NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

A NOTE ON SOME SINGLE SAMPLING PLANS REQUIRING THE INSPECTION OF A SMALL NUMBER OF ITEMS

By J. H. Curtiss

Cornell University[1]

In the practical application of sampling inspection plans it is often necessary to restrict the number of items (pieces, samples) inspected from each inspection lot to a relatively small number. For example, if many vendors are supplying a manufacturer with small lots of various kinds of material, the manufacturer will usually wish to have some check on his suppliers; however, he cannot afford to inspect large numbers of items from each lot. If sampling plans requiring the inspection of a small number of items are used, it is advantageous to know the characteristics of such plans. The present note offers several single sampling plans with sample size $n \leq 25$, together with their operating characteristic curves (OC curves) and average outgoing quality curves (AOQ curves).[2]

Single sampling plans for large lots may be described by the number n of items to be inspected, and the rejection number r. If r or more of the items inspected fail to meet some predetermined standard the lot is rejected; if less than r items fail to meet the standard the lot is accepted.

The OC curve (see Figures 1, 1A, 3 and 5) shows the relationship between the probability of rejecting a lot and the true quality of the lot. The quality of the lot is often measured by the "percent defective" in the lot; i.e., the proportion of material which does not meet some predetermined standard. It should be noted that the definition of OC curve given here is only one of several in common use. In particular, the vertical axis often gives the probability of "acceptance"; such a treatment would amount to an "inversion" of the curves given here. Another

[1] The material in this note was originally prepared as an office memorandum for the use of engineering technical personnel in a Government Bureau. The author wishes to express his appreciation to Mr. C. F. Mosteller for extensive editorial work on the original memorandum which has resulted in a revision more suitable for publication in the *Annals*.

[2] The OC and AOQ curves are often adequate to analyze single sampling plans because it is not customary to curtail single sampling even when the outcome of the inspection (acceptance or rejection) is determined before all the items are inspected. In other kinds of sampling plans (double, multiple, and sequential) where curtailing is often used after the first sample, curves for the average amount of sampling are also useful. However, if one is interested in the average amount of inspection, including detailing, as a manufacturer inspecting his own product might be, curves for the average amount of inspection would be useful in connection with any sampling plan.

common form would have the "percentage of presented lots (of quality indicated on the horizontal axis) that will be rejected (accepted)" as its vertical scale.

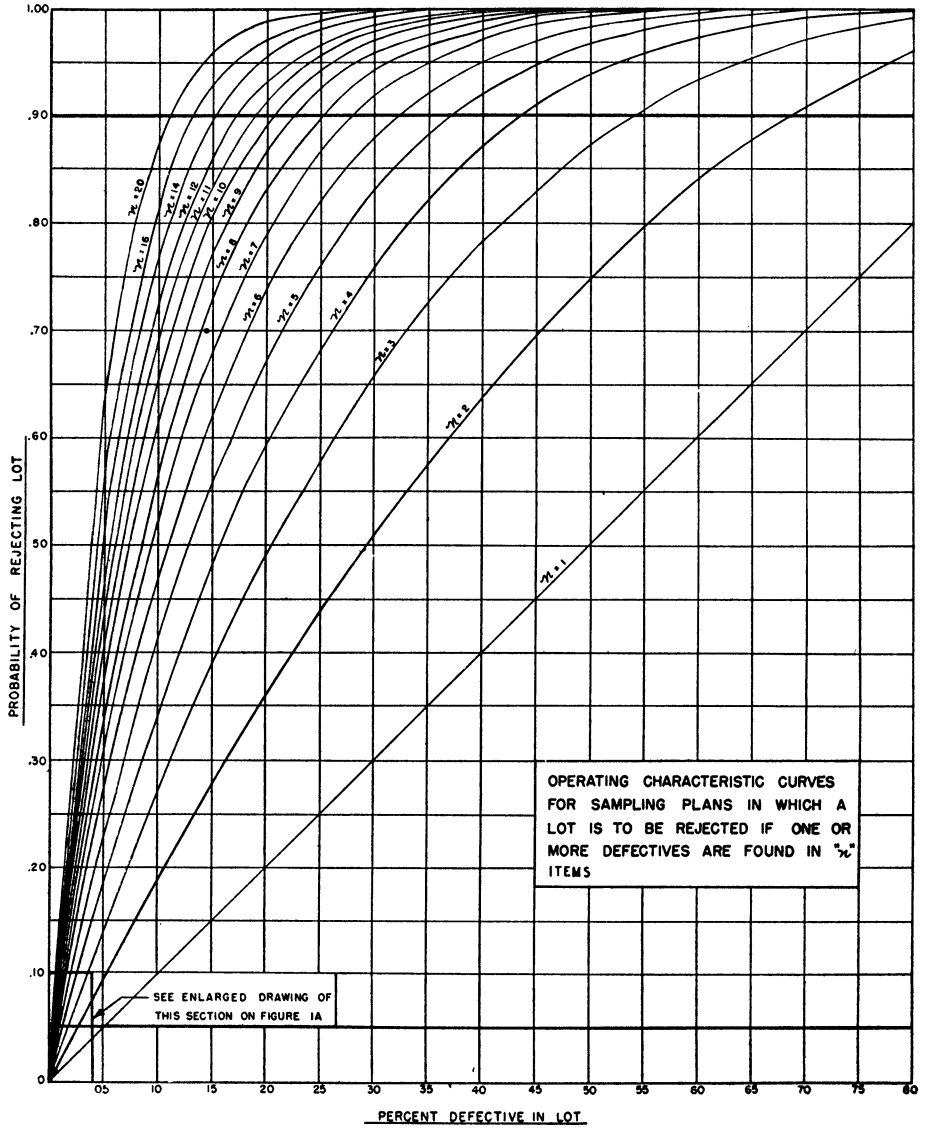

FIGURE 1

It has been assumed that the lots are so large that the samples can be regarded as being drawn from an infinite population, or to put it another way, that there

is no error in treating the samples as if they had been randomly drawn "with replacement".

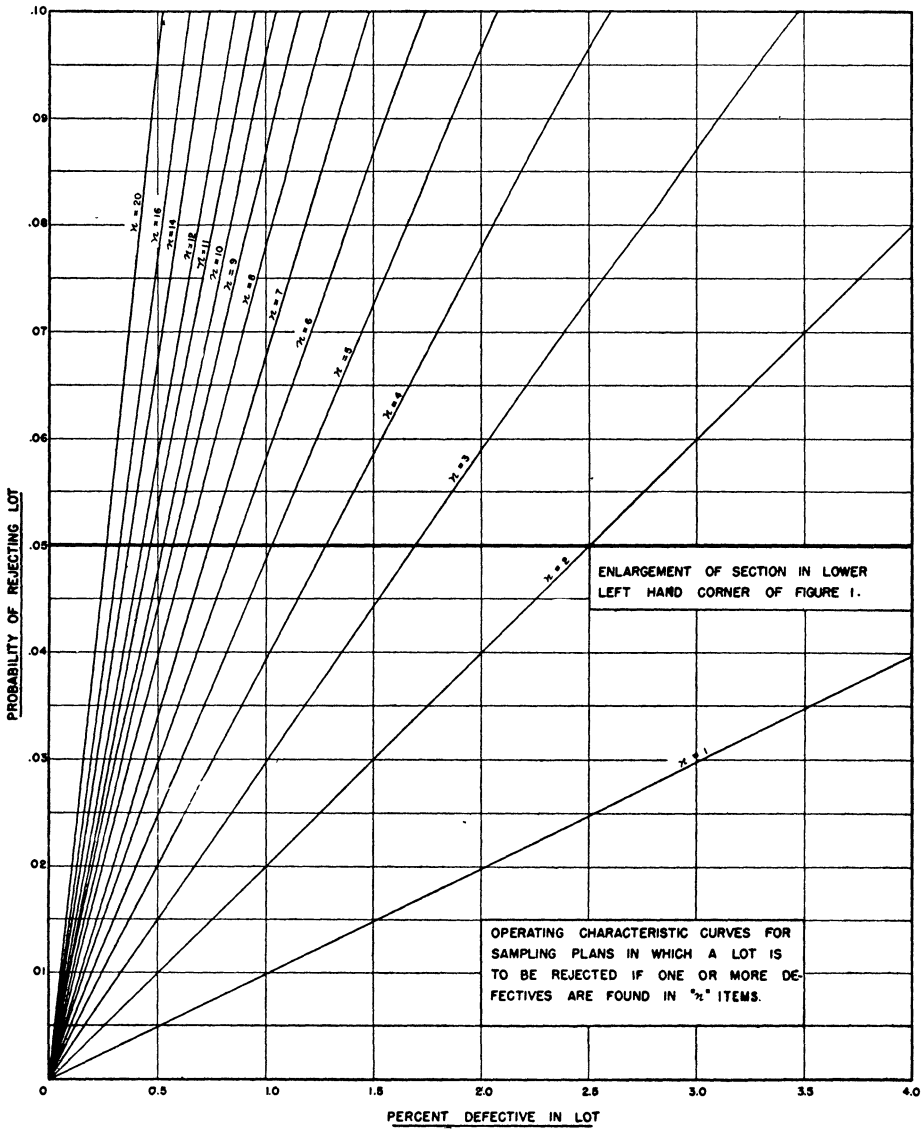

FIGURE 1A

Especial interest is often attached to the points where the curve crosses the 5% and the 90% probability levels. A rejection probability of 5% is frequently associated with a quality value that has been called the "acceptable quality level"

(abbreviated AQL), and in published sampling tables by Dodge and Romig,[3] a rejection probability of 90% is associated with a quality value which they call the "tolerance percent defective."

The average outgoing quality curve (AOQ curve, see Figures 2, 4 and 6) of a sampling plan shows the relationship between the long run average quality of the outgoing product *after* sampling inspection and the quality of the product as submitted for inspection. The quality of the product in each case is usually measured by the "percent defective" in the product.

SUPPLEMENT TO FIGURES 1 AND 1A.

Quality of Lot (measured in percent defective) corresponding to various probabilities of rejection, for sampling plans in which a lot is to be rejected if one or more defective items are found in a set of n random sample items

n	Probability of Rejection					
	.01	.05	.25	.50	.75	.90
	per cent	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*
1	01.00	05.00	25.00	50.00	75.00	90.00
2	00.50	02.53	13.40	29.29	50.00	68.38
3	00.34	01.70	09.14	20.63	37.01	53.58
4	00.25	01.28	06.94	15.91	29.29	43.77
5	00.20	01.02	05.59	12.95	24.21	36.90
6	00.17	00.85	04.68	10.91	20.63	31.87
7	00.14	00.73	04.03	09.43	17.97	28.03
8	00.12	00.64	03.53	08.30	15.91	25.01
9	00.11	00.57	03.14	07.41	14.28	22.57
10	00.10	00.51	02.84	06.70	12.95	20.57
11	00.09	00.47	02.58	06.11	11.84	20.40
12	00.08	00.43	02.37	05.61	10.91	17.46
14	00.07	00.36	02.03	04.83	09.43	15.17
16	00.06	00.32	01.78	04.24	08.30	13.40
20	00.05	00.26	01.43	03.41	06.70	10.88

The average outgoing quality is dependent upon the treatment of rejected lots. If rejected lots are cast aside once and for all, and are never resubmitted with all deficiencies corrected, then the average quality of the outgoing product after the sampling inspection tends to be the same as the average quality of the product submitted for inspection (provided that the quality of individual lots does not fluctuate too wildly). The only direct effect that the sampling inspection has in this case is to reduce the amount of the product which is accepted. However,

[3] H. E. Dodge and H. G. Romig, *Sampling Inspection Tables, Single and Double Sampling*, John Wiley and Sons, Inc., New York, 1944.

the situation is very different if a rejected lot is always resubmitted with all defective material removed or replaced with non-defective material. In this case,

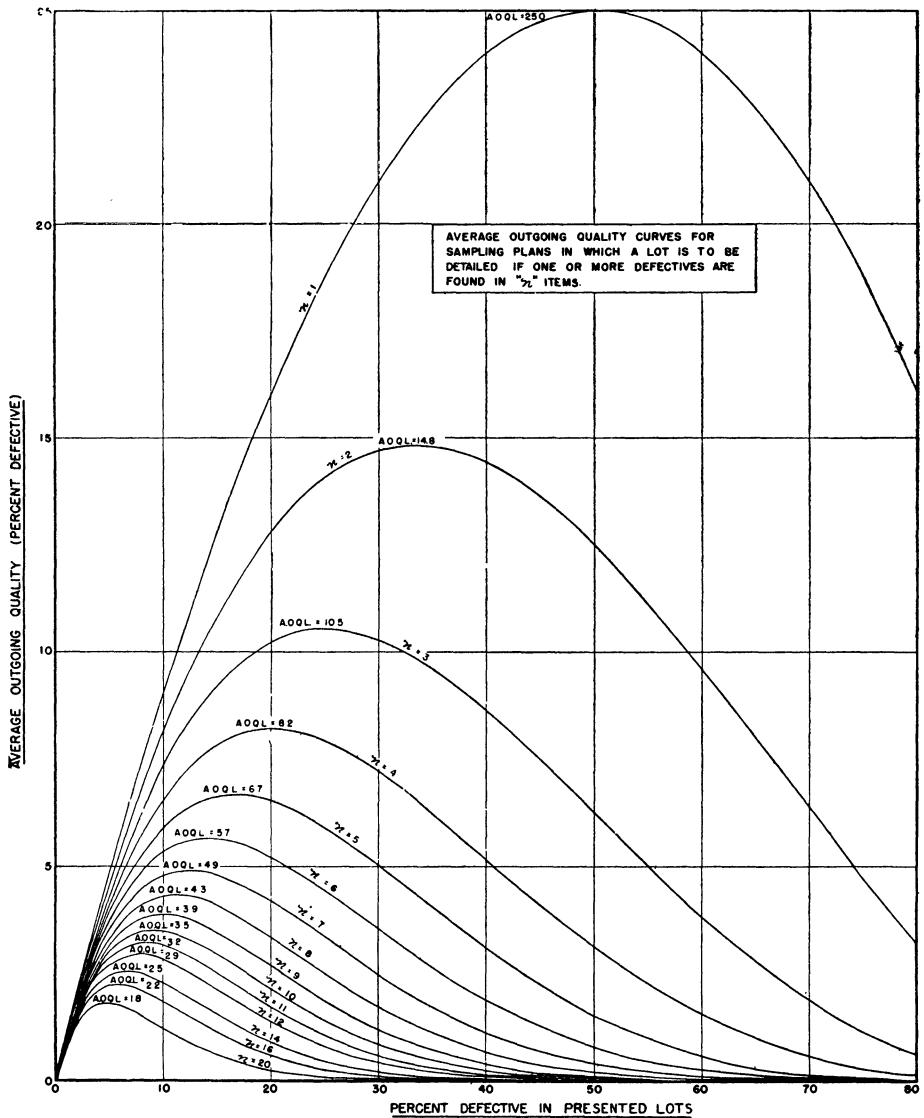

FIGURE 2

the average quality of the outgoing product after the sampling inspection will tend to be better than the average quality of the product submitted for inspection. In fact, if the submitted quality is very poor, the average outgoing quality

will theoretically tend to be very good, because so many of the lots are rejected and then detailed.

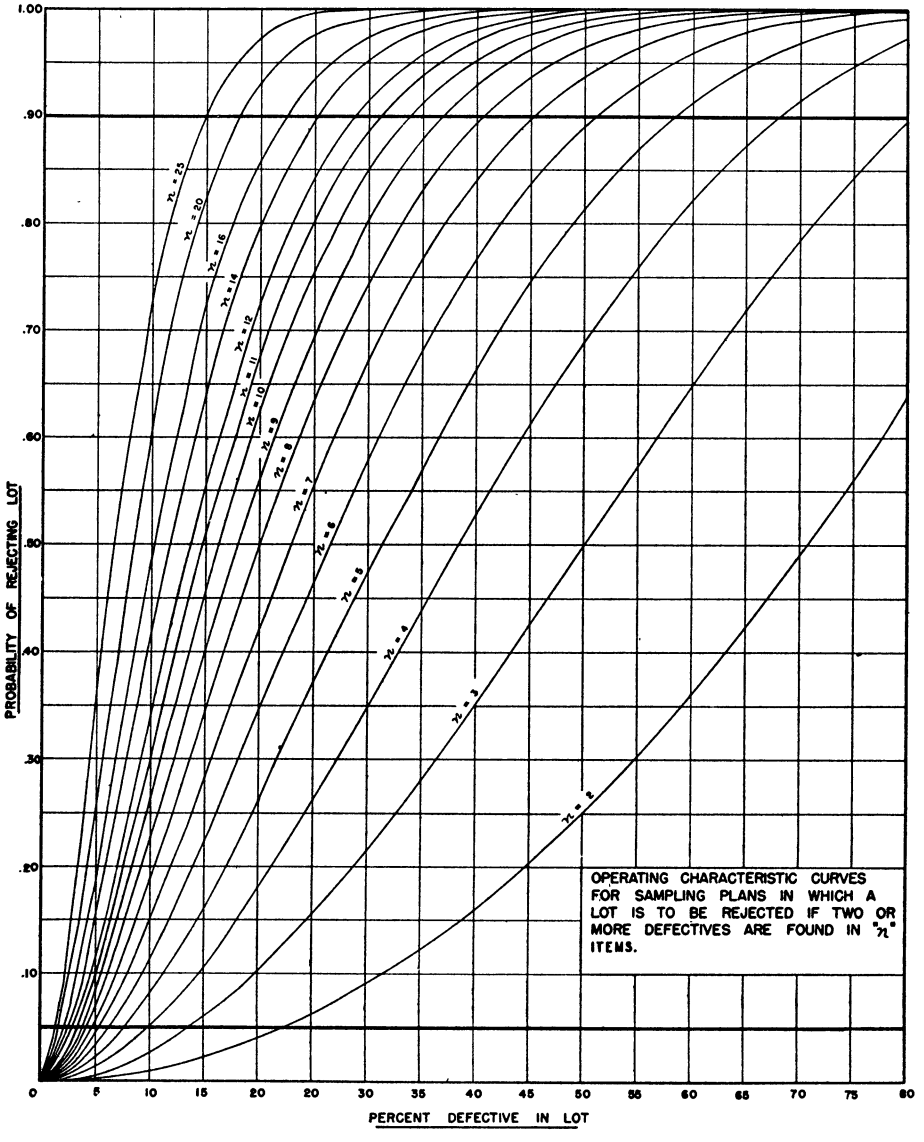

FIGURE 3

Under the assumption that each rejected lot will be detailed and resubmitted with all deficiencies corrected, a typical average outgoing quality curve starts at the origin, rises rapidly to a maximum, and falls off more slowly. The maxi-

mum average outgoing quality is called the average outgoing quality limit (AOQL) of the plan.

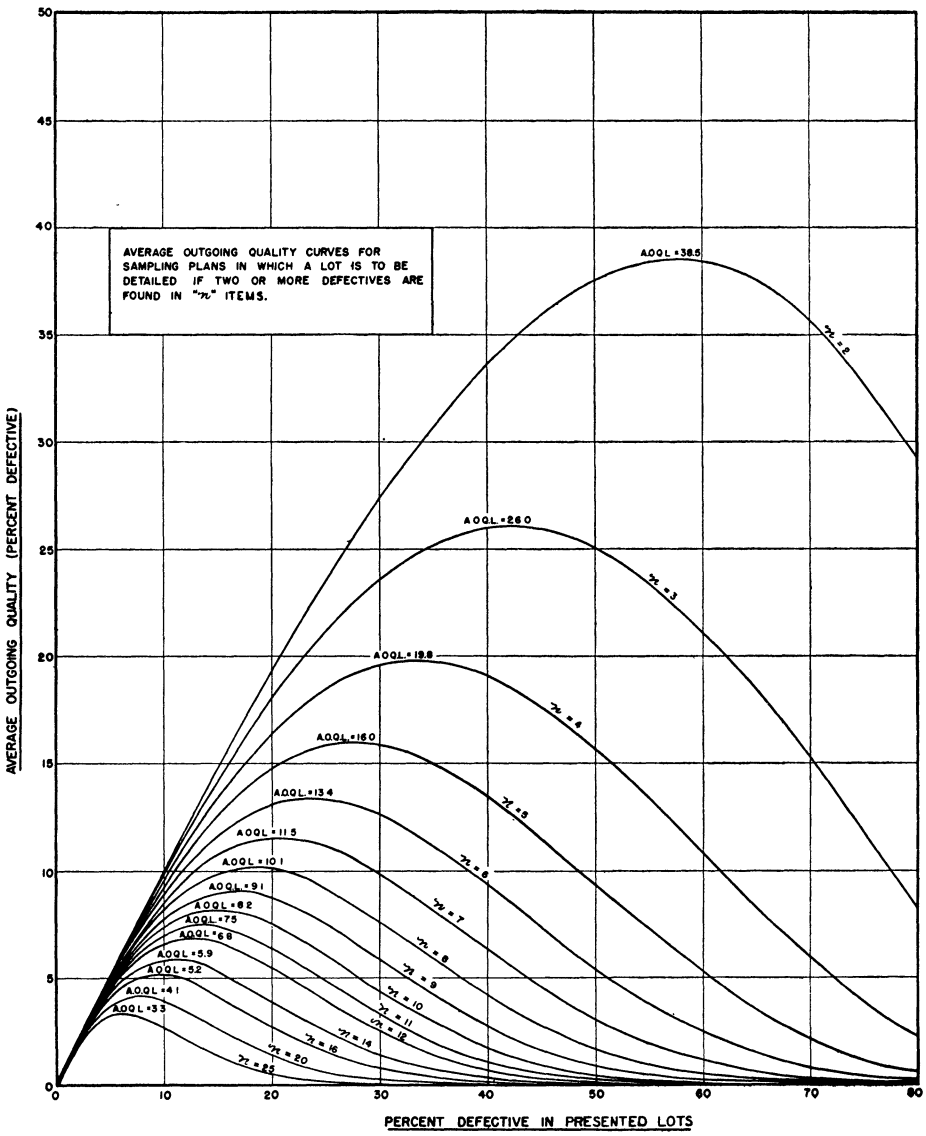

FIGURE 4

The graphs give the operating characteristic curves and average outgoing quality curves of certain single sampling plans. It is assumed the samples are taken at random without replacements from a lot which contains at least 10 times

the specified number of samples. In the case of the average outgoing quality curves, it is further assumed that rejected lots are always detailed and resub-

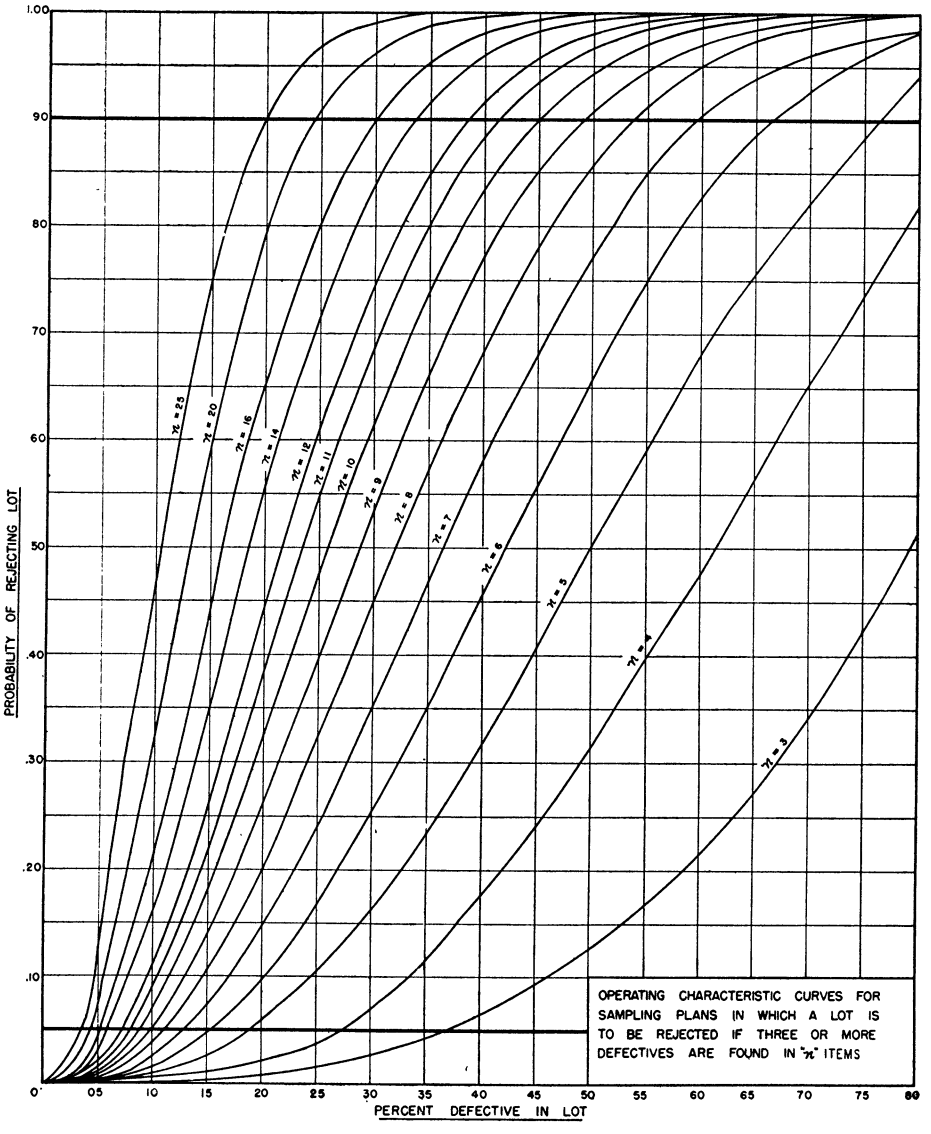

FIGURE 5

mitted with all the defective material replaced by non-defective material. An approximation has been made in the calculation of the AOQ curves which makes them upper bounds. If it is assumed that many lots of size N of exactly the

same quality of product p are being produced and that we are taking samples of size n from them, then it follows that AOQ $= p P_a (1 - n/N)$, where P_a is the probability of accepting a lot. The term n/N has been omitted; therefore these

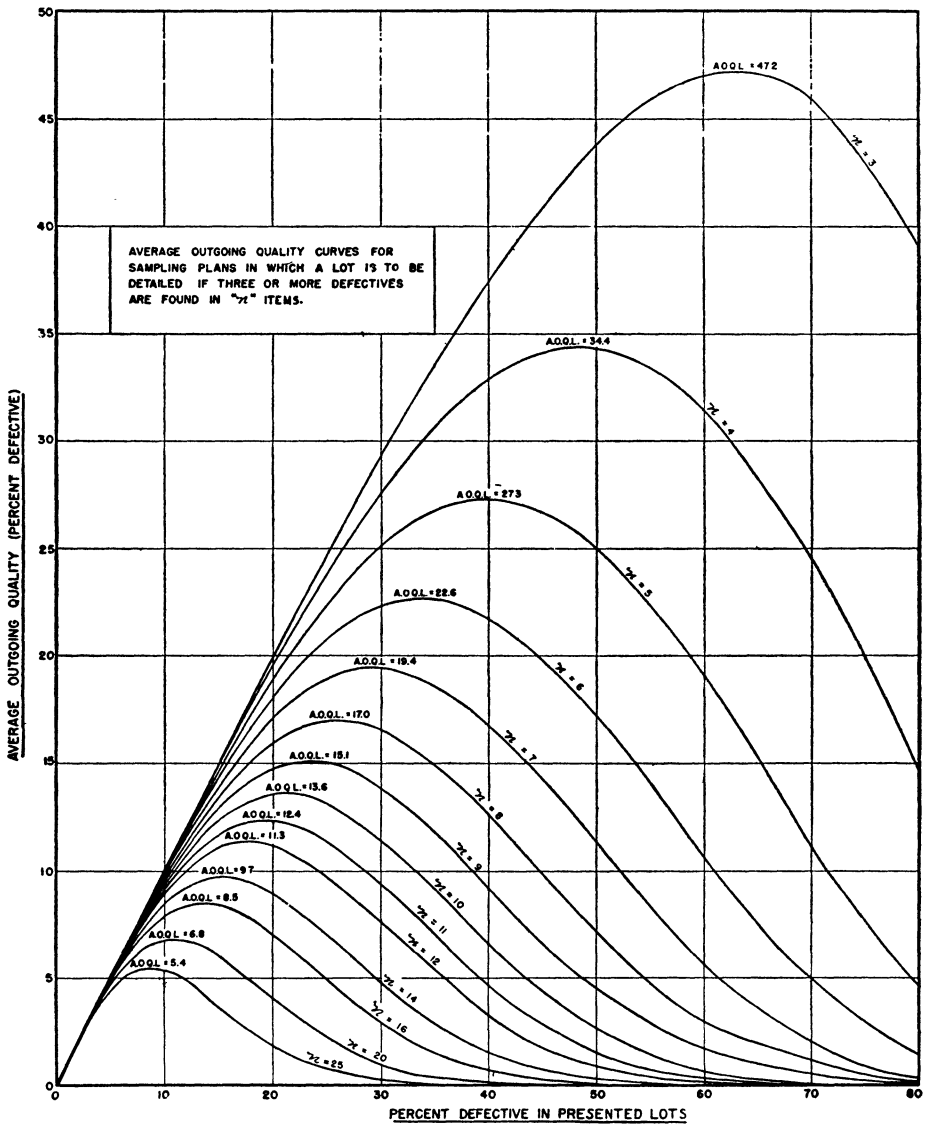

FIGURE 6

AOQ curves are too high, but are a good approximation provided only that the ratio of sample size to lot size is small. The condition mentioned earlier in this paragraph requires that $n/N \leq 0.1$.