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A t-TEST FOR THE SERIAL CORRELATION COEFFICIENT

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Summary. Let r be the sample serial correlation coefficient computed from a sample of size N drawn from a serially correlated process with parameter ρ. It is shown that the statistic

$$t = \frac{(r - \rho)\sqrt{N + 1}}{\sqrt{1 - r^2}}$$

is approximately distributed as Student's t with $N + 1$ degrees of freedom.

Introduction. Let (x_t) be a discrete process satisfying the stochastic difference equation

$$x_t = \rho x_{t-1} + u_t \qquad (t = 1, 2, \cdots)$$

where the u's are NID (0, 1) and ρ is an unknown parameter. If, considering a sample of size N, we assume that $x_{N+1} = x_1$, then the distribution of the x's is uniquely determined by that of the u's and the x's are said to be circularly correlated. The parameter ρ is called the (circular) serial correlation coefficient and may be estimated by

$$r = \frac{\sum_{t=1}^{N} x_t\, x_{t+1}}{\sum_{t=1}^{N} x_t^2}, \qquad (x_{N+1} = x_1).$$

Leipnik [1] obtained the following as an approximate (say $N > 20$) distribution for r

$$f(x) = \frac{(1 - x^2)^{(N-1)/2}}{\mathrm{B}\left(\frac{1}{2}, \frac{N + 1}{2}\right)(1 + \rho^2 - 2\rho x)^{N/2}}$$

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The t-transformation. We shall now make the change of variable from r to t in Leipnik's distribution. This change of variable could be made in one step; however, it seems more appealing to make a series of preliminary transformations. Let

$$z = \text{arc sin } x,$$

$$y = (\sin z - \rho)/\cos z$$

$$u = \frac{(x - \rho)\sqrt{N+1}}{\sqrt{1 - x^2}} = y\sqrt{N+1}$$

The density function for t is then

$$f(u) = \frac{1}{\sqrt{N+1}\,\text{B}\left(1 + \frac{u^2}{N+1}\right)^{(N+2)/2}} - \frac{\rho u}{(N+1)\text{B}\left(1 + \frac{u^2}{N+1}\right)^{(N+2)/2}\sqrt{1 + \frac{u^2}{N+1} - \rho^2}}$$

$$= \text{(say) } s_{N+1}(u) + h(u),$$

where $\text{B} = \text{B}(1/2, [N+1]/2)$.

Applications. The function $s_{N+1}(u)$ is immediately recognized as the density function for Student's t distribution with $N + 1$ degrees of freedom. Since $h(u)$ is an odd function, probabilities associated with absolute t value may be read directly from a standard table of the t distribution. For example a symmetric 95% confidence interval for ρ will be of the form

$$r + t_{.025}\sqrt{\frac{1 - r^2}{N+1}} < \rho < r + t_{.975}\sqrt{\frac{1 - r^2}{N+1}}$$

where $t_{.025}$ and $t_{.975}$ are the 2.5 and 97.5 percentile points of the t distribution with $N + 1$ degrees of freedom.

Probabilities not associated with symmetric intervals about the origin will require the evaluation of integrals of $h(u)$. A basic probability which might be considered is

$$\text{Prob } (a < t < \infty) = \int_a^\infty f(u)\,du = \int_a^\infty s_{N+1}(u)\,du + \int_a^\infty h(u)\,du.$$

The integral of $s_{N+1}(u)$ may be found from a table of the t distribution and need not concern us here. The problem then is to calculate

$$R(a) = \int_a^\infty h(u)\,du.$$

With some manipulation it may be shown that

$$R(a) = -\frac{1}{2\rho^N} I_x\left(\frac{N+1}{2}, \frac{1}{2}\right),$$

where $x = \rho^2[1 + a^2/(N + 1)]^{-1}$ and $I_x(\frac{1}{2}(N + 1), \frac{1}{2})$ is Karl Pearson's notation for the Incomplete Beta-Function as tabled in [2].

In the preceding discussion it has been assumed that the mean of the process (x_t) is known to be zero. If the mean must be estimated from the sample, the serial correlation coefficient will be

$$r' = \frac{\sum_{t=1}^{N} (x_t - \bar{x})(x_{t+1} - \bar{x})}{\sum_{t=1}^{N} (x_t - \bar{x})^2}, \qquad \bar{x} = \frac{\sum_{t=1}^{N} x_t}{N}.$$

All of the results concerning r also hold true for r' with N degrees of freedom rather than $N + 1$.

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GROUPS AND CONDITIONAL MONTE CARLO

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Summary. The conditional Monte Carlo technique advanced by Tukey *et al.* [1, 2] has been explained in analytic terms by Hammersley [3]. This note offers an alternative explanation, wherein the group-theoretic aspect of the problem plays the dominant role. The method is illustrated on an example simpler than that treated in [1, 2].

The framework. Throughout this note X will be a random vector in euclidean n-space $\mathfrak{X}$, having distribution function G. F will denote a distribution function absolutely continuous with respect to G, with Radon-Nikodym derivative $dF/dG \equiv w$, so that

$$F(M) = \int_M w(x)\, dG(x)$$

for all Borel sets M, and

$$\int \varphi(x)\, dF(x) = \int \varphi(x)w(x)\, dG(x)$$

for Borel functions φ. It is standard in this situation to call w a *weight* and to say that X (drawn from G) *with weight* $w(X)$ *is a sample from* F; thus for Borel φ we have

$$E_G(\varphi(X)w(X)) = E_F(\varphi(X))$$

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