

# NOTES

## TABLES FOR A NONPARAMETRIC TEST OF DISPERSION

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**1. Introduction.** Two samples are known to come from populations with the same mean (or median). Without reference to the form of the distribution function, we want to test the hypothesis that the populations are identical, and we simply count the number of points in one sample which lie outside the extreme values of the other sample. The device owes its origin to S. S. Wilks [1], who derived the basic formulas in a classic paper on tolerance limits. In the absence of the prior knowledge that the two populations have the same location, the test becomes merely a two-sample test of identity of the populations and rejection may imply difference in location as well as shape. (A one-sided modification of the test would, of course, be more appropriate for a specific test of location.)

**2. Test that the samples come from the same population.** We draw a random sample of  $n$  points and a random sample of  $m$  points from a population with a continuous distribution function. The probability that  $r$  points of the sample of  $m$  will lie outside the end values of the sample of  $n$  is [1]

$$(1) \quad \begin{aligned} P_r &= n(n-1) \frac{m!}{(m-r)!} \frac{(r+1)(n+m-r-2)!}{(n+m)!} \\ &= n(n-1) \binom{m}{r} B(n+m-1-r, r+2), \end{aligned}$$

where  $B$  is the complete Beta function.

For  $r_0 \leq m$ ,

$$(2) \quad \sum_{r=0}^{r_0} P_r = \sum_{r=0}^{r_0} n(n-1) \binom{m}{r} B(n+m-1-r, r+2)$$

is the probability that the value of  $r$  is not greater than  $r_0$ . Otherwise stated, one minus this quantity is the probability that the value of  $r$  is  $r_0 + 1$  or greater. We can therefore fix a probability level  $\epsilon$  and arrive at an  $r_0$  such that  $\sum_{r=0}^{r_0-1} P_r \leq \epsilon < \sum_{r=0}^{r_0} P_r$ .

Tables of  $r = r_0 + 1$  are given for  $\epsilon = 0.95$  and  $\epsilon = 0.99$  over the range  $n = 1, \dots, 50, m = 1, \dots, 50$ .

It may be worth pointing out that when  $r_0 = m$ , the hypothesis cannot be rejected whatever value  $r$  takes in the sample. Also a certain symmetry arises

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for  $n = m$ , where  $P_0 = n(n - 1)/2n(2n - 1)$ ,  $P_1 = n(n - 1)2n/2n(2n - 1)(2n - 2)$ , so that  $P_0 + P_1 = \frac{1}{2}$ . This is equivalent to saying that for equal samples of any size from the same population, whatever the form of its distribution function, the chances are even that not more than one point of the second sample shall lie outside the range of values of the first.

For large and approximately equal values of  $m$  and  $n$ , say  $m \approx n \approx N \gg 0$ ,  $P_r$  is approximately equal to  $N^2(r + 1)N^r/(2N)^{r+2} = (r + 1)/2^{r+2}$ . Then the critical values of  $r_0 + 1$  turn out to be,

$$\text{at } \epsilon = 0.95, r_0 + 1 = 7,$$

$$\text{at } \epsilon = 0.99, r_0 + 1 = 10.$$

#### REFERENCE

- [1] S. S. WILKS, "Statistical prediction with special reference to the problem of tolerance limits," *Ann. Math. Stat.*, Vol. 13 (1942), pp. 400-409.

5% values of  $r$

$n \backslash m$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	9	10	11	12	13	14	15	16	16	17	18	19	20	21	21	22	23
4	—	—	—	5	6	7	8	9	9	10	11	12	12	13	14	15	15	16	17	18	18	19	20	21
5	—	—	4	5	6	6	7	8	8	9	10	10	11	12	12	13	14	14	15	16	16	17	18	18
6	—	3	4	5	5	6	7	7	8	8	9	9	10	11	11	12	12	13	14	14	15	16	17	17
7	—	3	4	4	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	15	15
8	—	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	10	11	11	12	12	13	13	14
9	—	3	4	4	4	5	5	6	6	7	7	8	8	8	9	9	10	10	11	11	11	12	12	13
10	2	3	3	4	4	5	5	5	6	6	7	7	8	8	8	9	9	10	10	10	11	11	12	12
11	2	3	3	4	4	4	5	5	6	6	6	7	7	7	8	8	9	9	9	10	10	10	11	11
12	2	3	3	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	10	10	10	11
13	2	3	3	3	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10
14	2	3	3	3	4	4	4	5	5	5	6	6	6	6	7	7	7	8	8	8	9	9	9	10
15	2	3	3	3	4	4	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	9	9	9
16	2	3	3	3	3	4	4	4	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8	9
17	2	2	3	3	3	4	4	4	4	5	5	5	6	6	6	7	7	7	7	7	8	8	8	8
18	2	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	8	8	8	8
19	2	2	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	8	8	8
20	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	6	6	6	6	7	7	7	7	7
21	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7
22	2	2	3	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7
23	2	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7
24	2	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	6	6	6	6	6	6	7	7
25	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7
26	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6
27	2	2	2	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6
28	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	6	6	6	6
29	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6
30	2	2	2	2	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	6	6	6
31	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	6	6	6
32	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	5	6	6
33	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	5	5	5
34	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	5	5
35	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	5	5
36	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5
37	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5
38	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5
39	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5
40	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5
41	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5
42	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5
43	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5
44	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5
45	2	2	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5
46	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
47	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
48	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
49	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4
50	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4

The probability is less than 5% that  $r$  or more points of a sample of size  $m$  lie outside the extreme values of a sample of size  $n$  if the samples are drawn randomly from the same population, whatever its distribution.

5% values of  $r$

$m \backslash n$	26 27 28 29 30	31 32 33 34 35	36 37 38 39 40	41 42 43 44 45	46 47 48 49 50
2	— — — — —	— — — — —	— — — — 40	41 42 43 44 45	46 47 48 49 50
3	24 25 26 27 28	29 29 30 31 32	33 34 35 35 36	37 38 39 40 41	41 42 43 44 45
4	21 22 23 24 24	25 26 27 27 28	29 30 30 31 32	33 33 34 35 36	36 37 38 39 39
5	19 20 20 21 22	22 23 24 24 25	26 26 27 28 28	29 30 30 31 32	32 33 33 34 35
6	17 18 18 19 19	20 21 21 22 22	23 24 24 25 25	26 26 27 28 28	29 29 30 31 31
7	16 16 17 17 18	18 19 19 20 20	21 21 22 22 23	23 24 24 25 25	26 27 27 28 28
8	14 15 15 16 16	17 17 18 18 19	19 19 20 20 21	21 22 22 23 23	24 24 25 25 26
9	13 14 14 15 15	15 16 16 17 17	18 18 18 19 19	20 20 21 21 21	22 22 23 23 24
10	12 13 13 14 14	14 15 15 15 16	16 17 17 17 18	18 19 19 19 20	20 21 21 21 22
11	12 12 12 13 13	13 14 14 14 15	15 16 16 16 17	17 17 18 18 19	19 19 20 20 20
12	11 11 12 12 12	13 13 13 14 14	14 15 15 15 16	16 16 17 17 17	18 18 18 19 19
13	10 11 11 11 12	12 12 13 13 13	14 14 14 14 15	15 15 16 16 16	17 17 17 18 18
14	10 10 10 11 11	11 12 12 12 13	13 13 13 14 14	14 15 15 15 15	16 16 16 17 17
15	9 10 10 10 10	11 11 11 12 12	12 12 13 13 13	14 14 14 14 15	15 15 16 16 16
16	9 9 9 10 10	10 11 11 11 11	12 12 12 12 13	13 13 13 14 14	14 15 15 15 15
17	9 9 9 9 10	10 10 10 11 11	11 11 12 12 12	12 13 13 13 13	14 14 14 14 15
18	8 9 9 9 9	9 10 10 10 10	11 11 11 11 12	12 12 12 13 13	13 13 14 14 14
19	8 8 8 9 9	9 9 10 10 10	10 11 11 11 11	11 12 12 12 12	13 13 13 13 13
20	8 8 8 8 9	9 9 9 9 10	10 10 10 11 11	11 11 11 12 12	12 12 13 13 13
21	7 8 8 8 8	9 9 9 9 9	10 10 10 10 10	11 11 11 11 11	12 12 12 12 12
22	7 7 8 8 8	8 8 9 9 9	9 9 10 10 10	10 10 11 11 11	11 11 12 12 12
23	7 7 7 8 8	8 8 8 9 9	9 9 9 10 10	10 10 10 11 11	11 11 11 11 12
24	7 7 7 7 8	8 8 8 8 9	9 9 9 9 9	10 10 10 10 10	11 11 11 11 11
25	7 7 7 7 7	8 8 8 8 8	8 9 9 9 9	9 10 10 10 10	10 10 11 11 11
26	7 7 7 7 7	7 8 8 8 8	8 8 9 9 9	9 9 9 10 10	10 10 10 10 11
27	6 7 7 7 7	7 7 8 8 8	8 8 8 9 9	9 9 9 9 10	10 10 10 10 10
28	6 6 7 7 7	7 7 7 7 8	8 8 8 8 8	9 9 9 9 9	9 10 10 10 10
29	6 6 6 7 7	7 7 7 7 7	8 8 8 8 8	8 9 9 9 9	9 9 10 10 10
30	6 6 6 6 7	7 7 7 7 7	7 8 8 8 8	8 8 9 9 9	9 9 9 9 10
31	6 6 6 6 6	7 7 7 7 7	7 7 8 8 8	8 8 8 8 9	9 9 9 9 9
32	6 6 6 6 6	6 7 7 7 7	7 7 7 8 8	8 8 8 8 8	9 9 9 9 9
33	6 6 6 6 6	6 6 7 7 7	7 7 7 7 8	8 8 8 8 8	8 9 9 9 9
34	5 6 6 6 6	6 6 6 7 7	7 7 7 7 7	8 8 8 8 8	8 8 8 9 9
35	5 6 6 6 6	6 6 6 6 7	7 7 7 7 7	7 8 8 8 8	8 8 8 8 9
36	5 5 6 6 6	6 6 6 6 6	7 7 7 7 7	7 7 8 8 8	8 8 8 8 8
37	5 5 5 6 6	6 6 6 6 6	6 7 7 7 7	7 7 7 7 8	8 8 8 8 8
38	5 5 5 5 6	6 6 6 6 6	6 6 7 7 7	7 7 7 7 7	8 8 8 8 8
39	5 5 5 5 6	6 6 6 6 6	6 6 7 7 7	7 7 7 7 7	7 8 8 8 8
40	5 5 5 5 5	6 6 6 6 6	6 6 6 7 7	7 7 7 7 7	7 7 8 8 8
41	5 5 5 5 5	5 6 6 6 6	6 6 6 6 7	7 7 7 7 7	7 7 7 8 8
42	5 5 5 5 5	5 6 6 6 6	6 6 6 6 6	7 7 7 7 7	7 7 7 7 8
43	5 5 5 5 5	5 5 6 6 6	6 6 6 6 6	6 7 7 7 7	7 7 7 7 7
44	5 5 5 5 5	5 5 5 6 6	6 6 6 6 6	6 6 7 7 7	7 7 7 7 7
45	5 5 5 5 5	5 5 5 5 6	6 6 6 6 6	6 6 6 7 7	7 7 7 7 7
46	5 5 5 5 5	5 5 5 5 6	6 6 6 6 6	6 6 6 6 7	7 7 7 7 7
47	5 5 5 5 5	5 5 5 5 5	6 6 6 6 6	6 6 6 6 6	7 7 7 7 7
48	4 5 5 5 5	5 5 5 5 5	5 6 6 6 6	6 6 6 6 6	6 7 7 7 7
49	4 5 5 5 5	5 5 5 5 5	5 6 6 6 6	6 6 6 6 6	6 6 7 7 7
50	4 4 5 5 5	5 5 5 5 5	5 5 6 6 6	6 6 6 6 6	6 6 6 7 7

The probability is less than 5% that  $r$  or more points of a sample of size  $m$  lie outside the extreme values of a sample of size  $n$  if the samples are drawn randomly from the same population, whatever its distribution.

1% values of  $r$

$n \backslash m$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	23	24	25
4	—	—	—	—	—	—	—	—	—	11	12	13	14	15	16	17	17	18	19	20	21	22	23	24	
5	—	—	—	—	—	—	—	8	9	10	11	11	12	13	14	15	15	16	17	18	19	19	20	21	22
6	—	—	—	—	6	7	8	9	9	10	11	11	12	13	14	14	15	16	16	17	18	19	19	20	
7	—	—	—	5	6	7	7	8	9	9	10	11	11	12	13	13	14	15	15	16	17	17	18	19	
8	—	—	—	5	6	6	7	8	8	9	9	10	11	11	12	12	13	14	14	15	15	16	17	17	
9	—	—	4	5	6	6	7	7	8	8	9	10	10	11	11	12	12	13	13	14	15	15	16	16	
10	—	—	4	5	5	6	6	7	7	8	9	9	10	10	11	11	12	12	13	13	14	14	15	15	
11	—	—	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	12	13	13	14	14	
12	—	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	11	11	12	12	13	13	14
13	—	3	4	4	5	5	6	6	7	7	7	8	8	9	9	10	10	10	11	11	12	12	13	13	
14	—	3	4	4	5	5	6	6	6	7	7	8	8	8	9	9	10	10	10	11	11	12	12	12	
15	—	3	4	4	5	5	5	6	6	7	7	7	8	8	9	9	9	10	10	10	11	11	12	12	
16	—	3	4	4	4	5	5	6	6	6	7	7	7	8	8	9	9	9	10	10	10	11	11	11	
17	—	3	4	4	4	5	5	5	6	6	7	7	7	8	8	8	9	9	9	10	10	10	11	11	
18	—	3	3	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10	10	11	
19	—	3	3	4	4	5	5	5	6	6	6	7	7	7	8	8	8	8	9	9	9	10	10	10	
20	—	3	3	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8	8	9	9	9	10	10	
21	—	3	3	4	4	4	5	5	5	6	6	6	6	7	7	7	8	8	8	9	9	9	9	10	
22	—	3	3	4	4	4	5	5	5	5	6	6	6	7	7	7	7	8	8	8	9	9	9	9	
23	—	3	3	4	4	4	4	5	5	5	6	6	6	6	7	7	7	8	8	8	8	9	9	9	
24	2	3	3	4	4	4	4	5	5	5	5	6	6	6	7	7	7	7	8	8	8	8	9	9	
25	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	7	7	7	7	7	8	8	8	9	
26	2	3	3	3	4	4	4	4	5	5	5	6	6	6	6	7	7	7	7	7	8	8	8	8	
27	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	7	8	8	8	
28	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	7	8	8	8	
29	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	7	8	8	8	
30	2	3	3	3	4	4	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	8	8	8	
31	2	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	
32	2	3	3	3	3	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	
33	2	3	3	3	3	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	
34	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	7	
35	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	7	
36	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	7	
37	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	
38	2	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	6	6	6	6	6	6	7	7	
39	2	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	6	6	6	6	6	6	7	7	
40	2	2	3	3	3	3	4	4	4	4	4	4	5	5	5	5	6	6	6	6	6	6	6	7	
41	2	2	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	6	6	6	6	6	6	6	
42	2	2	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	
43	2	2	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	
44	2	2	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6	
45	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	6	
46	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	6	
47	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	6	
48	2	2	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	
49	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	
50	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6	

The probability is less than 1% that  $r$  or more points of a sample of size  $m$  lie outside the extreme values of a sample of size  $n$  if the samples are drawn randomly from the same population, whatever its distribution.

1% values of  $r$

$m \backslash n$	26 27 28 29 30	31 32 33 34 35	36 37 38 39 40	41 42 43 44 45	46 47 48 49 50
2	— — — — —	— — — — —	— — — — —	— — — — —	— — — — —
3	26 27 28 29 30	31 32 33 34 35	36 37 38 39 39	40 41 42 43 44	45 46 47 48 49
4	24 25 26 27 28	29 30 30 31 32	33 34 35 36 37	37 38 38 40 41	42 42 43 44 45
5	22 23 24 25 26	26 27 28 29 29	30 31 32 33 33	34 35 36 37 37	38 39 40 40 41
6	21 21 22 23 24	24 25 26 26 27	28 29 29 30 31	31 32 33 33 34	35 36 36 37 38
7	19 20 20 21 22	22 23 24 24 25	26 26 27 28 28	29 30 30 31 31	32 33 33 34 35
8	18 18 19 20 20	21 21 22 23 23	24 24 25 26 26	27 27 28 29 29	30 30 31 31 32
9	17 17 18 18 19	19 20 21 21 22	22 23 23 24 24	25 25 26 27 27	28 28 29 29 30
10	16 16 17 17 18	18 19 19 20 20	21 21 22 22 23	23 24 24 25 25	26 26 27 27 28
11	15 15 16 16 17	17 18 18 19 19	20 20 21 21 21	22 22 23 23 24	24 25 25 26 26
12	14 15 15 15 16	16 17 17 18 18	19 19 19 20 20	21 21 22 22 23	23 23 24 24 25
13	13 14 14 15 15	16 16 16 17 17	18 18 18 19 19	20 20 21 21 21	22 22 23 23 23
14	13 13 14 14 14	15 15 16 16 16	17 17 18 18 18	19 19 20 20 20	21 21 21 22 22
15	12 13 13 13 14	14 15 15 15 16	16 16 17 17 18	18 18 19 19 19	20 20 20 21 21
16	12 12 13 13 13	14 14 14 15 15	15 16 16 16 17	17 17 18 18 19	19 19 20 20 20
17	11 12 12 12 13	13 13 14 14 14	15 15 15 16 16	16 17 17 17 18	18 18 19 19 19
18	11 11 12 12 12	13 13 13 14 14	14 15 15 15 15	16 16 16 17 17	17 18 18 18 19
19	11 11 11 12 12	12 12 13 13 13	14 14 14 15 15	15 16 16 16 16	17 17 17 18 18
20	10 11 11 11 11	12 12 12 13 13	13 14 14 14 14	15 15 15 16 16	16 16 17 17 17
21	10 10 11 11 11	11 12 12 12 13	13 13 13 14 14	14 14 15 15 15	16 16 16 16 17
22	10 10 10 10 11	11 11 12 12 12	12 13 13 13 13	14 14 14 15 15	15 15 16 16 16
23	9 10 10 10 10	11 11 11 12 12	12 12 13 13 13	13 14 14 14 14	15 15 15 15 16
24	9 9 10 10 10	10 11 11 11 11	12 12 12 12 13	13 13 13 14 14	14 14 15 15 15
25	9 9 9 10 10	10 10 11 11 11	11 12 12 12 12	13 13 13 13 14	14 14 14 15 15
26	9 9 9 9 10	10 10 10 11 11	11 11 12 12 12	12 12 13 13 13	13 14 14 14 14
27	9 9 9 9 9	10 10 10 10 11	11 11 11 11 12	12 12 12 13 13	13 13 14 14 14
28	8 9 9 9 9	9 10 10 10 10	11 11 11 11 11	12 12 12 12 13	13 13 13 13 14
29	8 8 9 9 9	9 9 10 10 10	10 11 11 11 11	11 12 12 12 12	12 13 13 13 13
30	8 8 8 9 9	9 9 9 10 10	10 10 11 11 11	11 11 12 12 12	12 12 13 13 13
31	8 8 8 8 9	9 9 9 9 10	10 10 10 10 11	11 11 11 11 12	12 12 12 12 13
32	8 8 8 8 8	9 9 9 9 9	10 10 10 10 10	11 11 11 11 11	12 12 12 12 12
33	8 8 8 8 8	9 9 9 9 9	9 10 10 10 10	10 11 11 11 11	11 12 12 12 12
34	7 8 8 8 8	8 9 9 9 9	9 9 10 10 10	10 10 11 11 11	11 11 12 12 12
35	7 7 8 8 8	8 8 9 9 9	9 9 9 10 10	10 10 10 11 11	11 11 11 11 12
36	7 7 8 8 8	8 8 8 9 9	9 9 9 9 10	10 10 10 10 11	11 11 11 11 11
37	7 7 7 8 8	8 8 8 8 9	9 9 9 9 9	10 10 10 10 10	11 11 11 11 11
38	7 7 7 7 8	8 8 8 8 8	9 9 9 9 9	10 10 10 10 10	10 11 11 11 11
39	7 7 7 7 8	8 8 8 8 8	9 9 9 9 9	9 10 10 10 10	10 10 10 11 11
40	7 7 7 7 7	8 8 8 8 8	8 9 9 9 9	9 9 10 10 10	10 10 10 11 11
41	7 7 7 7 7	7 8 8 8 8	8 8 9 9 9	9 9 9 10 10	10 10 10 10 10
42	7 7 7 7 7	7 8 8 8 8	8 8 8 9 9	9 9 9 9 10	10 10 10 10 10
43	6 7 7 7 7	7 7 8 8 8	8 8 8 8 9	9 9 9 9 9	10 10 10 10 10
44	6 7 7 7 7	7 7 7 8 8	8 8 8 8 8	9 9 9 9 9	9 10 10 10 10
45	6 6 7 7 7	7 7 7 7 8	8 8 8 8 8	9 9 9 9 9	9 9 10 10 10
46	6 6 7 7 7	7 7 7 7 8	8 8 8 8 8	8 9 9 9 9	9 9 9 10 10
47	6 6 6 7 7	7 7 7 7 7	8 8 8 8 8	8 8 9 9 9	9 9 9 9 10
48	6 6 6 7 7	7 7 7 7 7	7 8 8 8 8	8 8 8 9 9	9 9 9 9 9
49	6 6 6 6 7	7 7 7 7 7	7 8 8 8 8	8 8 8 8 9	9 9 9 9 9
50	6 6 6 6 6	7 7 7 7 7	7 7 8 8 8	8 8 8 8 8	9 9 9 9 9

The probability is less than 1% that  $r$  or more points of a sample of size  $m$  lie outside the extreme values of a sample of size  $n$  if the samples are drawn randomly from the same population, whatever its distribution.