and von Mises tests. A comparison of the power functions of these tests would be of great interest. Almost nothing is known of the small-sample power of any of these tests. The large-sample power of the chi-square test is known. It is the author's conjecture that the limiting joint distribution of $Q(n)$ and $R_n(1; 1)$ is bivariate normal under the alternatives as well as under the hypothesis. If this conjecture could be proved, the asymptotic power of the proposed test would be known.

TABLES FOR OBTAINING NON-PARAMETRIC TOLERANCE LIMITS

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The general consideration of non-parametric tolerance limits had its origin with Wilks [10]. Wilks showed that for continuous populations, the distribution of P, the proportion of the population between two order statistics from a random sample, was independent of the population sampled, and was in fact a function only of the particular order statistics chosen. Wald [9] and Tukey [8] extended the method to multivariate populations, Tukey being responsible for the term "statistically equivalent block." Their work was extended further by Fraser [2], [3]. Murphy [4] presented graphs of minimum probable coverage by sample blocks determined by order statistics of a sample from a population with a continuous but unknown c.d.f. This note extends the results of Murphy, and tabularizes the results in a manner which eliminates or minimizes interpolation, particularly with respect to m, in a large number of cases. The form of Table I parallels the tables of Eisenhart, Hastay and Wallis [1] "Tolerance Factors for Normal Distributions."

Let P represent the proportion of the population between the r^{th} smallest and the s^{th} largest value in a random sample of n from a population having a continuous but unknown distribution function. Table I gives the largest value of $m = r + s$ such that we have confidence of at least that 100 P percent of the population lies between the r^{th} smallest and s^{th} largest in the sample. Note, that we may choose any $r, s \geqq 0$ such that $r + s = m$. We must, of course, decide upon the values of r and s independently of the observations in the sample. We obtain one-sided confidence intervals when we use $r = 0$ or $s = 0$ for a given m. The values of m are the largest such that

$$\gamma \leqq I_{1-P}(m, n - m + 1)$$

where I is the incomplete Beta function tabulated in [5] and [7].

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[1] Part of this work was done while the author was a guest worker at the National Bureau of Standards.

TABLE I

Values of $m = r + s$ such that we may assert with confidence at least γ that 100 P percent of a population lies between the rth smallest and the sth largest of a random sample of n from that population (continuous distribution function assumed)

n	P																								
	$\gamma = 0.50$					$\gamma = 0.75$					$\gamma = 0.90$					$\gamma = 0.95$					$\gamma = 0.99$				
	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99	.50	.75	.90	.95	.99
50	25	12	5	2	0	22	10	3	1	—	20	9	2	1	—	19	8	2	—	—	16	6	1	—	—
55	28	14	5	3	0	25	12	4	2	—	23	10	3	1	—	21	9	2	—	—	19	7	1	—	—
60	30	15	6	3	0	27	13	4	2	—	25	11	3	1	—	24	10	2	1	—	21	8	1	—	—
65	33	16	6	3	0	30	14	5	2	—	27	12	4	1	—	26	11	3	1	—	23	9	2	—	—
70	35	17	7	3	1	32	15	5	2	—	30	13	4	1	—	28	12	3	1	—	25	10	2	—	—
75	38	19	7	4	1	35	16	6	2	—	32	14	4	1	—	30	13	3	1	—	27	10	2	—	—
80	40	20	8	4	1	37	17	6	3	—	34	15	5	2	—	33	14	4	1	—	30	11	2	—	—
85	43	21	8	4	1	39	19	7	3	—	37	16	5	2	—	35	15	4	1	—	32	12	3	—	—
90	45	22	9	4	1	42	20	7	3	—	39	17	5	2	—	37	16	5	1	—	34	13	3	1	—
95	48	24	9	5	1	44	21	7	3	—	41	18	6	2	—	39	17	5	2	—	36	14	3	1	—
100	50	25	10	5	1	47	22	8	3	—	44	20	6	2	—	42	18	5	2	—	38	15	4	1	—
110	55	27	11	5	1	51	24	9	4	—	48	22	7	3	—	46	20	6	2	—	43	17	4	1	—
120	60	30	12	6	1	56	27	10	4	—	53	24	8	3	—	51	22	7	2	—	47	19	5	1	—
130	65	32	13	6	1	61	29	11	5	—	58	26	9	3	—	56	25	8	3	—	52	21	6	2	—
140	70	35	14	7	1	66	31	12	5	1	62	28	10	4	—	60	27	8	3	—	56	23	6	2	—
150	75	37	15	7	1	71	34	12	6	1	67	31	10	4	—	65	29	9	3	—	61	26	7	2	—
170	85	42	17	8	2	81	39	14	7	1	77	35	12	5	—	74	33	11	4	—	70	30	9	3	—
200	100	50	20	10	2	95	46	17	8	1	91	42	15	6	—	88	40	13	5	—	84	36	11	4	—
300	150	75	30	15	3	144	70	26	12	2	139	65	23	10	1	136	63	22	9	1	130	58	19	7	—
400	200	100	40	20	4	193	94	36	17	3	187	89	32	15	2	184	86	30	13	1	177	80	27	11	—
500	250	125	50	25	5	242	118	45	22	3	236	113	41	19	2	232	109	39	17	2	224	103	35	14	1
600	300	150	60	30	6	292	143	55	26	4	284	136	51	23	3	280	133	48	21	2	272	126	44	18	1
700	350	175	70	35	7	341	167	65	31	5	333	160	60	28	4	328	156	57	26	3	319	149	52	22	2
800	400	200	80	40	8	390	192	74	36	6	382	184	69	32	5	377	180	66	30	4	367	172	61	26	2
900	450	225	90	45	9	440	216	84	41	7	431	208	79	37	5	425	204	75	35	4	415	195	70	30	3
1000	500	250	100	50	10	489	241	94	45	8	480	233	88	41	6	474	228	85	39	5	463	219	79	35	3

TABLE II

Confidence γ with which we may assert that 100 P percent of the population lies between the largest and smallest of a random sample of n from that population (continuous distribution assumed)

n	$P = .50$	$P = .75$	$P = .90$	$P = .95$	$P = .99$	n	$P = .75$	$P = .90$	$P = .95$	$P = .99$
3	.50	.16	.03	.01	.00	17	.95	.52	.21	.01
4	.69	.26	.05	.01	.00	18	.96	.55	.23	.01
5	.81	.37	.08	.02	.00	19	.97	.58	.25	.02
6	.89	.47	.11	.03	.00	20	.98	.61	.26	.02
7	.94	.56	.15	.04	.00	25	.99	.73	.36	.03
8	.96	.63	.19	.06	.00	30	1.00–	.82	.45	.04
9	.98	.70	.23	.07	.00	40		.92	.60	.06
10	.99	.76	.26	.09	.00	50		.97	.72	.09
11	.99	.80	.30	.10	.01	60		.99	.81	.12
12	1.00–	.84	.34	.12	.01	70		.99	.87	.16
13		.87	.38	.14	.01	80		1.00–	.91	.19
14		.90	.42	.15	.01	90			.94	.23
15		.92	.45	.17	.01	100			.96	.26
		.94	.49	.19	.01					

Table II gives the confidence γ that 100 P percent of the population lies between the largest and smallest of a random sample of n.

In the case where we are dealing with a multivariate population, we take m to be the number of blocks (See Tukey [8]) excluded from the tolerance region.

REFERENCES

[1] C. E. Eisenhart, M. W. Hastay, and W. A. Wallis, *Techniques of Statistical Analysis*, McGraw-Hill Book Company Inc., New York, 1947, Chapter 2.

[2] D. A. S. Fraser, "Sequentially determined statistically equivalent blocks," *Ann. Math. Stat.*, Vol. 22 (1951), pp. 372–381.

[3] D. A. S. Fraser, "Nonparametric tolerance regions," *Ann. Math. Stat.*, Vol. 24 (1953), pp. 44–55.

[4] R. B. Murphy, "Nonparametric tolerance limits," *Ann. Math. Stat.*, Vol. 19 (1948), pp. 581–589.

[5] Karl Pearson, *Tables of the Incomplete Beta-function*, Cambridge University Press (1934).

[6] Harry G. Romig, "50–100 Binomial Tables," John Wiley and Sons, New York (1947).

[7] Catherine M. Thompson, "Tables of the Percentage Points of the Incomplete Beta-function," *Biometrika*, Vol. 32 (1941–42), pp. 151–181.

[8] J. W. Tukey, "Non-parametric estimation, II; Statistically equivalent blocks and tolerance regions, the continuous case," *Ann. Math. Stat.*, Vol. 18 (1947), pp. 529–539.

[9] A. Wald, "An extension of Wilks' method for setting tolerance limits," *Ann. Math. Stat.*, Vol. 14 (1943), pp. 45–55.

[10] S. S. Wilks, "Determination of sample sizes for setting tolerance limits," *Ann. Math. Stat.*, Vol. 12 (1941), pp. 91–96.

[11] Tables of the Cumulative Binomial Probability Distribution, *Annals of the Computation Laboratory of Harvard University*, Vol. XXXV (1955).

NONPARAMETRIC ESTIMATION OF SAMPLE PERCENTAGE POINT STANDARD DEVIATION

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1. Summary. The available data consists of a random sample $x(1) < \cdots < x(n)$ from a reasonably well-behaved continuous statistical population. The problem is to estimate the standard deviation of a specified $x(r)$ that is not in the tails of the sample. The estimates examined are of the form

$$a[x(r + i) - x(r - i)]$$

and the explicit problem consists of determining suitable values for a and i. The solution

$$a = (\tfrac{1}{2})(n + 1)^{-3/10}\{[r/(n + 1)][1 - r/(n + 1)]\}^{1/2}, \; i \doteq (n + 1)^{4/5}$$

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