# ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Chicago meeting of the Institute, April 28–29, 1950*)

## 1. The Distribution of the Quotient of Ranges in Samples from a Rectangular Population. Paul R. Rider, Washington University, St. Louis, Missouri.

The distribution of the quotient of the ranges of two independent, random samples from a continuous rectangular population is derived. The distribution is independent of the population range and can be used to test the hypothesis that two samples came from the same rectangular population just as the distribution of the variance ratio is used to test whether two samples came from the same normal population.

## 2. A Geometric Method for Finding the Distribution of Standard Deviations when the Sampled Population Is Arbitrary. (Preliminary Report). Paul Irick, Purdue University.

For an ordered random sample, $x_1 \leqq x_2 \leqq \cdots \leqq x_n$, chosen from a population, $f(x)$, $a \leqq x \leqq b$, let $r_i = x_{i+1} - x_i \geqq 0$, $i = 1, 2, \cdots, n-1$. Make the transformation

$$r_i = -\sqrt{\frac{i-1}{2i}}\, r'_{i-1} + \sqrt{\frac{i+1}{2i}}\, r'_i,$$

and call $U'$ the $1/n!$ portion of the $r'$ space bounded by the $n-1$ sphere and hyperplanes, $\sum_1^{n-1} r_i'^2 = 2ns^2$, $r'_i = \sqrt{\dfrac{i-1}{i+1}}\, r'_{i-1}$, $i = 1, 2, \cdots, n-1$, where $s$ is the sample standard deviation. The point density in $U'$, $\delta(r')$, is the transform of

$$\delta(r) = \int_{x_1=a}^{b-\Sigma r_i} f(x_1) f(x_1 + r_1) \cdots f(x_1 + r_1 + \cdots + r_{n-1})\, dx_1 .$$

Change to generalized polar coordinates and call $U$ the outer hyperspherical boundary of $U'$ whereon the density is designated by $\delta(\sqrt{2n}s, \varphi)$. Then $p(s)$, the probability law for $s$, is given by

$$p(s)\, ds = n!\, n^{n/2} s^{n-2}\, ds \int_{\varphi_1} \cdots \int_{\varphi_{n-2}} \delta(\sqrt{2n}s, \varphi) \sin^{n-3} \varphi_1 \cdots \sin \varphi_{n-3}\, d\varphi_{n-2} \cdots d\varphi_1,$$

where

$$\text{arc cos} \sqrt{\frac{n}{(n-i)(i+1)}} \leqq \varphi_i \leqq \text{arc cos} \left[ \sqrt{\frac{i-1}{i+1}} \Big/ \tan \varphi_{i-1} \right], \quad i = 1, 2, \cdots, n-2,$$

whenever $b$ is infinite. The distribution of sample range is readily found in $U'$ and is expressible in the same form as $p(s)$ with the same limits of integration. When $b$ is finite, the complete integral holds only for $0 \leqq s \leqq (b-a)/\sqrt{2n}$, there being $n^2/4$ connected arcs in $p(s)$ if $n$ is even, and $(n^2-1)/4$ arcs if $n$ is odd. The axes are rotated to give relatively simple formulas for $p(s)$ when $n \leqq 4$, the case of $n = 5$ also being discussed. The method readily produces previously reported results for $p(s)$. In the application of the method, particular attention has been paid to the Type III and polynomial Type I populations. The density function provides much information concerning the form of $p(s)$ for various populations, and contours of constant $\delta$ in $U'$ are of theoretical interest.

**3. Probability of a Correct Result with a Certain Rounding-off Procedure.** W. S. LOUD, University of Minnesota.

Consider the problem of the addition of $n$ numbers expressed in the base $B$ of numeration. Supposing each number known to arbitrary accuracy, to obtain the sum accurate to $k$ places, one may round off each number to $(k + 1)$ places, add, and round the sum to $k$ places. If the numbers are assumed uniformly distributed, the probability that the above procedure gives the correct result may be found explicitly by use of characteristic functions. If the base $B$ is odd, the result is $2(\pi B)^{-1}\int_0^\infty \sin^{n-1}u \sin^2 Bu\, u^{-n-1}\, du$, and if the base $B$ is even, $2(\pi B)^{-1}\int_0^\infty \sin^2 Bu \cos u\, u^{-n-1}\, du$. Both formulas have the asymptotic formula $6^{1/2}B(\pi n)^{-1/2}$ as $n$ becomes infinite.

**4. Analysis of a One-person Game. (Preliminary Report).** W. M. KINCAID, University of Michigan.

The problem of allocation of supplies is one which arises in many military and economic connections. The present report discusses a game constructed as a model of a simple situation of this type. The player is given a supply of cards, and receives payments for giving these up when certain random events occur during the period of play.

The optimal strategy, which maximizes the expected value of these payments, is governed by certain critical times such that the player's response to a particular event depends on whether it occurs before or after one of these times.

## NEWS AND NOTICES

*Readers are invited to submit to the Secretary of the Institute news items of interest*

### Personal Items

Dr. Leo A. Aroian, on leave from Hunter College, is acting as a Research Physicist in charge of computations at the Hughes Aircraft Co., Department of Electronics and Guided Missiles, Culver City, California.

Dr. Ralph A. Bradley from McGill University, Montreal, Canada will join the staff as Associate Professor in the Department of Statistics at Virginia Polytechnic Institute on July 1, 1950. He will devote the majority of his time to research on rank order statistics.

Dr. E. R. Dalziel has relinquished his post as Assistant Master at Technical School, New Zealand, to become Senior Engineer with the Overseas Telecommunication Commission, Australia.

On September 1, Dr. David Duncan from the University of Sydney, Sydney, Australia, will join the statistical staff of Virginia Polytechnic Institute as Associate Professor of Statistics. He will devote the majority of his time to teaching.

Dr. C. H. Fischer has been promoted to the rank of Professor of Actuarial Mathematics in the Department of Mathematics and Professor of Insurance in the School of Business Administration, University of Michigan, Ann Arbor, Michigan.