mum likelihood on the joint density, and from (1) we see that this obviously produces (2).

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ABSTRACTS OF PAPERS

(Additional abstracts of papers presented at the Washington meeting of the Institute, March 7–9, 1957)

1. The K-Visit Method of Consumer Testing, George E. Ferris, General Foods Corporation, (By Title).

When testing a pair of products for consumer preference the problem of how to treat or interpret no preference votes arises. A method is described of collecting data from a given number of consumers by repeated visits to them, or by obtaining repeated judgments from them in stores, which enables the estimation of the true proportion of those consumers in the population who have a preference for either product and of those who cannot discriminate or have no preference. For the model assumed, the maximum likelihood estimators of the above proportions are derived, their variance-covariance matrix is obtained, and a way of testing the appropriateness of the model is indicated. A decision theoretical formulation is suggested. (Received March 8, 1957; revised March 12, 1957.)

2. Factorial Treatments in Group Divisible Incomplete Block Designs, Clyde Y. Kramer and Ralph A. Bradley, Virginia Polytechnic Institute.

Methods of incorporating factorial treatment combinations in group divisible incomplete block designs are given. The factorial treatment combinations are so matched with the basic treatments in the association matrices of the designs that the sums of squares for the factorial effects can be obtained as functions of the original treatment estimators. It is shown first how a two-factor factorial may be incorporated into group divisible incomplete block designs. Single degree of freedom contrasts are obtained for the effects in much the usual way as for factorials in complete block designs. Multifactor factorials and partial factorials are discussed, and a method of obtaining estimates and tests of significance of the effects is given. (Received March 18, 1957.)

3. Iterative Experimentation, G. E. P. Box, Statistical Techniques Group, Princeton University.

Scientific research is usually an iterative process. The cycle: conjecture–design–experiment–analysis leads to a new cycle of conjecture–design–experiment–analysis, and so on. It is helpful to keep this picture of the experimental method in mind when considering statistical problems. Although this cycle is repeated many times during an investigation, the experimental environment in which it is employed and the techniques appropriate for design and analysis tend to change as the investigation proceeds. Broadly speaking, one or more of the following four phases can be detected in most investigations: (*a*) a screening phase in which an attempt is made to isolate the important variables; (*b*) a descriptive phase in which the effects of the variables and the positions of interesting regions of the space of the variables are empirically determined; (*c*) a phase leading from (*b*) to (*d*); (*d*) a theoretical phase in which an attempt is made to understand the actual mechanism of the process studied. The roles which statistical methods should properly play in assisting the iterative process at these various phases of experimentation were briefly discussed. (Received March 14, 1957.)

(Abstracts of papers for the Atlantic City meeting of the Institute, September 10–13, 1957)

4. On the Joint Estimation of the Spectra, Cospectrum and Quadrature Spectrum of a Two-dimensional Stationary Gaussian Process, NATHANIEL ROY GOODMAN, New York University (introduced by Leon H. Herbach).

The probability structure of a real two-dimensional stationary (zero mean) Gaussian process $[x(t), y(t)]$, $-\infty < t < \infty$, is specified (in the absolutely continuous case) by $f_{xx}(\lambda)$, $f_{yy}(\lambda)$ the spectral densities of the $x(t)$ and $y(t)$ processes respectively, $c(\lambda)$ the cospectral density, and $q(\lambda)$ the quadrature spectral density $(-\infty < \lambda < \infty)$. The paper treats the problem of jointly estimating (in a suitable sense) $f_{xx}(\lambda), f_{yy}(\lambda)$, $c(\lambda)$, $q(\lambda)$ from a finite part of a sample function of the $[x(t), y(t)]$, $-\infty < t < \infty$, process. An approximation to the joint sampling distribution of the estimators for $f_{xx}(\lambda), f_{yy}(\lambda)$, $c(\lambda)$, $q(\lambda)$ is obtained. This approximate sampling distribution, termed a complex Wishart distribution, serves as the starting point in the derivation of approximate sampling distributions of estimators for functions of $f_{xx}(\lambda), f_{yy}(\lambda)$, $c(\lambda)$, $q(\lambda)$. The paper was motivated by the need of experimenters in fields such as micrometeorology, oceanography, electrical engineering, and aeronautical engineering to statistically estimate "parameters" characterizing their particular physical systems and to treat the sampling variability of estimators for the "parameters." In many cases the "parameters" to be estimated are functions of the densities $f_{xx}(\lambda), f_{yy}(\lambda)$, $c(\lambda)$, $q(\lambda)$ of a real two-dimensional stationary (zero mean) Gaussian process. (Received March 29, 1957.)

5. The Significance Probability of the Smirnov Two-Sample Test, J. L. HODGES, JR., University of California, (By Title).

The approximation $\tilde{P}$ for the significance probability P of Smirnov's two-sample test, based on the asymptotic theorem published by Smirnov in 1939, has been shown (Drion) to be accurate when the sample sizes m and n are equal. For this case the error of $\tilde{P}$ is of order $1/n$ (Gnedenko). Example: $m = n = 12$, $P = 0.032$, $\tilde{P} = 0.034$. A small numerical investigation shows that the accuracy is much poorer when $m \neq n$. Example: $n = 12$, $m = 13, P = 0.024$, $\tilde{P} = 0.054$. An asymptotic theory is developed for $n \to \infty$ with $m - n > 0$ bounded. The error of $\tilde{P}$ is now of order $1/\sqrt{n}$, and to this order is oscillatory as a function of P. This implies that no expression of the simple kind recently published by Korolyuk can be correct. An auxiliary table makes easy the computation of accurate estimates for P when $m - n$ is small and $m \leqq 30$. (Received April 8, 1957).

6. Nonparametric Mean and Variance Estimation from Truncated Data, JOHN E. WALSH, Lockheed Aircraft Corporation.

This paper considers situations where a known number of the smallest values of a sample and a known number of the largest values have been truncated. The problem is to obtain an estimate of the population mean, an estimate of the standard deviation of this estimate of the mean, and an estimate of the population standard deviation. This paper derives a nonparametric estimate for each of these three cases. These estimates are approximately valid for most continuous statistical populations of practical interest when a small number of sample values are truncated and the sample size is not too small. The mean estimate consists of a linear function of the ordered values of the truncated sample, while each standard deviation estimate is the square root of a quadratic function of these observations. (Received April 10, 1957.)

7. Distinguishability of Sets of Distributions (The Case of Independent and Identically Distributed Chance Variables.), W. HOEFFDING, University of North Carolina, and J. WOLFOWITZ, Cornell University.

Let $\mathfrak{I}$ be a class of tests, based on a sequence of independent chance variables with the common distribution F (assumed to belong to a set $\mathfrak{F}$ of distributions), for testing whether F belongs to one of two disjoint subsets, $\mathcal{G}$ and $\mathcal{H}$, of $\mathfrak{F}$. We consider the cases where $\mathfrak{I}$ is either the class of all tests which terminate with probability one if $F \in \mathfrak{F}$, or the class of all fixed sample size tests, or one of several classes intermediate between these two. The sets $\mathcal{G}$ and $\mathcal{H}$ are said to be distinguishable in $\mathfrak{I}$ if, for every $\epsilon > 0$, there exists a test in $\mathfrak{I}$ such that the error probability is $<\epsilon$ for all $F \in \mathcal{G} \cup \mathcal{H}$. It is shown that if there exists a test in $\mathfrak{I}$ such that the sum of the maximum error probability in $\mathcal{G}$ and the maximum error probability in $\mathcal{H}$ is less than 1, then $\mathcal{G}$ and $\mathcal{H}$ are distinguishable in $\mathfrak{I}$. Sufficient conditions and necessary conditions for the distinguishability of two sets are expressed in terms of certain distance functions. Certain simple necessary conditions for distinguishability are found to be also sufficient if the class of distributions is suitably restricted. (Received May 20, 1957.)

8. An Extension of Box's Results on the Use of the F Distribution in Multivariate Analysis, SEYMOUR GEISSER AND SAMUEL W. GREENHOUSE, National Institute of Mental Health.

The mixed model in a 2-way analysis of variance is characterized by fixed classification, e.g. treatments, and a random classification, e.g. plots. Under the usual analysis of variance assumptions the proper error for the fixed effect is the fixed $\times$ random interaction component, and the resulting ratio has the F-distribution. If we have individuals instead of plots as the random component and the treatments are correlated, then Box has shown that one may still use the same F-ratio as before as a test of treatment effects; however, the F-ratio does not have the requisite F-distribution, but it can be shown that it is distributed approximately like an F-distribution but with modified degrees of freedom. Box did this for one group of individuals; the authors have extended the Box technique to g groups of individuals and give the modified F-distribution for the tests of treatment effects and treatment $\times$ group interaction effects. (Received May 24, 1957.)

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest

Personal Items

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