Thus the simple symmetric s.p.'s of size $2n$ can also be characterized by the vectors $(a_1, \cdots, a_{n-1})$ satisfying (7); or more precisely the vectors $(a_1, \cdots, a_{n-1}, 0, 0, 0, a_{n-1}, \cdots a_1)$ satisfying (7) characterize simple symmetric s.p.'s of size $2n$. The a's in fact represent the "distances" of its boundary points from the points on the line $x + y = 2n$. From known results [3. p. 170], the number of simple symmetric s.p.'s of size $2n$ is $n^{-1}\binom{3n}{n-1}$.

Evidently, a 1:1 correspondence similar to (8) yields a characterization of any simple s.p. of size n in terms of the "distances" of its boundary points from the line $x + y = n$. The vectors $(a_1, \cdots, a_{n+1})$ depend on both $(t_1, \cdots, t_{n-1})$ and $(b_1, \cdots, b_{n-1})$ in this case, but the method as well as the conditions satisfied by $(a_1, \cdots, a_{n+1})$ can be easily derived. Since the lattice-theoretic ideas developed in [2, 3] yield a simple 1:1 correspondence between the vectorial representations (using boundary points) of simple s.p.'s of size n and simple symmetric s.p.'s of size $2n$, we obtain without further calculations another proof of our theorem. The characterization (7) of s.p.'s and their interpretation as a distributive lattice applies with little change to other problems in probability theory, and yields a unified approach for rederiving and extending many results. [cf., 2].

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AN INEQUALITY FOR BALANCED INCOMPLETE BLOCK DESIGNS

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1. Summary. For a resolvable balanced incomplete block design, R. C. Bose [1] obtained the inequality $b \geqq v + r - 1$, and P. M. Roy [2] and W. F. Mikhail [3] proved this inequality without the assumption of resolvability, but with the weaker assumption that v is a multiple of k. In this note an alternative and simpler proof of Roy's theorem is given.

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2. Proof. A B.I.B. design is an arrangement of v treatments in b blocks of size $k < v$ such that (i) every block contains k distinct treatments, (ii) every treatment occurs in r blocks and (iii) any two treatments occur together in λ blocks.

The parameters satisfy

$$bk = vr, \tag{2.1}$$

$$r(k - 1) = \lambda(v - 1), \tag{2.2}$$

$$b \geqq v, \qquad r \geqq k. \tag{2.3}$$

From (2.2) we have

$$r/(v - 1) = \lambda/(k - 1) = (r - \lambda)/(v - k). \tag{2.4}$$

If now we assume that v is a multiple of k, $v = nk$, we have from (2.4)

$$\begin{aligned} r/(v - 1) = (r - \lambda)/(v - k) &= (r - \lambda)/(k(n - 1)), \\ (r(n - 1))/(v - 1) &= (r - \lambda)/k \end{aligned} \tag{2.5}$$

Putting $v = nk$ in (2.1), we have $b = nr$, so that (2.5) can be rewritten as

$$(r - \lambda)/k = (b - r)/(v - 1). \tag{2.6}$$

Rewriting (2.2) after expansion we have $r - \lambda = rk - v\lambda$, and $(r - \lambda)/k = r - n\lambda$. Thus

$$(r - \lambda)/k = (b - r)/(v - 1) = r - n\lambda. \tag{2.7}$$

Since n, r, λ are all integers, $r - n\lambda$ is an integer, from which it follows that the other two ratios in (2.7) are integers. It can easily be seen that they must be positive integers since $r > \lambda$ and k is a positive integer. Therefore

$$(b - r)/(v - 1) \geqq 1$$

and $b \geqq v + r - 1$.

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