and get the same result, so that we can also reach the point where $m(x)$ takes its last zero, which can be sometimes important. (By a similar process, we can reach the first zero, when $m(x) \geqq 0$, for every value of x exceeding that zero point.)

If we do not know the value of the constant, we can use the next Theorem, which imposes, however, sharper conditions on $m(x)$.

THEOREM. *Let the following Conditions be fulfilled.*

$$|m(x+1) - m(x)| < L|x| + K. \tag{14}$$

$$\sigma^2(x) \leqq \sigma^2 < \infty. \tag{15}$$

$$\text{If } x < x_0, \text{ then } \bar{D}m(x) = 0; \text{ while if } x > x_0, \text{ then } \underline{D}m(x) > 0. \tag{16}$$

$$\text{For every } \delta > 0, \quad \inf_{\delta < x - x_0 < \infty} \underline{D}m(x) > 0. \tag{17}$$

If we choose a_n, c_n, δ_n such that:

$$a_n > 0, \quad \sum a_n = \infty, \quad \sum a_n^2 < \infty, \quad \sum a_n^2/c_n^2 < \infty,$$

$$\delta_n > 0, \quad \delta_n \to 0, \quad \sum a_n\delta_n = \infty;$$

and if we define: $x_{n+1} = x_n - a_n\{[y(x_n + c_n) - y(x_n)]/c_n - \delta_n\}$, then $x_n \to x_0$ w.p.1. and in mean square.

The problem of finding the point where $m(x)$ stops being a constant, was suggested by Gutmann [3].

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THE USE OF THE RANGE IN PLACE OF THE STANDARD DEVIATION IN STEIN'S TEST

BY WILLIAM KNIGHT

University of New Brunswick

A two sample procedure for obtaining a confidence interval of predetermined length for the mean, μ, of a normal distribution with unknown variance, σ^2, was devised by Stein [4] and generalized by Wormleighton [5]. In this procedure a first

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sample of size n_1 is taken following which a second sample of size n_2 is taken, n_2 being a function of the standard deviation of the first sample. If the choice of n_2 must be made under field conditions, computation of the standard deviation may not be practical, whereas the range is easily obtained. It is the purpose of this note to present a range-based Stein procedure.

Let s be any estimate of σ obtained from the first sample which is statistically independent of the mean of that sample, $\bar{x}_1$, and for which the distribution of s/σ is independent of σ. (In particular, both standard deviation and range [3] have these properties.) Let

$$u = (\bar{x}_1 - \mu)(n_1)^{\frac{1}{2}}/s, \tag{1}$$

and let u_a be the ath quantile of its distribution.

The size of the second sample will be $n_2 = n(s) - n_1$ where n is any measureable function of s for which $n(s) \geqq n_1$. It will be shown that

$$\text{Prob}\{\bar{x} - \mu \leqq su_a/n(s)^{\frac{1}{2}}\} = a, \tag{2}$$

where $\bar{x}$ is the mean of the total sample (first and second combined); from this, confidence intervals are immediate, and it is seen that the same tables will be used for the two as for the one sample procedure. (If s is the range, tables can be found in references [1] and [3].) Proof of (2) follows Wormleighton [5]. For a ratio, y/s, to be distributed as u it is sufficient that the conditional distribution of y given s be $N(0, \sigma^2)$. As the conditional distribution, given s, of the mean of the first sample is $N(\mu, \sigma^2/n_1)$, and the conditional distribution, given s, of the mean of the second sample, if one be taken, is $N(\mu, \sigma^2/[n(s) - n_1])$, the conditional distribution, given s, of the mean of the total sample is $N(\mu, \sigma^2/n(s))$, and that of $(\bar{x} - \mu)[n(s)]^{\frac{1}{2}}$ is $N(0, \sigma^2)$. Equation (2) holds when the distribution of s/σ is dependent on σ, but in such a case u_a will also depend on σ.

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