NOTES

NOTE ON THE DISTRIBUTION OF LOCALLY MAXIMAL ELEMENTS IN A RANDOM SAMPLE[1]

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Glasgow's formula for the second factorial moment of this distribution [1] is considerably more complicated than it need be. We have elsewhere [2] and [3] obtained a formula requiring just one summation, over the fixed range $0 \leqq s \leqq h - 1$, thus eliminating the summation over the ever-increasing range $0 \leqq s \leqq m$.

Following Glasgow's notation, let β be the number of locally k-maximal elements in a permutation of the first n integers. Our formula, for the variance of β, is

$$\text{var}(\beta) = (n+1)C_k, \qquad n \geqq 2k,$$

where

$$C_k = \frac{-2k(5k+3)}{(2k+1)(k+1)^2} + \frac{8}{k+1}\sum_{s=0}^{k-1}\frac{1}{k+s+2}.$$

Using the expected value of β given in [1], we find that

$$E(\beta^{(2)}, n) = \text{var}(\beta) + E(\beta)(E(\beta) - 1)$$
$$= (n+1)C_k + (2n-k+1)(2n-2k)/(k+1)^2.$$

Both referees have pointed out that Glasgow's formula can be reduced to ours. In fact, the summation in his equation (3.8) can be performed, yielding

$$\frac{2(m+1)(7k^2+10k+3+4km+2m)}{(k+1)^2(2k+1)(2k+m+2)}.$$

REFERENCES

[1] M. O. Glasgow, "Note on the factorial moments of the distribution of locally maximal elements in a random sample," *Ann. Math. Stat.*, Vol. 30 (1959), pp. 586–90.

[2] M. Freimer, B. Gold, and A. L. Tritter, "On a mathematical model for a Morse code translator," Lincoln Laboratory Group Report 34-61, November 1, 1957. (Not generally available.)

[3] M. Freimer, B. Gold, and A. L. Tritter, "The Morse distribution," *IRE Transactions on Information Theory*, IT-5 (1959), pp. 25–31.

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