On Saturday afternoon the members of the three societies were the guests of Miss Margaret Trumbull Corwin, Dean of the College, New Jersey College for Women, at an informal reception at the Dean's House. On Sunday evening an informal buffet supper for the mathematical organizations was served at Wood Lawn, the Alumnae House of the New Jersey College for Women. Later the same evening the Department of Music presented a Musicale in the Music Building.

EDWIN G. OLDS,
Secretary

REPORT ON THE SECOND MEETING OF THE PITTSBURGH CHAPTER OF THE INSTITUTE

The second meeting of the Pittsburgh Chapter of the Institute of Mathematical Statistics was held at Carnegie Union, Carnegie Institute of Technology, on Saturday, October 9, 1943. Thirty-four persons attended the meeting, including the following eight members of the Institute:

W. O. Clinedinst, G. G. Eldredge, K. L. Fetters, H. J. Hand, G. E. Niver, F. G. Norris, E. G. Olds, E. M. Schrock.

At the morning session Mr. Charles E. Young, Westinghouse Electric and Manufacturing Company, presented a paper entitled "Analysis of Cyclical Fluctuations." The program for the afternoon session consisted of a paper entitled "Use of orthogonal coordinates in linear regression," presented by Mr. W. O. Clinedinst, National Tube Company. Mr. F. G. Norris, President of the Pittsburgh Chapter, acted as chairman for both sessions.

HOWARD HAND,
Secretary of the Pittsburgh Chapter

ABSTRACTS OF PAPERS

(Presented Monday, September 13, 1943, at the New Brunswick Meeting of the Institute)

Asymptotic Distributions of Ascending and Descending Runs. JACOB WOLFOWITZ, Columbia University.

Let $a_1, a_2, \cdots, a_N$ be any permutation of N unequal numbers. Let there be assigned to each permutation the same probability. An element $a_i (1 < i < N)$ is called a turning point if a_i is greater than or less than both a_{i-1} and a_{i+1}. Let a_j and a_{j+k} be consecutive turning points; they are said to determine a "run" of length k. The author obtains the asymptotic distributions of a large class of functions of these runs. An example of his results is the following: It is proved that the following are asymptotically normally distributed: (a) the total number of runs; (b) $R(p)$, the number of runs of length p; (c) $R(p)$ and $R(q)$ jointly. Similar results are obtained for runs defined by any of a large set of criteria, of which the one given above is of value in statistical applications.

On the Plotting of Statistical Observations. E. J. GUMBEL, The New School for Social Research.

It is well known that there exist two step functions corresponding to a continuous variate. We may attribute to the m-th observation the ranks m or $m - 1$. To obtain one and only one serial number m, which will, in general, not be integer, we attribute to x_m an adjusted frequency $m - \Delta$, namely the probability of the most probable m-th value. The correction Δ for the rank thus introduced depends upon the distribution. If the variate is unlimited and possesses a mode, Δ increases for increasing values of the variate from zero up to unity. The correction is important for small numbers of observations. For large numbers of observations and for the ogive it is sufficient to choose $\Delta = \frac{1}{2}$. The calculation of Δ allows a correct plotting of all observations (including the first and last) on probability paper (equiprobability test). For the return periods, the ranks m and $m - 1$ correspond to the observed exceedance and recurrence intervals. The correction Δ leads to adjusted return periods which pass for increasing values of the variate from the exceedance to the recurrence intervals, provided the variate is unlimited and possesses a single mode. The asymptotic standard error of the partition values may be used to construct confidence bands for the ogive, the equiprobability test, and the return periods. This control for the fit between theory and observation may be applied to all observations which are not extreme.

On a Measure-Theoretic Problem Arising in the Theory of Non-Parametric Tests. HENRY SCHEFFÉ, Princeton University.

Let $F(x)$ be the cumulative distribution function of a univariate population. Denote a sample from the population by the sample point, $E = (x_1, x_2, \cdots, x_k)$ and let w be a Borel region in the sample space. How can we characterize w in order that $Pr\{E \text{ in } w\}$ be independent of $F(x)$ for all F in a given class of distribution functions? For various classes of F necessary conditions and sufficient conditions are found. For example, if the boundary of w is a null set, a necessary and sufficient condition for w to have the desired property for all absolutely continuous $F(x)$ is that it have the following structure except on a null set: For every point E in the sample space, M of the $k!$ points obtained by permuting the coordinates of E are in w and the remaining $k! - M$ are not $(0 < M < k!)$.

On a General Class of "Contagious" Distributions. W. FELLER, Brown University.

This paper is concerned with some properties of a class of contagious distributions which contains, among others, some distributions studied by Greenwood and Yule, Polya, and Neyman, respectively.

On the Statistical Treatment of Linear Stochastic Difference Equations. H. B. MANN AND A. WALD, Columbia University.

For any integer t let $x_{1t}, \cdots, x_{rt}$ be a set of r random variables which satisfy the system of linear stochastic difference equations $\sum_{j=1}^{r} \sum_{k=0}^{p_{ij}} \alpha_{ijk} x_{j,t-k} + \alpha_i = \epsilon_{it}$ $(i = 1, \cdots, r)$. The coefficients α_{ijk} and α_i are (known or unknown) constants and the vectors $\epsilon_t = (\epsilon_{1t}, \cdots, \epsilon_{rt})$ $(t = 1, 2, \cdots, \text{ad inf.})$ are independently distributed random vectors each having the same distribution. It is assumed that $E(\epsilon_{it}) = 0$. The problem dealt with in this paper is to estimate the unknown coefficients α_{ijk} and α_i on the basis of Nr observations x_{it} $(i = 1, \cdots, r; t = 1, \cdots, N)$. The statistics used as estimates of the unknown coefficients are identical with the maximum likelihood estimates if ϵ_t is normally distributed. The joint limiting distribution of these estimates is obtained without assuming normality of the distribution of ϵ_t.

An Exact Test for Randomness in the Non-Parametric Case Based on Serial Correlation. A. WALD AND J. WOLFOWITZ, Columbia University.

Let $X_1, \cdots, X_n$ be n chance variables, about the distribution of which nothing is known. Let the problem be to test the (null) hypothesis that $X_1, \cdots, X_n$ are independently distributed with the same distribution function. It is shown that an exact test of this hypothesis based on the serial correlation coefficient can be made. For this purpose the distribution of the serial correlation coefficient in the sub-population consisting of all possible permutations of the observed values is employed. Under the null hypothesis, this distribution is independent of the distribution function of $X_i (i = 1, \cdots, n)$. Several exact moments are obtained and asymptotic normality is proved.