

BOOK REVIEW

The Theory of Games and Economic Behavior *John von Neumann and Oskar Morgenstern.* Princeton University Press, 1947; Second Edition, Pp. xviii, 641. \$10.00

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This review is devoted to the second edition of a book which from its first appearance was acknowledged to be a major contribution in the field of theory of rational behavior. As is pointed out in the Preface, "the second edition differs from the first in some minor respects only". The main change is the addition of a proof (of "measurability" of utility) omitted in the first edition.

The book's objective is to solve the problem of rational behavior in a very general type of situation.

It is, therefore, not surprising that its results are of relevance in many fields of knowledge, among them economics and statistical inference.

In both economics and statistics the problem of rational behavior is a fundamental one. Thus one of the classical problems treated by the economic theory is that of profit maximization by a firm. The firm is assumed to be maximizing its net profit which is a function of prices of the product, materials used, etc., as well as the quantities used and produced. In the simplest case prices are taken as given; more generally they are assumed to be functions (known to the firm) of the quantities sold and purchased. But assuming this function to be known presupposes the knowledge of behavior of other firms. This procedure has for a long time been regarded as highly unsatisfactory; it is analogous to elaborating the theory of rational behavior of a poker player on the assumption that he knows the strategy of the other players!

It is the type of situation in which not only the behavior of various individuals, but even their strategies, are interdependent, that is treated by von Neumann and Morgenstern. The essence of their solutions is to base the optimal strategy on the *minimax principle*. As applied to a game, the principle requires that one should choose a strategy which minimizes the maximum loss that could be inflicted by the opponent.

The minimax principle, when applied by both players need not, in general, lead to a stable solution. To ensure the existence of such a solution the authors are led to the postulate that the choice of strategies be made through a random process. The minimax to be found is that of the *mathematical expectation* of the loss in the game. The latter postulate is of a restrictive nature² since it implies that the game is played for numerical ("measurable") stakes and that

¹ On leave to the United Nations Economic Commission for Europe.

² See Jacob C. Marschak, "Neumann's and Morgenstern's New Approach to Static Economics", *The Journal of Political Economy*, Vol. LIV (1946).

the second and higher moments of the probability distribution of the losses are immaterial. This restriction, however, has permitted the authors to go deeper in other directions. Given the great complexity of the problem, even in its restricted version, the authors' decision can hardly be criticized. One could only wish that similar considerations had made the authors more tolerant towards other work in the field of economics than is shown in some sections of the book.

The readers of the *Annals* will be particularly interested in the connection between the *Theory of Games* and the theory of statistical inference.

As has been pointed out by Abraham Wald³ the problem faced by the statistician is somewhat similar to that of a player in a game of strategy. The theory of statistical inference may be viewed as a theory of rational behavior of the statistician. His "strategy" consists in adopting an optimal test or estimate, more generally an optimal decision function. This optimal decision function must be chosen without the knowledge of the "a priori" distribution of the population parameters. Wald's basic postulate of minimization of maximum risk is equivalent to regarding the statistician as a player in a game of strategy, with "Nature" as the other player. The optimal decision function is chosen in a way which (as shown by Wald) is equivalent to assuming the "least favorable" a priori distribution of the parameters. As Wald says, "we cannot say that Nature wants to maximize [the statistician's risk]. However, if the statistician is completely ignorant as to Nature's choice, it is perhaps not unreasonable to base the theory of a proper choice of [the decision function] on the assumption that Nature wants to maximize (the statistician's risk)".

It may be noted, however, that statistical inference, as seen by Wald, is a relatively simple game since it involves only two players and is of the zero-sum variety.

The admiring and enthusiastic reception given to the book's first edition would make any further general appraisal somewhat anticlimatic. Suffice it to say that a good deal of valuable work has already been stimulated by the *Theory of Games*, both in the field of social sciences and in mathematics.

³Abraham Wald, "Statistical Decision Functions which Minimize the Maximum Risk", *Annals of Mathematics*, Vol. 46, (1945).