BOOK REVIEWS

Sequential Analysis *Abraham Wald.* John Wiley and Sons, Inc. pp. vi, 212, $4.00.

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The development of sequential analysis as a new tool of statistics is by and large the work of Abraham Wald. This fact in itself would make the appearance of a book by him on this subject an important event. However, Wald in this book did more than discuss the present status of sequential theory. He has, in fact, written a very lucid treatise on the general subject of statistical inference—a treatise which is likely to have great influence on statistical thinking.

While this book is not written for the mathematically untrained, a knowledge of differential and integral calculus will suffice to follow all the arguments except perhaps for some sections in the appendix where the more complicated proofs have been placed.

The main body of this book is divided into 3 parts and 11 chapters. Part I, covering chapters 1 to 4 inclusive, deals with the general theory of the sequential probability ratio test. Chapter 1 introduces in an elementary fashion the notion of probability distributions, tests of hypotheses and the Neyman-Pearson theory of two-valued decisions based on a fixed sample size. In Chapter 2, the general notion of a sequential test procedure is introduced and the operating characteristics of such tests are discussed. Chapter 3 deals with the sequential probability ratio test for testing a single hypothesis against a single alternative. Here the boundaries of this sequential criterion are expressed in terms of the risks, the operating characteristic and the average sample number functions are developed and bounds are obtained for the errors arising from truncation and neglect of excess over the boundaries. Chapter 4 presents a sequential theory for testing simple and composite hypotheses against a set of alternatives. The fundamental idea introduced is the concept of a weight function in the parameter space which permits handling composite hypotheses, or simple hypotheses with many alternatives, by means of the sequential probability ratio test.

Part II of this book, consisting of chapters 5 to 9 inclusive, deals with the applications of sequential analysis to special problems. Chapter 5 contains a discussion of the binomial case with specific reference to lot-by-lot acceptance inspection. Of special interest in this chapter is the derivation of the exact characteristic function for a large class of tests and the development of upper and lower limits for the effect of grouping on the OC and ASN curves. Chapter 6 deals with the problem of double dichotomies. A procedure for testing the difference between the parameters of two binomial distributions is developed

for the fixed size as well as the sequential procedure. Chapters 7, 8, and 9 are concerned with the application of sequential analysis to the normal distribution. In these chapters the sequential probability ratio test is applied to hypotheses concerning the mean of a normal distribution when the variance is known, when the variance is not known (non-central t case) and hypotheses concerning the variance when the mean is known and when the mean is not known.

Part III consists of two short chapters and deals with multi-valued decisions and sequential interval estimation. The results in these chapters are not definitive answers to the two outstanding problems in statistical inference but are merely suggestive of a possible approach to them. Nevertheless, from the point of view of stimulating future research these 2 chapters are perhaps the most valuable sections of this book. The reader, having been exposed in the previous chapters to various tests the outcome of which is a two-valued decision, is naturally led in Chapter 10 to the consideration of tests the outcome of which is a multi-valued decision. The notion of a risk function, introduced elsewhere by the author in the non-sequential case, is again used as the main tool in handling multi-valued decisions sequentially. In Chapter 11 the important problem of setting up confidence intervals of fixed length by means of a sequential procedure is discussed and a possible method for accomplishing this is indicated.

As was previously noted, the main theorems on sequential analysis are contained in the Appendix and since they have all been previously published in the Annals they will not be mentioned in the present review. The Appendix, together with the main body of the book form a fairly exhaustive treatment of sequential theory. A notable exception to this is the lack of any mention of the published research on sequential point estimation. This is probably accounted for by the fact that this research came too late to be included in the book. Other minor omissions that may be noted are references to the generalization of the Fundamental Identity to more than one dimension and other theorems on sequences of functions of random vectors which have appeared in print. Also no mention is made of the similarity of sequential analysis to the problems of the random walk and the gambler's ruin. This, in the opinion of the reviewer, is regrettable.

This book will make a very suitable companion to the book *Sequential Analysis of Statistical Data: Applications* prepared by the Statistical Research Group, Columbia University (see review by J. W. Tukey, *Ann. of Math. Stat. Vol.* xviii, 1947). While there is some overlap in the material covered, the two books differ in emphasis. Wald's book, though not highly technical, is more in the nature of a textbook on the theory and application of sequential analysis. The SRG book on the other hand, was prepared mainly for statisticians who may wish to use sequential analysis in practice. The latter book is therefore more detailed and puts less emphasis on the theoretical aspects of the sequential procedure.

The book is surprisingly free of typographical errors which is a tribute to the high quality of the editorship.