines provide a most economical and accurate method of recording and analyzing, non-mathematically, observational data. The punched-card system is especially effective in constructing frequency distributions and correlation tables when the data are very numerous.

The recent development of the Automatic Multiplying Punch is unquestionably the most important contribution to the mechanics of mathematical statistics since the invention of adding and multiplying machines. By enabling one to compute moments and product-moments exactly and without the grouping of variates about class-marks, corrections such as those due to Sheppard are unnecessary for practical computations. Indeed, it is even possible to evaluate linear functions of one or more variables with this machine, and subsequently print the graduated values on a scroll —these results being "rounded off" to any desired number of decimal places.

In this article I have described only a few of the various machines employed in statistical and accounting practice. Readers may secure additional information from the International Business Machines Corporation, Tabulating Machine Division, 270 Broadway, New York City.

B. A. C. 6-17-34.