From well known limit theorems,

$$\lim_{\lambda\to\infty} A_\lambda = \lim_{\lambda\to\infty} \Pr\{X_\lambda \leqq \lambda\} = \lim_{\lambda\to\infty} \Pr\left\{\frac{X_\lambda - \lambda}{\sqrt{\lambda}} \leqq 0\right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-y^2/2}\, dy = \tfrac{1}{2}. \tag{9}$$

The second portion of inequality (2) follows directly from (8) and (9). The remainder is a consequence of these, of the fact that $A_{n,\lambda}$ decreases monotonically from $A_{n,n}$ to $A_{n,n+1}$ in the interval $[n, n+1)$, and of

$$A_{n,n+1} > A_{0,1} = e^{-1}, \qquad n = 1, 2, \cdots . \tag{10}$$

If an additional term is included in the sum, we note that

$$b_\lambda = \sum_{j=0}^{[\lambda]+1} \frac{\lambda^j e^{-\lambda}}{j!} \geqq \tfrac{1}{2} \qquad \text{for all } \lambda \geqq 0, \tag{11}$$

since $b_\lambda > A_{n+1,n+1}$ for $\lambda \leqq n + 1$. A final reformulation of part of (2) is

$$\Pr\{X_\lambda \leqq \lambda\} > \Pr\{X_\lambda > \lambda\}, \qquad \text{all integral } \lambda > 0. \tag{12}$$

NOTE ON LINEAR HYPOTHESES WITH PRESCRIBED MATRIX OF NORMAL EQUATIONS

By John E. Maxfield and Robert S. Gardner

Naval Ordnance Test Station Inyokern,

The existence theorem proven in this note relates to the problem of finding an experimental design leading to the analysis determined by the given rational matrix A of the normal equations. The matrix B found by the method used in the proof always has an interpretation as specifying the rational values of some set of regression variables. In the interesting case in which the entries of A are integers, so are the entries of B, but B is not in general interpretable as an analysis of variance. The transpose of a matrix A will be denoted by A^T.

Theorem. *Let A be a symmetric positive semidefinite matrix with rational integral entries. There exist a rational integer a and a matrix B having rational integral entries such that $BB^T = a^2A$.*

Proof. There exists a nonsingular matrix P such that $P^TAP = D$, a diagonal matrix, where P and D have rational entries ([1], p. 56). Then $(P^T)^{-1}$ has rational entries. Let a_1 be the least common denominator of the entries of P^T, a_2 of $(P^T)^{-1}$, and a_3 of D. Let $a^{1/2} = a_1a_2a_3$. Then $a^{1/2}P^Ta^{1/2}AP = aD$. If A is positive semidefinite, then aD has only positive integers or zeros on its diagonal. Let B_1 be a $n \times 4n$ matrix composed of four diagonal $n \times n$ matrices placed side by side,

Received February 8, 1954.

$(S_1S_2S_3S_4)$, such that $ad_{ii} = \sum_{k=1}^{4} {}_ks_{ii}^2$, where ${}_ks_{ii}$ is the entry in the ith column of S_k. This can always be done by the 4 square problem ([2], p. 235). Let $B = a^{1/2}(P^T)^{-1}B_1$, then B has integral entries, since B_1 and $a^{1/2}(P^T)^{-1}$ have integral entries. Then

$$P^TB = a^{1/2}B_1, \qquad P^TBB^TP = aB_1B_1^T = a^2D,$$

Thus $BB^T = a(P^T)^{-1}a^{1/2}Da^{1/2}P^{-1}$. But $A = (P^T)^{-1}DP^{-1}$. Thus $BB^T = a^2A$.

REFERENCES

[1] C. C. MacDuffee, *The Theory of Matrices*, Chelsea Publishing Co., 1946.
[2] Edmund Landau, *Vorlesungen uber Zahlentheorie*, Vol. 1, Chelsea Publishing Co., 1949.

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Iowa City Meeting of the Institute, November 26–27, 1945)

1. The Distribution of the Number of Components of a Random Mapping Function. (Preliminary Report.)

Leo Katz, Michigan State College, and Jay E. Folkert, Michigan State College and Hope College.

A set H, of N elements, is mapped into itself by a function f. The most general function takes each point into an arbitrary number of points of the set. A function is said to be random if the r_i images of the point x_i may with equal probability be any subset of r_i points of H. A subset h of H is a *component* of the mapping if it is a minimal subset such that $f(h) \subset h$ and $f^{-1}(h) \subset h$. Every mapping f decomposes H into a number of disjoint components. The probability distribution of the number of components of a random mapping, where only the numbers of images of each point are known in advance, is obtained. The probability distribution of the number of components is also obtained for a variant case in which the mapping is hollow in the sense that no point maps into itself. The two distributions are obtained through a modification of the King-Jordan-Frechét formula. For each case two specializations are considered; first, one in which the multiplicity of images is the same for each point of the set, and second, where this common multiplicity is unity (so that the function f is single-valued). Numerical examples and approximations to the exact distribution are considered. This work was supported by the Office of Naval Research.

2. Approximate Sequential Tests for Hypotheses about the Proportion of a Normal Population to One Side of a Given Number.

William Kruskal, University of Chicago.

It is sometimes of interest to test the hypothesis that the proportion of a given population exceeding a given number U is p_0 against the hypothesis that this proportion is p_1. This testing situation has been called that of testing for one-sided fraction defective. If the population is normal then the problem is to test the hypothesis $(U - \mu)/\sigma = K_0$ against the hypothesis $(U - \mu)/\sigma = K_1$. (Here μ is the mean, σ^2 the variance, and K_i the unit-normal deviate exceeded with probability p_i.) A simple translation puts this in the form: $\mu/\sigma = K_0$ vs. $\mu/\sigma = K_1$. If a sequential test is desired, it is very reasonable to base it on the sequence of Student t values computed from the first n observations. Application of the Wald sequential probability-ratio method to this sequence gives a procedure that may be called the WAGR test (after Wald, Arnold, Goldberg, and Rushton). Another sequential test