

groups, an efficient estimate will be $m' = \frac{M \left[\bar{u} - \frac{s_{uv}}{s_v^2} \left(\bar{v} - \frac{N}{M} \right) \right]}{N}$. The efficiency

of m' relative to the conventional estimate $\frac{M\bar{u}}{N}$ is $(1 - \rho_{uv}^2)^{-1}$, which ordinarily would seem to be quite large. This is easily extended to the case II is divided into l strata with M_i groups comprising N_i individuals in the i^{th} stratum, when a random sample of m_i out of the M_i groups in each stratum is taken. Let v_{ij} be the number of individuals in the j^{th} group of the i^{th} stratum and u_{ij} denote the sum of the values of x for these v_{ij} individuals. The estimate of m_x becomes

$$m'' = \frac{\sum_{i=1}^l M_i \left[\bar{u}_i - \frac{s_{u_i v_i}}{s_{v_i}^2} \left(\bar{v}_i - \frac{N_i}{M_i} \right) \right]}{N}$$

If $\sum_{i=1}^l m_i = m$ is fixed, the large sample variance of m'' will be a minimum if m_i is proportional to $M_i \sigma_{u_i} \sqrt{1 - \rho_i^2}$, where ρ_i is the correlation between u and v in the i^{th} stratum.

In conclusion, the writer wishes to thank Professor A. Wald for his advice and encouragement, and Mr. Henry Goldberg for several suggestions.

REFERENCES

- [1] J. NEYMAN, "On the two different aspects of the representative method," *Journal of the Royal Statistical Society*, Vol. 97 (1934), pp. 558-606.
- [2] R. C. GEARY, "The frequency distribution of the quotient of two normal variates," *Roy. Stat. Soc. Jour.*, Vol. 93 (1930), pp. 442-446.
- [3] W. A. SHEWHART, *Economic Control of Quality of Manufactured Product*, New York, (1931), pp. 182-183.
- [4] H. C. CARVER, "Fundamentals in the theory of sampling," *Annals of Math. Stat.*, Vol. 1 (1930), pp. 110-112.
- [5] W. G. COCHRAN, "The use of the analysis of variance in enumeration by sampling," *Jour. Amer. Stat. Assoc.*, Vol. 34 (1939), pp. 492-510.
- [6] J. L. DOOB, "The limiting distribution of certain statistics," *Annals of Math. Stat.*, Vol. 6 (1935), p. 166.

SIGNIFICANCE LEVELS FOR THE RATIO OF THE MEAN SQUARE SUCCESSIVE DIFFERENCE TO THE VARIANCE

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For purposes of practical application in connection with significance tests a tabulation of the argument corresponding to certain percentage points of the probability integral is usually more convenient than that of the probability integral for equal intervals of the argument. A table of probabilities for the

Values of $\frac{\delta^2}{g^2}$ for Different Levels of Significance

#	Values of k'					Values of k'					Values of k'				
	P = .001	P = .01	P = .05	P = .95	P = .99	P = .001	P = .01	P = .05	P = .95	P = .99	P = .001	P = .01	P = .05	P = .95	P = .99
4	.7864	.8341	1.0406	4.2927	4.4992	4.5469	1.0115	1.2469	1.4746	2.6587	2.8864	3.1219	2.6587	2.8864	3.1219
5	.5201	.6724	1.0255	3.9745	4.3276	4.4799	1.0245	1.4817	1.4817	2.6473	2.8720	3.1046	2.6473	2.8720	3.1046
6	.4361	.6738	1.0682	3.7318	4.1262	4.3639	1.0369	1.2667	1.4855	2.6365	2.8583	3.0882	2.6365	2.8583	3.0882
7	.4311	.7163	1.0919	3.5748	3.9504	4.2356	1.0488	1.2761	1.4951	2.6262	2.8451	3.0725	2.6262	2.8451	3.0725
8	.4612	.7575	1.1228	3.4486	3.8139	4.1102	1.0603	1.2852	1.5014	2.6163	2.8324	3.0574	2.6163	2.8324	3.0574
9	.4973	.7974	1.1524	3.3476	3.7025	4.0027	1.0714	1.2940	1.5075	2.6068	2.8202	3.0429	2.6068	2.8202	3.0429
10	.5351	.8363	1.1803	3.2642	3.6091	3.9093	1.0822	1.3025	1.5135	2.5977	2.8085	3.0289	2.5977	2.8085	3.0289
11	.5717	.8706	1.2062	3.1938	3.5294	3.8283	1.0927	1.3108	1.5193	2.5889	2.7973	3.0154	2.5889	2.7973	3.0154
12	.6062	.9033	1.2301	3.1335	3.4603	3.7574	1.1029	1.3188	1.5249	2.5804	2.7865	3.0024	2.5804	2.7865	3.0024
13	.6390	.9336	1.2521	3.0812	3.3996	3.6944	1.1128	1.3266	1.5304	2.5722	2.7760	2.9898	2.5722	2.7760	2.9898
14	.6702	.9618	1.2725	3.0352	3.3458	3.6375	1.1224	1.3342	1.5357	2.5643	2.7658	2.9776	2.5643	2.7658	2.9776
15	.6999	.9880	1.2914	2.9943	3.2977	3.5858	1.1317	1.3415	1.5408	2.5567	2.7560	2.9658	2.5567	2.7560	2.9658
16	.7281	1.0124	1.3090	2.9577	3.2543	3.5386	1.1407	1.3486	1.5458	2.5494	2.7466	2.9545	2.5494	2.7466	2.9545
17	.7548	1.0352	1.3253	2.9247	3.2148	3.4952	1.1494	1.3554	1.5506	2.5424	2.7376	2.9436	2.5424	2.7376	2.9436
18	.7801	1.0566	1.3405	2.8948	3.1787	3.4552	1.1577	1.3620	1.5552	2.5357	2.7289	2.9332	2.5357	2.7289	2.9332
19	.8040	1.0766	1.3547	2.8675	3.1456	3.4182	1.1657	1.3684	1.5596	2.5293	2.7205	2.9232	2.5293	2.7205	2.9232
20	.8265	1.0954	1.3680	2.8425	3.1151	3.3840	1.1734	1.3745	1.5638	2.5232	2.7125	2.9136	2.5232	2.7125	2.9136
21	.8477	1.1131	1.3805	2.8195	3.0869	3.3523	1.1807	1.3802	1.5678	2.5173	2.7044	2.9044	2.5173	2.7044	2.9044
22	.8677	1.1298	1.3923	2.7982	3.0607	3.3228	1.1877	1.3856	1.5716	2.5117	2.6977	2.8956	2.5117	2.6977	2.8956
23	.8866	1.1456	1.4035	2.7784	3.0362	3.2953	1.1944	1.3907	1.5752	2.5064	2.6908	2.8872	2.5064	2.6908	2.8872
24	.9045	1.1606	1.4141	2.7599	3.0133	3.2695	1.2010	1.3957	1.5787	2.5013	2.6842	2.8790	2.5013	2.6842	2.8790
25	.9215	1.1748	1.4241	2.7426	2.9919	3.2452	1.2075	1.4007	1.5822	2.4963	2.6777	2.8709	2.4963	2.6777	2.8709
26	.9378	1.1883	1.4336	2.7264	2.9718	3.2222	1.2139	1.4057	1.5856	2.4914	2.6712	2.8630	2.4914	2.6712	2.8630
27	.9535	1.2012	1.4426	2.7112	2.9528	3.2003	1.2202	1.4107	1.5890	2.4866	2.6648	2.8553	2.4866	2.6648	2.8553
28	.9687	1.2135	1.4512	2.6969	2.9348	3.1794	1.2264	1.4156	1.5923	2.4819	2.6585	2.8477	2.4819	2.6585	2.8477
29	.9835	1.2252	1.4594	2.6834	2.9177	3.1594	1.2324	1.4203	1.5955	2.4773	2.6524	2.8403	2.4773	2.6524	2.8403
30	.9978	1.2363	1.4672	2.6707	2.9016	3.1402	1.2383	1.4249	1.5987	2.4728	2.6465	2.8331	2.4728	2.6465	2.8331
							1.2442	1.4294	1.6019	2.4684	2.6407	2.8260	2.4684	2.6407	2.8260
							1.2500	1.4339	1.6051	2.4640	2.6350	2.8190	2.4640	2.6350	2.8190
							1.2558	1.4384	1.6082	2.4596	2.6294	2.8120	2.4596	2.6294	2.8120

ratio of the mean square successive difference δ^2 to the variance s^2 , $P\left(\frac{\delta^2}{s^2} < k\right) = \int_0^k \omega(\delta^2/s^2) d(\delta^2/s^2)$, where $\omega(\delta^2/s^2)$ is the distribution of δ^2/s^2 ,¹ has been published recently² with k as argument. The following table of values of δ^2/s^2 for $P = .001, .01$ and $.05$ has been computed from it by interpolation.

Since the distribution of δ^2/s^2 , $\omega(\delta^2/s^2)$, is symmetric³ about $E(\delta^2/s^2)$, $P(\delta^2/s^2 < k) = P(\delta^2/s^2 > k')$ if $E(\delta^2/s^2) - k = k' - E(\delta^2/s^2)$, where $E(\delta^2/s^2) = 2n/(n - 1)$.³ The upper levels are rarely of practical use, since large values of the ratio, δ^2/s^2 , could arise only from a somewhat artificial set of observations, such as alternately high and low values of the observed variable.

The computation of this table of significance levels was made at the suggestion of Lt. Col. L. E. Simon.

¹ For determination of $\omega(\delta^2/s^2)$ cf. JOHN VON NEUMANN, "Distribution of the ratio of the mean square successive difference to the variance," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 367-395.

² B. I. HART, "Tabulation of the probabilities for the ratio of the mean square successive difference to the variance," *Annals of Math. Stat.*, Vol. 13 (1942) p. 213.

³ Loc. cit.¹ p. 372 for proof of symmetry and evaluation of $E(\delta^2/s^2)$.

A CORRECTION

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In my article "Notes on the Distribution of Roots of a Polynomial with Random Complex Coefficients" which appeared in the June 1942 issue of the *Annals of Mathematical Statistics*, the symbol $\sum_{p=1}^n \sum_{q=p+1}^n$ in formulas (13), (14), and (15) should be replaced by $\prod_{p=1}^n \prod_{q=p+1}^n$.