or $[f^*(a', b', w_3, \cdots, w_n), g^*(a', b', w_3, \cdots, w_n)]$, is the center of gravity of the points (a, b) for which the cube C, shifted to have center at $(a, b, 0, \cdots, 0)$, contains the point $(a', b', w_3, \cdots, w_n)$. However, it is equivalent to state that the estimate of (α, β) is the center of gravity of the points (a, b) for which the line $y = a + bx$ is a possible regression line for the observed points $(x_1, y_1), \cdots, (x_n, y_n)$, i.e., for which $y = a + bx$ is within $\delta/2$ vertically of each point $(x_1, y_1), \cdots, (x_n, y_n)$.

It is of interest to note that the estimates of α and β are $\bar{y}$ and $\sum y_i x_i / \sum x_i^2$ plus corrections which depend only on the deviations from the usual regression line. This is essentially the invariance requirement.

The methods of Section 3 up to formulas (7) and (8) may be applied in much the same manner to any regression problem for which the errors are a sample from some given fixed distribution.

REFERENCE

[1] David Blackwell and M. A. Girshick, *Theory of Games and Statistical Decisions*, John Wiley & Sons, New York, 1954.

ON DISCRETE VARIABLES WHOSE SUM IS ABSOLUTELY CONTINUOUS[1]

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1. Summary. If $\{Z_n\}$, $n = 1, 2, \ldots$ is a stationary stochastic process with D states $0, 1, \cdots, D - 1$, and $X = \sum_1^\infty Z_k/D^n$, Harris [1] has shown that the distribution of X is absolutely continuous if and only if the Z_n are independent and uniformly distributed over $0, 1, \cdots, D - 1$, i.e., if and only if the distribution of X is uniform on the unit interval. In this note we show that if $\{Z_n\}$, $n = 1, 2, \cdots$ is any stochastic process with D states $0, 1, \cdots, D - 1$ such that $X = \sum_1^\infty Z_n/D^n$ has an absolutely continuous distribution, then the conditional distribution of $R_k = \sum_{n=1}^\infty Z_{k+n}/D^n$ given $Z_1, \cdots, Z_k$ converges to the uniform distribution on the unit interval with probability 1 as $k \to \infty$. It follows that the unconditional distribution of R_k converges to the uniform distribution as $k \to \infty$. Since if $\{Z_n\}$ is stationary the distribution of R_k is independent of k, the result of Harris follows.

2. Proof of the theorem.

Theorem. *If $\{Z_n\}$, $n = 1, 2, \cdots$ is a sequence of random variables, each assuming only values $0, 1, \cdots, D - 1$ such that $X = \sum_1^\infty Z_n/D^n$ has an absolutely continuous distribution, and*

$$0 < \lambda \leqq 1, \textit{ then } U_k(\lambda) = P\left(\sum_1^\infty Z_{k+n}/D^n < \lambda \mid Z_1, \cdots, Z_k\right) \to \lambda$$

with probability 1 *as* $k \to \infty$.

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PROOF. Say X has density p with respect to Lebesgue measure on the unit interval. Then

$$U_k(\lambda) = \lambda\, d(y_k(X), \lambda D^{-k})/d(y_k(X), D^{-k}),$$

where $y_k(s) = mD^{-k}$ for $mD^{-k} \leqq s < (m+1)D^{-k}$, $m = 0, 1, \cdots, D^k - 1$, and $d(a, h) = h^{-1} \int_a^{a+h} p(s)\, ds$.

We must show that

$$\lambda d(y_k(s), \lambda D^{-k})/d(y_k(s), D^{-k}) \to \lambda$$

for almost all s (Lebesgue measure) for which $p(s) > 0$, and this will follow from

$$(1) \qquad d(y_k(s), \lambda D^{-k}) \to p(s) \qquad \text{a.e.}$$

Now a basic theorem of real variable theory asserts that

$$(2) \qquad d(s, h) \to p(s) \qquad \text{a.e.}$$

as $h \to 0$. Let $a_k(s) = (s - y_k(s))/\lambda D^{-k}$

Then

$$\begin{aligned} d(y_k(s), \lambda D^{-k}) &= a_k(s)\, d(s, y_k(s) - s) + [1 - a_k(s)]\, d(s, y_k(s) + \lambda D^{-k} - s) \\ (3) \qquad &= a_k(s)[d(s, y_k(s) - s) - d(s, y_k(s) + \lambda D^{-k} - s)] \\ &\qquad + d(s, y_k(s) + \lambda D^{-k} - s). \end{aligned}$$

Since $a_k(s)$ is bounded, letting $k \to \infty$ in (3) and using (2) yields (1), and the proof is complete.

REFERENCE

[1] T. E. HARRIS, "On chains of infinite order," *Pacific J. Math.*, Vol. 5 (1955), pp. 707–724.

A PROOF THAT THE SEQUENTIAL PROBABILITY RATIO TEST (S.P.R.T.) OF THE GENERAL LINEAR HYPOTHESIS TERMINATES WITH PROBABILITY UNITY

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1. Introduction. It can be shown [1] [2] that the S.P.R.T. of the general linear hypothesis resolves itself into the following form of procedure: Continue sampling at stage (n) if

$$(1) \qquad \frac{\beta}{1-\alpha} < e^{-\lambda^{(n)}/2} M\left(\alpha(n), \gamma; \frac{\frac{1}{2}\lambda^{(n)} G^{(n)}}{1 + G^{(n)}}\right) < \frac{1-\beta}{\alpha} \cdots;$$

otherwise accept or reject the null hypothesis depending upon whether the left-hand or right-hand inequality is violated.

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