It is clear from the definitions (5) that both $u(x)$ and $v(x)$ are bounded and nonincreasing functions of x, and hence by (6) and (2)

$$\Pr[t_k \leqq T_k \text{ for all } k = 1, \cdots, n] \geqq \int_{-\infty}^{\infty} u(x)\, dF_1(x) \cdot \int_{-\infty}^{\infty} v(x)\, dF_1(x)$$

$$= \Pr[T_k \leqq t_k \text{ for all } k \,\varepsilon\, A] \cdot \Pr[T_k \leqq t_k \text{ for all } k \,\varepsilon\, B],$$

which proves (3).

THEOREM. *If $A_1, \cdots, A_r$ form a partition of the set of integers $1, \cdots, n$ into any number r of disjoint subsets, then*

$$\Pr[T_k \leqq t_k \textit{ for all } k = 1, \cdots, n] \geqq \prod_{j=1}^{r} \Pr[T_k \leqq t_k \textit{ for all } k \,\varepsilon\, A_j].$$

In particular, setting $r = n$, $A_j = \{j\}$, $j = 1, \cdots, n$, (1) holds.

PROOF. Induction on r, using Lemma 2.

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Pasadena meeting of the Institute, June 18–19, 1954)

1. The Integral of a Symmetric Unimodal Function over a Symmetric Convex Set and Some Probability Inequalities. T. W. ANDERSON, Columbia University and Stanford University.

The integral over an interval of fixed length of a symmetric unimodal function is maximized if the interval is centered at the origin; in fact, the value of the integral is a nonincreasing function of the distance of the midpoint of the interval from the origin. A generalization of this result to n-space is the following: *Theorem* 1. *Let E be a convex set in n-space, symmetric about the origin. Let $f(x) \geqq 0$ be a function such that $f(x) = f(-x)$, $\{x \mid f(x) \geqq u\}$ is convex for every $u(0 \leqq u \leqq \infty)$, and $\int_E f(x)\, dx < \infty$ (in the Lesbesgue sense). Then $\int_E f(x + ky)\, dx \geqq \int_E f(x + y)\, dx$ for $0 \leqq k \leqq 1$.* A direct consequence is that the distribution of $X + Y$ is more spread out than the distribution of X. *Theorem* 2. *Let X be a random vector with density $f(x)$ satisfying the conditions of Theorem* 1; *let Y be an independent random vector; and let E be a convex set, symmetric about the origin. Then* $\Pr \{X + kY \,\varepsilon\, E\} \geqq \Pr \{X + Y \,\varepsilon\, E\}$ *for* $0 \leqq k \leqq 1$. Inequalities are derived for distributions of functions of random variables such as ΣX_i^2 and $\max_{1 \leqq i \leqq n} |X_i|$ and corresponding functionals of stochastic processes. Another application is to show that certain tests of location parameters are unbiased. (Work supported by the Office of Naval Research.)

2. The Spectral Method of Hypothesis Testing Concerning Continuous Gaussian Stationary Random Processes. R. C. DAVIS, Hughes Tool Company.

Present rigorous methods of hypothesis testing concerning Gaussian stationary random functions depending upon a continuous parameter—in which a process is observed only during a finite time interval of duration T—have been based upon an analysis carried out in the time domain. In order to determine the sample decision function by this method for testing even a simple hypothesis against a single alternative, it is necessary to solve

two homogeneous integral equations of the second kind. On the other hand, by using Cramér's representation of a continuous stationary process (possessing a continuous covariance function) and obtaining by convolution the resultant spectrum obtained in finite observation time, the likelihood ratio can be obtained explicitly without solving any integral equations. Moreover, the spectral method is useful in making approximate calculations when one is limited by practical considerations to the obtaining of "information" only in a finite pass band of frequencies.

3. Note on the Distribution of a Definite Quadratic Form. JAMES PACHARES, Naval Air Missile Test Center, Point Mugu.

An expression is derived for the distribution of a definite quadratic form in independent $N(0, 1)$ variates which depends only on the value of the determinant of the form and on the moments of a quadratic form whose matrix is the inverse of the original quadratic form. This expression is an alternating series which converges absolutely and is such that if we stop with any even power of the series we have an upper bound and if we stop with any odd power of the series a lower bound to the cumulative distribution function. If $Q_n = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij} x_i x_j$, $Q_n^* = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}^{-1} x_i x_j$, where $x_1, \cdots, x_n$ are independent $N(0, 1)$ variates, $a_{ij} > 0, i, j = 1, \cdots, n$, and $F_n(t) = \Pr(Q_n \leqq t)$, then $F_n(t) = t^{n/2} \mid A \mid^{-1/2} \sum_{k=0}^{\infty} (-t)^k E(Q_n^*)^k / k! \, \Gamma(k + 1 + n/2)$, where $\mid A \mid$ is the determinant of the matrix (a_{ij}), and $E(Q_n^*)^k$ is the kth moment of Q_n^*.

4. On Simultaneous Analysis of Variance Test. (By Title.) K. V. RAMACHANDRAN, University of North Carolina.

In situtations involving the testing of the significance of k mean squares the usual method of analysis gives tests which are not independent. Instead of combining these hypotheses and using an F test, one would prefer to make simultaneous decisions regarding the hypotheses. In these situations a simultaneous analysis of variance test has been proposed by M. N. Ghosh. In this paper we have shown that the power of this test is a monotonic function of the deviation parameters. If s_i^2 is the mean square corresponding to the hypothesis H_i ($i = 1, 2, \cdots, K$) then in the case when all s_i^2's are based on the same number of degrees of freedom ($d. f.$) n, it is shown how the simultaneous test could be carried out using the distribution of the Studentized largest chi-square $u_n = [\max(s_1^2, s_2^2, \cdots, s_k^2)]/s_0^2$ where s_0^2 is an independent chi-square variable with m $d. f.$ The exact distribution of u_n has been obtained and upper 5 per cent points tabulated. In the general case when n_i is the number of $d. f.$ associated with s_i^2, certain recurrence relations have been obtained to facilitate the computation of the percentage points.

5. The Optimum Character of a Certain Wald Sequential Test. J. V. BREAKWELL, North American Aviation, Inc. Introduced by T. E. Harris.

A recent paper by the author in the Journal of the Operations Research Society of America contains formulas and graphs with the aid of which the parameters of a Wald-type sequential test for the fraction of defectives may be chosen so as to minimize the maximum risk, the loss functions and the cost of test being assumed to have certain linear forms. This "optimization" was carried out only with the aid of certain approximations, valid for large average sample numbers, equivalent to regarding the random walk associated with the sequential test as a continuous curve.

The present paper demonstrates that the first variations of both acceptance-probability and average sample number corresponding to arbitrary (small) deformations of the straight boundaries of the continuous random walk are identical with those corresponding to parallel shifts of these boundaries by certain weighted average deformations. The paper proceeds to demonstrate that the first variation in maximum risk is zero when the boundaries undergo arbitrary deformations accompanied by an appropriate tilting of both boundaries to insure that the maximum risk associated with rejecting a good product balances that due to accepting a poor product.

This result is a strong indication that the optimum Wald-type sequential test is optimum among all possible sequential tests in the light of the assumed risk function and to the extent that the continuous random walk is a valid approximation.

6. An Optimum Decision Procedure for Ranking Means of Normal Populations. (By Title.) K. C. SEAL, University of North Carolina.

Among the infinite class of decision procedures (as suggested in "On a property of a class of decision procedures for ranking means of normal populations" (abstract), Eastern Regional meeting, Gainesville, Florida, March, 1954) it is shown that the optimum rule may be taken to correspond to $c_i = 1/n$ $(i = 1, \cdots, n)$. This rule has the following desirable property: If among $(n + 1)$ given normal populations n populations are identical with $N(\xi, \sigma^2)$ and if the remaining population is $N(\xi + \delta, \sigma^2)$ then the probability of (i) either retaining the most desirable population $N(\xi + \delta, \sigma^2)$ when $0 < \delta < \infty$ in the selected group, (ii) or rejecting the most undesirable population $N(\xi + \delta, \sigma^2)$ when $-\infty < \delta < 0$ from the group is approximately maximum for the above rule. (This research was supported in part by the United States Air Force under Contract AF 18(600)-83.)

7. On the Central Limit Theorem for d_n Dependent Variables. (By Title.) P. H. DIANANDA, University of North Carolina and University of Malaya.

Let $(A)\{X_{n,i}\}(i = 1, \cdots, \nu_n; n = 1, 2, \cdots)$ be a double sequence of random variables ($r. v.$'s). If the $r. v.$'s $(X_{n,1}, \cdots, X_{n,a})$ and $(X_{n,b}, \cdots, X_{n,\nu_n})$ are independent whenever $b - a > d_n$, then the sequence (A) is said to be d_n-dependent. Let $S_n = X_{n,1} + \cdots + X_{n,\nu_n}$, $s'_n = E(S_n^2)$, $s_n = E(X_{n,1}^2 + \cdots + X_{n,\nu_n}^2)$ and $d'_n = d_n + 1$. Suppose that (A) is a d_n-dependent sequence of $r. v.$'s with zero means and finite variances such that as $n \rightarrow \infty$ (1) $\liminf (s'_n/s_n\sqrt{d'_n} > 0$ and (2) for every fixed $\epsilon > 0$, $(1/s_n'^2)\sum_{i=1}^{\nu_n} \int_{|x|>\epsilon s_n/\sqrt{d'_n}} x^2 \, dF_{n,i}(x) \rightarrow 0$, where $F_{n,i}(x)$ is the distribution function of $X_{n,i}$ $(i = 1, \cdots, \nu_n; n = 1, 2, \cdots)$. Then S_n/s'_n is asymptotically distributed as a standardized normal variable. This result improves the main result of an earlier paper, "On the central limit theorem for m-dependent variables," (abstract to appear in *Ann. Math. Stat.*) in three directions: (i) bounded variances are replaced by finite variances, (ii) m-dependent $r. v.$'s are replaced by d_n-dependent $r. v.$'s and (iii) the sequence $X_1, X_2, \cdots$, is replaced by the sequence (A). In proving this theorem and its vector analogue the methods of the previous paper are used in modified forms. In the case of independent $r. v.$'s $(d_n = 0)$ (1) is automatically satisfied and (2) reduces to the Lindeberg condition. The main theorem is thus an extension of Lindeberg's theorem.

8. Estimation of a Selection Function. (Preliminary Report.) DOUGLAS G. CHAPMAN, University of Washington.

Let X, Y be random variables with probability density functions $f(x)$, $r^{-1}\varphi(x)f(x)$ respectively where $0 \leqq \varphi(x) \leqq 1$ and $r = \int_{-\infty}^{\infty} \varphi(x)f(x)\,dx$. $\varphi(x)$ is known as a "selection function". Numerous studies have been made about "selected" distributions where the selection function is unknown and reduces to the characteristic function of a semi-infinite interval ("truncation" on the right or left) or where the selection function is determined by the experimenter. In many situations the selection function is unknown and the statistician must obtain point or interval estimates of φ from $n + m$ independent observations $(x_1 \cdots x_n \; y_1 \cdots y_m)$ of (X, Y). Large sample procedures are derived for such estimates under several assumptions as to the functional form of φ and of f. The estimation of r, the degree of selection is also studied. Consideration is also given to the problem of estimating the selection function for grouped data.

9. On the Power of a Distribution-free One-Sided Test of Fit Against Stochastically Comparable Alternatives. (Preliminary Report.) Z. W. BIRNBAUM AND ERNEST M. SCHEUER, University of Washington.

Let the hypothesis H and the alternative G be continuous cumulative distribution functions, and F_n the empirical distribution function corresponding to a sample of size n of a random variable with distribution G. It is known that, if $H = G$, the statistic $D_n^+ = \sup_{(s)} \{H(s) - F_n(s)\}$ has a probability distribution independent of H. In the present paper a sharp lower bound is obtained for the power of a test of fit based on D_n^+, for the class of alternatives G such that $\sup_{(s)} \{H(s) - G(s)\} = \delta$ is a given positive number and $H(s) \geqq G(s)$ for all s.

10. A Statistical Method of Determining Relationships between Test Specification Limits and Performance Specification Limits. BERL D. LEVENSON, Hughes Aircraft Company.

In many production processes the problem arises of determining whether or not a particular parameter of a number of similar devices conforms to a predetermined performance specification. When sufficiently accurate measurement techniques are available, performance characteristics may be determined quite readily. However, when sizable errors are introduced by measurement techniques, the determination of performance characteristics obviously becomes more complex. In many of these instances the *measured values* of the parameters are classified in accordance with *test specification limits* with the objective of properly classifying the *actual values* of the parameters relative to *performance specification limits*. It is the purpose of this article to explore the nature of the relationships which arise as a result of this type of test procedure. The discussion is confined to the problem of measurement of a single parameter. A three dimensional geometric analog has been devised as an aid in visualizing the general as well as specific solutions to the problem. Observations have been made based upon specific calculations for some special cases with normal distributions. It is shown that in many instances it may be highly desirable to establish test specification limits outside of performance specification limits.

11. A Method of Specification, Testing, and Evaluation of Missile Systems. E. J. ALTHAUS, S. C. MORRISON, AND W. R. TATE, Hughes Aircraft Company.

In tests of complex systems, test errors may be of the order of magnitude of the tolerances on the parameter being measured. Such large errors not only make uncertain the quality of accepted product, but also cause inefficiency through rejection of good product in test.

A procedure is discussed for minimizing both these adverse effects. Rejection of good product in test is reduced by the use of relaxed test tolerances, while quality is assured by tight adjustment tolerances and by quality control analysis of test data. Specification in statistical terms and advance estimation of test errors are considered part of a planned production scheme.

Characteristic aspects of the program are the following: (1) Production procedure is set up on the basis of statistical predictions of error distributions. (2) Test error (including component instability) is quantitatively made a part of quality control.

12. Discovery Sampling. JAMES R. CRAWFORD, Lockheed Aircraft Corporation.

A new method of acceptance sampling by use of the range of the truncated range of a small sample is developed. The purpose is to take advantage of the increased information afforded by variables inspection yet minimize the knowledge of mathematics needed by the inspector. The technique under investigation is that of adding and subtracting the value of the sample range, or a portion thereof, to its upper and lower values, respectively. If the sum and difference are still within the prescribed tolerances the lot is accepted. Plans of this type are found to yield approximately the same results as attribute sampling plans requiring twice the sample size. The inspector need only know addition and subtraction for their use.

13. **Evaluation of Quality through Demerit Rating System.** Harry G. Romig, International Telemeter Corporation.

Where inspection and tests are made for a series of specified requirements, these results must be properly analysed to obtain maximum benefits. Such requirements cover characteristics and other features, termed Inspection Items. When such Items are inspected by the Method of Attributes it is economical and efficient to classify them with respect to their importance or seriousness into definite classes, such as Critical, Major, Minor and Incidental. Various classifications, such as three-fold, four-fold, and five-fold, with their assigned Demerit Weights are discussed and the mathematical relations pertaining to their use are developed. Various uses of these systems in evaluating the quality of different processes, products and activities are presented. The nature of these distributions depicted by the multinomial and approximations thereto are described. It is shown how to use Demerits, Demerits-per-Unit and Indexes for single components, subassemblies, assemblies and Systems, as well as shops and composite plants for evaluating performance quality-wise. Various weighting systems are introduced and evaluated. Procedures for setting up control charts with prescribed limits are given. Finally it is shown how to combine variables results with attributes data to obtain over-all quality ratings for any desired sequence of operations.

14. **On Structural Fatigue under Random Loading.** John W. Miles, Department of Engineering, UCLA, and Douglas Aircraft Company.

Experience has shown that the fluctuating loads induced by a jet may cause fatigue failure of aircraft structural components. In order to throw some light on this and similar problems, the stress spectrum and the "equivalent fatigue stress" of an elastic structure subjected to random loading are studied. The analysis is simplified by assuming the structure to have only a single degree of freedom and by using the concept of cumulative damage, the results being expressed in terms of quantities that can be directly measured. As an example, a similarity expression for the probable value of the equivalent fatigue stress of a panel subjected to jet buffeting is derived.

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest

Personal Items

P. C. Clark has been appointed Executive Vice-President of Hunter Spring Company.

Dr. Edward P. Coleman, formerly Visiting Professor, has been appointed Professor in the Department of Engineering, University of California at Los Angeles.

Dr. R. N. Bradt of Stanford University has been appointed to an assistant professorship at the University of Kansas.