4. The frequency function. The quadratic form appearing in the exponent in the expression for the frequency function can now be written as

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{R_{ij}\,x_i\,x_j}{R\sigma_i\,\sigma_j} = \frac{2-(n-2)p}{2p[2-(n-1)p]}\left(\frac{x_1^2}{\sigma_1^2}+\frac{x_n^2}{\sigma_n^2}\right)$$
$$+\frac{1}{p}\left(\frac{x_2^2}{\sigma_2^2}+\frac{x_3^2}{\sigma_3^2}+\cdots+\frac{x_{n-1}^2}{\sigma_{n-1}^2}\right)$$
$$+\frac{1}{2[2-(n-1)p]}\left(\frac{x_1\,x_n}{\sigma_1\,\sigma_n}+\frac{x_n\,x_1}{\sigma_n\,\sigma_1}\right)$$
$$-\frac{1}{2p}\left(\frac{x_1\,x_2}{\sigma_1\,\sigma_2}+\frac{x_2\,x_1}{\sigma_2\,\sigma_1}+\frac{x_2\,x_3}{\sigma_2\,\sigma_3}+\frac{x_3\,x_2}{\sigma_3\,\sigma_2}+\cdots+\frac{x_n\,x_{n-1}}{\sigma_n\,\sigma_{n-1}}\right)$$
$$=\frac{1}{p}\left[\frac{2-(n-2)p}{2[2-(n-1)p]}\left(\frac{x_1^2}{\sigma_1^2}+\frac{x_n^2}{\sigma_n^2}\right)+\sum_{i=2}^{n-1}\frac{x_i^2}{\sigma_i^2}-\sum_{i=1}^{n-1}\frac{x_i\,x_{i+1}}{\sigma_i\,\sigma_{i+1}}\right]$$
$$+\frac{1}{2-(n-1)p}\left(\frac{x_1\,x_n}{\sigma_1\,\sigma_n}\right).$$

5. Maximum likelihood. The expression z is the likelihood of getting a particular set of values of the variables $x_1, x_2, \cdots, x_n$. It is often important to regard the r_{ij} and the σ_i as parameters and to determine them so that the likelihood will be a maximum. If we assume $\sigma_1 = \sigma_2 = \cdots = \sigma_n = \sigma$, then

$$z = \frac{1}{(2\pi)^{n/2}\sigma^n\sqrt{R}}\exp\left\{-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{R_{ij}\,x_i\,x_j}{R\sigma^2}\right\}.$$

The question, in our case, now becomes, What values of p and σ will make z a maximum for given x_i? Necessary conditions are that $\frac{\partial z}{\partial p} = 0$ and $\frac{\partial z}{\partial \sigma} = 0$. Since R_{ij} and R are given in terms of p, the process of differentiation can be carried out (first take the logarithm of z), and values of p and σ necessary for a maximum determined. It is, of course, possible that z has no maximum, and the sufficiency of these values must be tested. The computations for the general case are laborious, though straightforward. Furthermore, because of the complicated nature of the coefficients in the equation to be solved for p, the general solution is not readily obtainable. This equation is, however, of third degree, and it can be solved in any particular case.

TABLE OF NORMAL PROBABILITIES FOR INTERVALS OF VARIOUS LENGTHS AND LOCATIONS

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1. Introduction. The probability associated with a particular finite range of values is often desired. The usual tables of normal areas gives values for $\int_0^x$ or as in the table by Salvosa [1], $\int_{-\infty}^{x}$. The WPA table [2] gives $\int_{-x}^{x}$. The author has deposited with Brown University a table of $\int_{x-\frac{1}{2}l}^{x+\frac{1}{2}l}$ for values of $x[0(.1)\ 5.0]$ and values of $l[0(.1)\ 10.0]$. The values in the table may be interpreted as the probability that an observation from a normal population with unit variance will fall in an interval of length l whose midpoint is a distance x from the mean. These values can be obtained by a simple computation from the existing tables. Since values were being used frequently, the present table was constructed. Microfilm or photostat copies may be obtained upon request to the Brown University Library.

2. Computation. The values were obtained by finding the difference between the integrals $\int_{-\infty}^{x-\frac{1}{2}l}$ and $\int_{-\infty}^{x+\frac{1}{2}l}$ as given to six decimal places in Salvosa's table. Being differences, the values are subject to an error of 1 unit in the sixth place. For values of $x + \frac{1}{2}l$ greater than 5, the values can be obtained by computing $1 - \int_{-\infty}^{x-\frac{1}{2}l}$. The search for errors was aided by computing column sums; i.e.

$$\sum_{i=1}^{50} \int_{x_i-\frac{1}{2}l}^{x_i+\frac{1}{2}l} + \frac{1}{2}\int_{-\frac{1}{2}l}^{\frac{1}{2}l} = .5\ n, \tag{1}$$

where i represents the row number and n represents the column number. For example, $n = 17$ corresponds to column for $l = 1.7$. The approximation becomes poorer as n increases but the sums were still useful for checking purposes.

3. Example. The table has been used in studies of the expected proportion of a line covered by intervals dropped on it according to some normal probability function. Let $P_n(x)$ be the probability that the point x is covered at least once when n intervals are dropped on the x-axis. H. E. Robbins [3] gives the expression:

$$E(F) = \frac{1}{L}\int_0^L P_n(x)\ dx, \tag{2}$$

for the expected proportion of a line of length L covered at least once by these intervals.

Let $f(x)\ dx$ be the probability that an interval falls with its center in dx and l be the length of the interval. The probability that a point x will be covered by one interval dropped on the x-axis is:

$$g(x) = \int_{x-\frac{1}{2}l}^{x+\frac{1}{2}l} f(t)\ dt. \tag{3}$$

When n intervals are dropped, the probability that x is covered at least once is

$$P_n\ (x) = 1 - (1 - g(x))^n, \tag{4}$$

and

$$E(F) = 1 - \frac{1}{L}\int_0^L (1 - g(x))^n \, dx. \tag{5}$$

When k groups of n_i intervals are dropped according to, say normal distributions with different means,

$$P_n(x) = 1 - \prod_{i=1}^{k} (1 - g_i(x))^{n_i}. \tag{6}$$

Where

$$g_i(x) = \int_{x-\frac{1}{2}l}^{x+\frac{1}{2}l} f_i(t) \, dt \tag{7}$$

and we obtain

$$E(F) = 1 - \frac{1}{L}\int_0^L \prod_{i=1}^{k} (1 - g_i(x))^{n_i} \, dx. \tag{8}$$

The values $g(x)$ are those given in the table and are useful in evaluating the integrals in (5) and (8) by numerical methods.

REFERENCES

[1] Luis R. Salvosa, "Tables of Pearson's Type III functions," *Annals of Math. Stat.*, Vol. 1 (1930), p. 191.

[2] National Bureau of Standards, *Tables of Probability Functions*, Vol. 2 (1942).

[3] H. E. Robbins, "On the measure of a random set," *Annals of Math. Stat.*, Vol. 15, (1944).

CORRECTION TO "A NOTE ON THE FUNDAMENTAL IDENTITY OF SEQUENTIAL ANALYSIS"

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In the paper cited in the title (*Annals of Math. Stat.*, Vol. 18 (1947), pp. 593–596), the proof of Lemma 3 is incorrect. The following correct proof is due to Mr. C. R. Blyth of the Institute of Statistics, University of North Carolina.

It is easy to establish the equation

$$P(n = N|F)[\varphi(t_0)]^{-N} = P(n = N|G)E_{n=N}[\exp(-t_0 Z_N)|G],$$

where $E_{n=N}(u|G)$ denotes the conditional expectation of u under the condition that $n = N$ for any fixed integer N. By Wald [2], equations (2.4) and (2.6), there exists a finite constant C independent of N which dominates the expected values $E_{n=N}[\exp(-t_0 Z_N)|G]$ for every N. Thus

$$P(n = N|F)[\varphi(t_0)]^{-N} \leqq C \cdot P(n = N|G). \tag{A}$$